

# Parametric

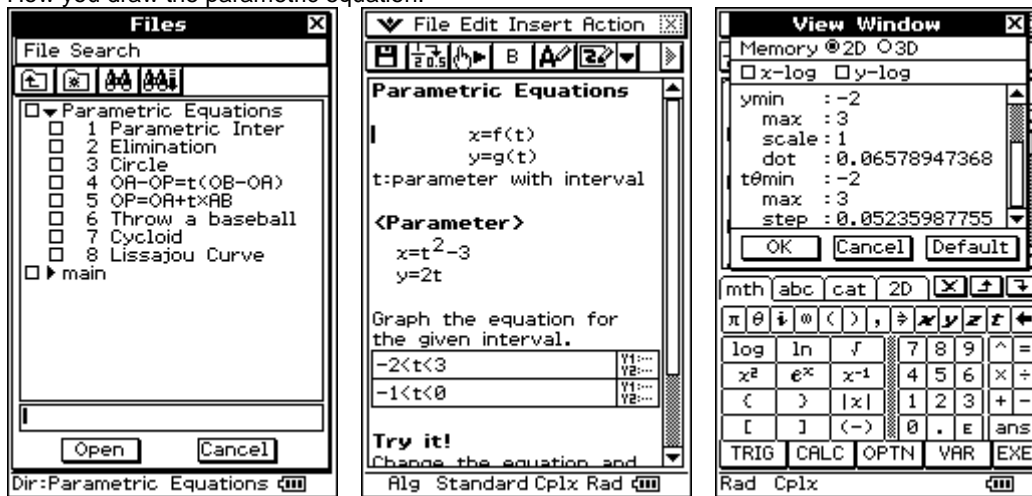
This week we are looking at parametric equations and how the intervals change the graph. We show how to compute the time it takes for the baseball to hit the ground and parametric equations that pass through two points. Play with the equations by changing the values. Can you predict what the graph will look like?

**This file includes eActivities on:**

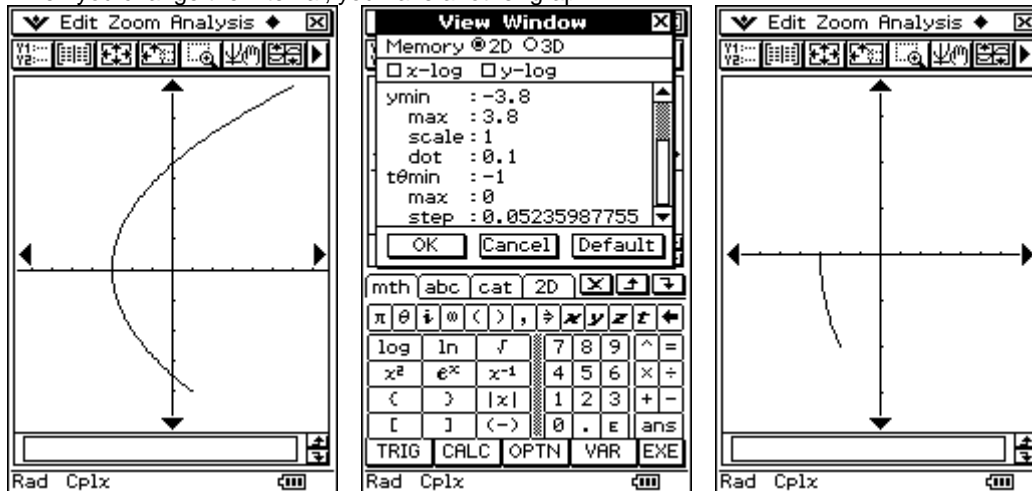
- 1 Parametric Interval** How you draw the parametric equation. When you change the interval, you have another graph.
- 2 Elimination** Eliminate the parameter--find the equation. When drawing the parametric graph, you can use the with-function (I) to choose the interval to graph.
- 3 Circle** Draw the circle using parametric equations. Change the interval and just that part of the circle will be graphed.
- 4 Line  $OA-OP=t(OB-OA)$**  Find the parametric equations that pass through two points.
- 5 Line  $OP=OA+tAB$**  Another look at finding the parametric equations that pass through two points.
- 6 Throw a baseball** Want to find out when the baseball will hit the ground?
- 7 Cycloid** Draw the basic Cycloid curve and then try your own!
- 8 Lissajou Curve** Lissajou Curve can also be written using parametric equations. Adjust the equation and enjoy the graphs.

## 1 Parametric Interval

How you draw the parametric equation.

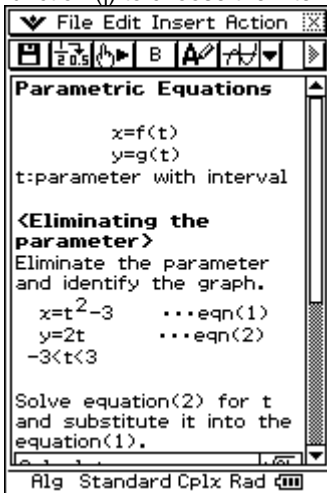


When you change the interval, you have another graph.



