

# Polar & Complex

This week, we are exploring the relationship between polar and complex coordinates. We will start with changing complex numbers into their trigonometric form. We will then learn how to multiply and divide these complex trigonometric numbers, use de Moivre's theorem and finally, find the nth root of a complex number.

This file includes eActivities on:

- 1\_ Trigonometric Form Transform complex numbers to their trigonometric form.
- 2\_ Calculation (Multiplication and Division) Multiply and divide trigonometric complex numbers.
- 3\_ Calculation Power Use de Moivre's Theorem to harness the power of complex numbers.
- 4\_ nth Root Find the nth root of complex numbers.

## De Moivre's Theorem

### 1\_ Trigonometric Form

Transform complex numbers to their trigonometric form.

### 2\_ Calculation (Multiplication and Division)

Multiply and divide trigonometric complex numbers.

### 3\_Calculation Power

Use de Moivre's Theorem to harness the power of complex numbers.

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Trigonometric Form of Complex Number  
<De Moivre's Theorem>

$$z^n = r^n (\cos(n\theta) + i \sin(n\theta))$$

<Example>  
Find  $z^5$  when  $z = 1 + \sqrt{3}i$ .

$$|1 + \sqrt{3}i| = \sqrt{1+3} = 2$$

$$\theta = \tan^{-1}\left(\frac{\sqrt{3}}{1}\right) = \frac{\pi}{3}$$

Therefore,  
 $z^5 = 2^5 \left(\cos\left(\frac{5\pi}{3}\right) + i \sin\left(\frac{5\pi}{3}\right)\right)$

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Therefore,  
 $z^5 = 2^5 \left(\cos\left(\frac{5\pi}{3}\right) + i \sin\left(\frac{5\pi}{3}\right)\right)$

Calculator (Fix2)

Hint to use CAS.  
 CAS Calculator (Fix2)

$1 + \sqrt{3}i \Rightarrow z$

arg(z)  $\frac{\pi}{3}$

|z| 2

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### 4\_nth Root

Find the nth root of complex numbers.

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Trigonometric Form of Complex Number  
nth Root

If  $z = r(\cos\theta + i\sin\theta)$ , the nth root is

$$\sqrt[n]{r} \left( \cos\frac{\theta + 2\pi k}{n} + i \sin\frac{\theta + 2\pi k}{n} \right)$$

$k = 0, 1, \dots, n-1$

<Example>  
Find the 4th roots of  $z = 3(\cos(\frac{\pi}{4}) + i\sin(\frac{\pi}{4}))$ .

Calculator

$z_1 = \sqrt[4]{3} \left( \cos\frac{\pi}{16} + i \sin\frac{\pi}{16} \right)$

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$z_1 = \sqrt[4]{3} \left( \cos\frac{\pi}{16} + i \sin\frac{\pi}{16} \right)$

$z_2 = \sqrt[4]{3} \left( \cos\frac{9\pi}{16} + i \sin\frac{9\pi}{16} \right)$

$z_3 = \sqrt[4]{3} \left( \cos\frac{17\pi}{16} + i \sin\frac{17\pi}{16} \right)$

$z_4 = \sqrt[4]{3} \left( \cos\frac{25\pi}{16} + i \sin\frac{25\pi}{16} \right)$

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### De Moivre's Theorem

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DeMoivre's Theorem

For  $z = r(\cos(\theta) + i\sin(\theta))$  and any positive integer n.

$$z^n = r^n (\cos(n\theta) + i\sin(n\theta))$$

.....Try it!.....  
 To find  $(1+i)^{10}$ .

$1+i \Rightarrow z$

compToTrig(z)  $1+i$

$$\sqrt{2} \cdot \left( \cos\left(\frac{\pi}{4}\right) + i \sin\left(\frac{\pi}{4}\right) \right)$$

compToTrig(z^10)

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