

Function and Graph

Learn how functions will look when stretched, mirrored, mixed, and balanced. Study and then design your own transformations

This file includes eActivities on:

Asymptotes - What is the difference between vertical and horizontal asymptotes?

Composition f(g) - Functions of functions

Inverse Composition - Can you discover the inverse of compositions?

Inverse Function - What about inverse of functions?

Reflection - Is the mirror on the x-axis or y-axis?

Stretch Perform - A few warm ups on your function.

Symmetry - Observe symmetry with respect to the y-axis, origin, or x-axis

Translation (Shift) - Functions on the move.

Asymptotes

Look at the difference between horizontal and vertical asymptotes.

The image shows two side-by-side screenshots of a software interface. The left window is titled "Asymptotes" and contains definitions for horizontal and vertical asymptotes. The horizontal asymptote section shows $\lim_{x \rightarrow -\infty} (f(x)) = \text{Constant}$ and $\lim_{x \rightarrow +\infty} (f(x)) = \text{Constant}$, with an example $f(x) = 1/x^2$ and asymptote $y = 0$. The vertical asymptote section shows $\lim_{x \rightarrow a^-} (f(x)) = \pm\infty$ or $\lim_{x \rightarrow a^+} (f(x)) = \pm\infty$. The right window is titled "Vertical asymptote" and shows the same limit definitions, with the example $f(x) = 1/x^2$ and a graph of the function. The graph shows a hyperbola with two branches in the first and third quadrants, approaching the x-axis and y-axis as asymptotes.

Composition f(g)

What happens when you take the function of a function? Are they interchangeable?

The image shows a screenshot of a software interface titled "Composition f=g". It displays the formula $(f \circ g)(x) = f(g(x))$. Below this, it says "Try it!" and defines two functions: $f(x) = \sqrt{x}$ and $g(x) = x + 10$. It then shows the composition $f(g(x)) = \sqrt{x+10}$ and $g(f(x)) = \sqrt{x} + 10$.

Inverse Composition

What happens when you take the inverse of a function within a function? Or, vice versa?

The first screenshot shows the calculator's 'Inverse Composition' screen. It displays the following text:

$$f(f^{-1}(x)) = x$$

$$f^{-1}(f(x)) = x$$

<Example>
 When $f(x) = \frac{x}{x+1}$,
 $g(x) = \frac{x}{1-x}$,
 Define $f(x) = \frac{x}{x+1}$ done
 Define $g(x) = \frac{x}{1-x}$ done

The second screenshot shows the calculator's 'Inverse Composition' screen with the following text:

Define $g(x) = \frac{x}{1-x}$ done
 $f(g(x)) = \frac{x}{(x-1) \cdot \left(\frac{x}{x-1} - 1\right)}$
 simplify(ans) x
 $g(f(x)) = \frac{-x}{(x+1) \cdot \left(\frac{x}{x+1} - 1\right)}$
 simplify(ans) x
 Then $f(x)$ and $g(x)$ are inverses.

Inverse Function

Follow the steps to find the inverse of a function.

The first screenshot shows the calculator's 'Inverse Function' screen. It displays the following text:

$$f^{-1}(b) = f(a) \text{ when } f(a) = b$$

<Example>
 When $f(x) = \frac{x-1}{x+2}$,
 f^{-1} satisfies the equation
 $x = \frac{y-1}{y+2}$.
 $(y+2) \cdot x = y-1$
 $x \cdot y + 2 \cdot x = y-1$
 $x \cdot y - y = -2 \cdot x - 1$
 $y \cdot (x-1) = -2 \cdot x - 1$
 $y = \frac{-(2 \cdot x + 1)}{x-1}$

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 $x \cdot y - y = -2 \cdot x - 1$
 $y \cdot (x-1) = -2 \cdot x - 1$
 $y = \frac{-(2 \cdot x + 1)}{x-1}$
 Then the inverse function
 is $y = \frac{-(2 \cdot x + 1)}{x-1}$.

Reflection

The functions are reflected across the x-axis or y-axis. When you understand how, you can create your own reflections.

The first screenshot shows the calculator's 'Reflection' screen. It displays the following text:

Across the x-axis
 $y = -f(x)$

Across the y-axis
 $y = f(-x)$

Graph $y1 = \dots$ $y2 = \dots$

Try your own!
 Graph $y1 = \dots$ $y2 = \dots$

The second screenshot shows the calculator's 'Edit Zoom Analysis' screen. It displays the following text:

Sheet1 | Sheet2 | Sheet3

$y1 = \sqrt{x}$ [—]
 $y2 = -\sqrt{x}$ [.....]
 $y3 = \sqrt{-x}$ [—]
 $x4 = \dots$
 $x5 = \dots$
 $x6 = \dots$

The graph shows three functions: $y = \sqrt{x}$ (solid line), $y = -\sqrt{x}$ (dotted line), and $y = \sqrt{-x}$ (dashed line).

Stretch

See how much the function stretches and in what direction.

Stretch

Horizontal Stretch
 $y=f\left(\frac{x}{c}\right) : c \neq 0$

Graph Y1:---
Y2:---

Vertical Stretch
 $y=c \cdot f(x)$

Graph Y1:---
Y2:---

Try your own!

Graph Y1:---
Y2:---

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Edit Zoom Analysis

Sheet1 | Sheet2 | Sheet3

$y1=x^2$ [—]

$y2=\left(\frac{x}{2}\right)^2$ [.....]

$y3=\left(\frac{x}{0.5}\right)^2$ [.....]

$x4:$ []

Rad Cplx ☰

Symmetry

Perfect symmetry is possible with respect to the y-axis, x-axis, or origin. Recognize the formula and create your own.

Symmetry

Symmetry with respect to the y-axis.
 $f(-x)=f(x)$: even

Example: $f(x)=x^2$

Graph Y1:---
Y2:---

$f(-x)=f(x)$ ☑

Symmetry with respect to the origin.
 $f(-x)=-f(x)$: odd

Example: $f(x)=x^3$

Graph Y1:---
Y2:---

$f(-x)=-f(x)$ ☑

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Edit Zoom Analysis

Example: $f(x)=x^3$

Graph Y1:---
Y2:---

$f(-x)=-f(x)$ ☑

Symmetry with respect to the x-axis. When (x,y) is on the graph, $(x,-y)$ is also on the graph.

Example: $x=y^2$

Graph Y1:---
Y2:---

Try your own!

Graph Y1:---
Y2:---

Calculator ☑

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Translation (Shift)

Learn how to move functions horizontally and/or vertically.

Translation (Shift)

Horizontal Translation
 $y=f(x \pm c)$

Graph Y1:---
Y2:---

Vertical Translation
 $y=f(x) \pm c$

Graph Y1:---
Y2:---

Try your own!

Graph Y1:---
Y2:---

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Edit Zoom Analysis

Sheet1 | Sheet2 | Sheet3

$y1=x^2$ [—]

$y2=(x+2)^2$ [.....]

$y3=(x-2)^2$ [.....]

$x4:$ []

$x5:$ []

$x6:$ []

$x7:$ []

Rad Cplx ☰