## 2 Elimination

Eliminate the parameter--find the equation. When drawing the parametric graph, you can use the with-function (()) to choose the interval to graph.



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Parametric Equations

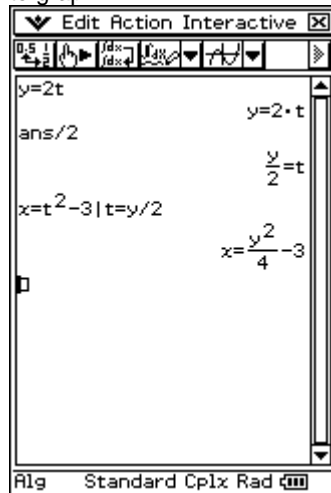
$x=f(t)$   
 $y=g(t)$   
 t:parameter with interval

**<Eliminating the parameter>**  
 Eliminate the parameter and identify the graph.

$x=t^2-3$  ...eqn(1)  
 $y=2t$  ...eqn(2)  
 $-3<t<3$

Solve equation(2) for t and substitute it into the equation(1).

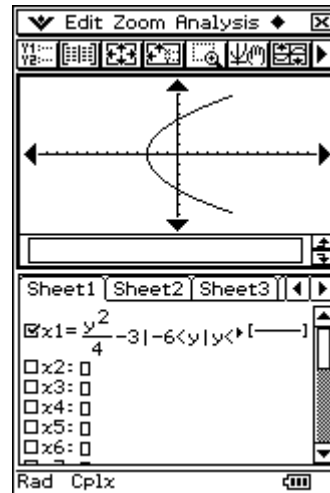
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Edit Action Interactive

$y=2t$   
 $ans/2$   
 $y=2 \cdot t$   
 $\frac{y}{2}=t$   
 $x=t^2-3|t=y/2$   
 $x=\frac{y^2}{4}-3$

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Graph of a parabola opening to the right on a Cartesian coordinate system.

Sheet1 Sheet2 Sheet3

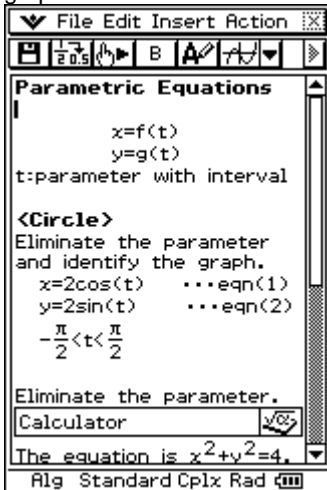
$x1=\frac{y^2}{4}-3|-6<y|y<[ ]$

x2:   
 x3:   
 x4:   
 x5:   
 x6:

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## 3 Circle

Draw the circle by the parametric equations. Change the interval and just that part of the circle will be graphed.



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Parametric Equations

$x=f(t)$   
 $y=g(t)$   
 t:parameter with interval

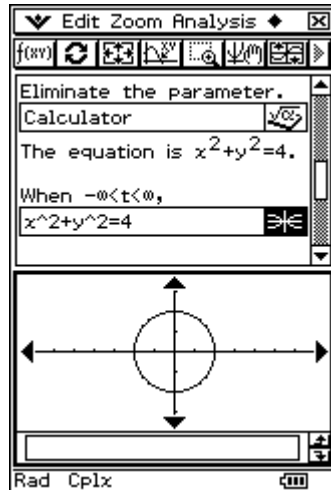
**<Circle>**  
 Eliminate the parameter and identify the graph.

$x=2\cos(t)$  ...eqn(1)  
 $y=2\sin(t)$  ...eqn(2)  
 $-\frac{\pi}{2}<t<\frac{\pi}{2}$

Eliminate the parameter.  
 Calculator  $\sqrt{\cos}$

The equation is  $x^2+y^2=4$ .

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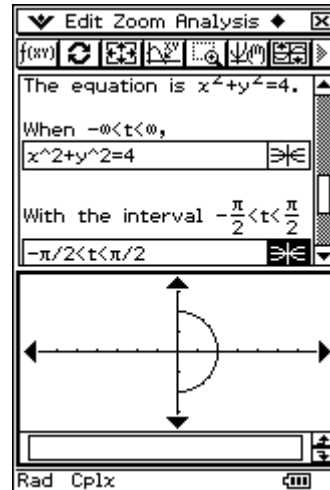
Eliminate the parameter.  
 Calculator  $\sqrt{\cos}$

The equation is  $x^2+y^2=4$ .

When  $-\omega<t<\omega$ ,  
 $x^2+y^2=4$

Graph of a full circle centered at the origin on a Cartesian coordinate system.

