

Conics – Hyperbola

This week we bring you Hyperbolas, part III in our conics series. As you have done with parabolas and ellipses, so can you do with hyperbolas. Put the equation in standard form, solve, and graph. Try G-Solve, expand, solve, or “Fit into conics Form” functions. Explore your own equations algebraically and graphically.

This file includes eActivities on:

Hyperbola 1 x-(0,0) Start with the basics, a hyperbola with its center at the origin.

Hyperbola 2 y-(0,0) If you know the positions of the foci and length of the conjugate axis, do you know how to graph the hyperbola?

Hyperbola 3 x-(h,k) Learn how easy it is to drag n’ drop equations in to the “Fit into Conics Form” function.

Hyperbola 4 y-(h,k) How do you solve and graph a hyperbola when all you know are the vertices and point?

Hyperbola 1 x-(0,0)

Start with the basics, a hyperbola with its center at the origin.

Hyperbola 1 with Center (0,0)

Focal axis is x-axis.

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Foci $(\pm c, 0)$
 Vertices $(\pm a, 0)$
 Semimajor axis a
 Semiminor axis b
 Asymptotes $y = \pm \frac{b}{a}x$
 Relation $c^2 = a^2 + b^2$

<Example>

When $4x^2 - 9y^2 = 36$, find the vertices and the foci.

We find the standard form $\frac{x^2}{9} - \frac{y^2}{4} = 1$.

Then $a = \pm 3$, $b = \pm 2$ and $c = \pm \sqrt{13}$.

Calculator

Thus Vertices are $(\pm 3, 0)$.
 Foci are $(\pm \sqrt{13}, 0)$

Conics Graph

Try your own.

Hyperbola 2 y-(0,0)

If you know the positions of the foci and length of the conjugate axis, do you know how to graph the hyperbola?

Hyperbola 2 with Center (0,0)

Focal axis is y-axis.

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

Foci $(0, \pm c)$
 Vertices $(0, \pm a)$
 Semimajor axis a
 Semiminor axis b
 Asymptotes $y = \pm \frac{a}{b}x$
 Relation $c^2 = a^2 + b^2$

<Example>

When the foci are $(0, -3)$ and $(0, 3)$, and the conjugate axis has length 4, find the equation.

From the length of conjugate axis, $b = \frac{4}{2} = 2$.

From the relation $c^2 = a^2 + b^2$, $a = \pm \sqrt{5}$.

Calculator

The equation is, $\frac{y^2}{5} - \frac{x^2}{4} = 1$

Conics Graph

Calculator

The equation is, $\frac{y^2}{5} - \frac{x^2}{4} = 1$

Conics Graph

Try your own.

solve($c^2 = a^2 + b^2, a | b = 2 | c =$
 $\{a = -\sqrt{5}, a = \sqrt{5}\}$)

Hyperbola 3 x-(h,k)

Learn how easy it is to drag n' drop equations in to the "Fit into Conics Form" function.

The first screenshot shows the 'Hyperbola 3 with Center (h,k)' screen. It displays the standard form equation $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ and lists properties: Foci $(h \pm c, k)$, Vertices $(h \pm a, k)$, Semimajor axis a , Semiminor axis b , Asymptotes $y = \pm \frac{b}{a}(x-h) + k$, and Relation $c^2 = a^2 + b^2$.

The second screenshot shows the 'Select Conics Form' dialog box with various conic equation options. The first option, $X=A(Y-K)^2+H$, is selected.

The third screenshot shows the 'Fit into Conics Form' function. It displays the equation $\frac{(x+2)^2}{2} - \frac{(y-5)^2}{4} = 1$ and the resulting parameters: $a = \pm\sqrt{2}$, $b = \pm 2$, and $c = \pm\sqrt{6}$. A graph of the hyperbola is shown below the text.

Hyperbola 4 y-(h,k)

How do you solve and graph a hyperbola when all you know are the vertices and point?

The first screenshot shows the 'Hyperbola 4 with Center (h,k)' screen. It displays the standard form equation $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$ and lists properties: Foci $(h, k \pm c)$, Vertices $(h, k \pm a)$, Semimajor axis a , Semiminor axis b , Asymptotes $y = \pm \frac{a}{b}(x-h) + k$, and Relation $c^2 = a^2 + b^2$.

The second screenshot shows a text-based problem: 'When the vertices are (2,1) and (2,-1), and passes through the point (5,√2), find the equation.' It identifies the center as (2,0) and h=2, k=0, a=1. It shows the equation form $\frac{y^2}{1^2} - \frac{(x-2)^2}{b^2} = 1$.

The third screenshot shows the 'Edit Action Interactive' screen. It shows the equation $\frac{(\sqrt{2})^2}{1^2} - \frac{(5-2)^2}{b^2} = 1$ being solved. The result is $b = \pm 3$.