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The equation is  $x^2+y^2=4$ .

When  $-\omega<t<\omega$ ,  
 $x^2+y^2=4$

With the interval  $-\frac{\pi}{2}<t<\frac{\pi}{2}$   
 $-\pi/2<t<\pi/2$

Graph of a semi-circle on the right side of a Cartesian coordinate system.

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#### 4 Line $OA-OP=t(OB-OA)$

Find the parametric equations that pass through two points.

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Parametric Equations

$x=f(t)$   
 $y=g(t)$   
t:parameter with interval

<Line through A & B>  
 $OA-OP=t(OB-OA)$

Find the parametric equation through A=(-2,2) and B=(3,4).  
A=(-2,2) & B=(3,4)

Study the vector on the line.  
Vector analysis

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A=(-2,2) & B=(3,4)

Study the vector on the line.  
Vector analysis

Vector AB on the line has the relation  $OA+AB=OB$ .  
Then  $AB=OB-OA$ .

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Add the point P on the line.  
 $P=(x,y)$

When we have the point  $P=(x,y)$  and vector PA,  
 $PA=t \times AB$ .  
Then we have  
 $OA-OP=t(OB-OA)$

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#### 5 Line $OP=OA+t \times AB$

Another look at finding the parametric equations that pass through two points.

File Edit Insert Action

Parametric Equations

$x=f(t)$   
 $y=g(t)$   
t:parameter with interval

<Line through A & B>  
 $OP=OA+t \times AB$

Find the parametric equation through A=(-2,2) and B=(3,4).  
Geometry

The equation is  
 $OP=OA+t \times AB$ .  
It follows

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The equation is  
 $OP=OA+t \times AB$ .  
It follows

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \end{bmatrix} + t \times \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 5 \cdot t - 2 \\ 2 \cdot t + 2 \end{bmatrix}$$

Calculator

As the result,  
 $x=5t-2$   
 $y=2t+2$   
 $-0 < t < 0$

Try your own.

Geometry

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File Edit View Draw

t:parameter with interval

<Line through A & B>  
 $OP=OA+t \times AB$

Find the parametric equation through A=(-2,2) and B=(3,4).  
Geometry

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#### 6 Throwing a baseball

Want to find out when the baseball will hit the ground?

File Edit Insert Action

<Throwing a baseball>

Tap ----->

The path of the object is modeled by

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} v_0 \times \cos(\theta) \times t \\ -\frac{g}{2} t^2 + v_0 \times \sin(\theta) \times t + y_0 \end{bmatrix}$$

$v_0$ : The initial velocity  
 $g$ : Gravity  
 $t$ : Time(second)

<Example>

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Tap ----->

The path of the object is modeled by

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} v_0 \times \cos(\theta) \times t \\ -\frac{g}{2} t^2 + v_0 \times \sin(\theta) \times t + y_0 \end{bmatrix}$$

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Graph

The ball hits the ground when  $t = \frac{\sqrt{2}}{2}$  second.

Calculator ( $y=0$ )

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## 7 Cycloid

Draw the basic Cycloid curve and then try your own!

**Parametric Equations**

$$x=f(t)$$

$$y=g(t)$$

t:parameter with interval

**<Cycloid>**

$$x=r \cdot (t-\sin(t))$$

$$y=r \cdot (1-\cos(t))$$

$$t>0$$

Graph  $0 < t < 4\pi$

**Try it!**  
Change r and try to draw your own.  
Change the interval and try to draw your own.

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**Edit Zoom Analysis**

Sheet1 Sheet2 Sheet3

xt1=1\*(t-sin(t))

yt1=1\*(1-cos(t))

xt2:0

yt2:0

xt3:0

yt3:0

xt4:0

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**View Window**

Memory @2D @3D

x-log  y-log

ymin : -1.4

max : 6.2

scale : 1

dot : 0.1

t0min : 0

max : 4π

step : 0.05235987755

OK Cancel Default

mth abc cat 2D

π	θ	i	o	( )	,	→	←	↖	↗	↘	↙
log	ln	f	7	8	9	^	=				
x²	eˣ	x⁻¹	4	5	6	×	÷				
(	)	x	1	2	3	+	-				
[	]	(-)	0	.	E	ans					
TRIG	CALC	OPTN	VAR	EXE							

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## 8 Lissajou Curve

Lissajou Curve can also be written using parametric equations. Adjust the equation and enjoy the graphs.

**Lissajou Curve**

$$x=\sin(n\cdot t)$$

$$y=\sin(m\cdot t)$$

$$0 \leq t \leq 2\pi$$

Try it!

Sheet1 Sheet2 Sheet3

xt1=sin(3\*t)

yt1=sin(4\*t)

xt2:0

yt2:0

xt3:0

yt3:0

xt4:0

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**Edit Zoom Analysis**

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