

Vector 1

We hope that you are already on friendly terms with vectors, but just in case, we have created eight eActivities showing some of their properties. Have fun playing on your ClassPad with these helpful hints.

This file includes eActivities on:

Vector Addition Are the rules the same?

Vector Angle 1 Find the angle of the vector.

Vector Angle 2 Do you know how to find the angle between two vectors?

Vector Dot Product What is a dot product?

Vector Points & Length From here to there.

Vector Projection Have you ever tried projecting vectors?

Vector Scalar X Simple with and without your ClassPad.

Vector Unit How to find the unit vector.

Vector Addition

As simple as $2 + 3 = 5$.

Vector Addition

<Example>
When $r = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$, $s = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$.

$$r+s = \begin{bmatrix} 2 \\ 2 \end{bmatrix} + \begin{bmatrix} -1 \\ 3 \end{bmatrix}$$

$r+s$ (Vector $r=t$.)

Try your own.

Geometry window

Calculator

Alg Standard Cplx Rad

Vector Addition

<Example>
When $r = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$, $s = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$.

$$r+s = \begin{bmatrix} 2 \\ 2 \end{bmatrix} + \begin{bmatrix} -1 \\ 3 \end{bmatrix}$$

$r+s$ (Vector $r=t$.)

Diagram showing vectors r and s being added to find their resultant $r+s$ on a grid.

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Vector Angle 1

Find the degree or direction of vectors.

Vector Angle

$$r = \begin{bmatrix} r_x \\ r_y \end{bmatrix} = \begin{bmatrix} |r| \cos(\theta) \\ |r| \sin(\theta) \end{bmatrix}$$

<Example>
Vector r

When $r = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$, the relation for r_x is $3 = \begin{bmatrix} 3 \\ 2 \end{bmatrix} \cos(\theta)$.

Then, $3 = \text{norm} \left(\begin{bmatrix} 3 \\ 2 \end{bmatrix} \right) \cos(\theta)$

Alg Standard Cplx Deg

Vector Angle

$$r = \begin{bmatrix} r_x \\ r_y \end{bmatrix} = \begin{bmatrix} |r| \cos(\theta) \\ |r| \sin(\theta) \end{bmatrix}$$

<Example>
Vector r

Diagram showing a vector r in the first quadrant of a coordinate system.

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Vector Angle

Then, $3 = \text{norm} \left(\begin{bmatrix} 3 \\ 2 \end{bmatrix} \right) \cos(\theta)$

$$\cos(\theta) = \frac{3}{\text{norm} \left(\begin{bmatrix} 3 \\ 2 \end{bmatrix} \right)}$$

Thus,

$$\theta = \cos^{-1} \left(\frac{3}{\text{norm} \left(\begin{bmatrix} 3 \\ 2 \end{bmatrix} \right)} \right)$$

$\theta = 33.69006753$ (Degree)

Try your own.

Geometry

Calculator

Alg Standard Cplx Deg

Vector Angle 2

Determine the angle between two vectors

File Edit Insert Action

Vector Angle 2

$$\cos(\theta) = \frac{r \cdot s}{|r||s|}$$

<Example>

Geometry

When $r = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, $s = \begin{bmatrix} -1 \\ -3 \end{bmatrix}$, the angle between two vectors is

$$\cos(\theta) = \frac{r \cdot s}{|r||s|} = \frac{\text{dotP}(r,s)}{\text{norm}(r)\text{norm}(s)}$$

$$\theta = \cos^{-1}\left(\frac{\text{dotP}(r,s)}{\text{norm}(r)\text{norm}(s)}\right)$$

Alg Standard Cplx Deg

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When $r = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, $s = \begin{bmatrix} -1 \\ -3 \end{bmatrix}$, the angle between two vectors is

$$\cos(\theta) = \frac{r \cdot s}{|r||s|} = \frac{\text{dotP}(r,s)}{\text{norm}(r)\text{norm}(s)}$$

Alg Standard Cplx Deg

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When $r = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, $s = \begin{bmatrix} -1 \\ -3 \end{bmatrix}$, the angle between two vectors is

$$\cos(\theta) = \frac{r \cdot s}{|r||s|} = \frac{\text{dotP}(r,s)}{\text{norm}(r)\text{norm}(s)}$$

$$\theta = \cos^{-1}\left(\frac{\text{dotP}(r,s)}{\text{norm}(r)\text{norm}(s)}\right)$$

then

$$\theta = \cos^{-1}\left(\frac{\text{dotP}\left(\begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ -3 \end{bmatrix}\right)}{\text{norm}\left(\begin{bmatrix} 2 \\ 1 \end{bmatrix}\right)\text{norm}\left(\begin{bmatrix} -1 \\ -3 \end{bmatrix}\right)}\right)$$

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Vector Dot Product

Learn to find the product of vectors by hand and with your ClassPad.

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Vector Dot Product

$$r \cdot s = r_x s_x + r_y s_y$$

<Example>

When $r = [3 \ 4]$, $s = [5 \ 2]$, the dot product is $r \cdot s = 3 \cdot 5 + 4 \cdot 2 = 23$

Another way is $\text{dotP}([3 \ 4], [5 \ 2])$

Try your own.

Calculator

Alg Standard Cplx Deg

Edit Action Interactive

the dot product is $r \cdot s = 3 \cdot 5 + 4 \cdot 2 = 23$

Another way is $\text{dotP}([3 \ 4], [5 \ 2])$

Try your own.

Calculator

$\text{dotP}([\] [\]), [\] [\])$

Alg Standard Cplx Deg

Vector Points & Lengths

Vectors from A to B and B to A.

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Vector Points and Length

<Example>

Point $A = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$, $B = \begin{bmatrix} -1 \\ -3 \end{bmatrix}$.

Point on Geometry

The initial point: A
The terminal point: B
The vector is $\begin{bmatrix} -3 \\ -5 \end{bmatrix}$.

vector r (AB)

The initial point: B
The terminal point: A

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The initial point: B
The terminal point: A
The vector is $\begin{bmatrix} 3 \\ 5 \end{bmatrix}$.

vector s (BA)

The length is $\sqrt{(2 - (-1))^2 + (2 - (-3))^2} = \sqrt{34}$

Try your own.

Geometry

Calculator

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The length is $\sqrt{(2 - (-1))^2 + (2 - (-3))^2} = \sqrt{34}$

Try your own.

Geometry

Calculator

$\sqrt{(\)^2 + (\)^2}$

Alg Standard Cplx Rad

Vector Projection

Look at projecting vectors.

File Edit Insert Action

Vector Projection

$$\text{Proj}_{r,s} = \left(\frac{r \cdot s}{|s|^2} \right) s$$

<Example>

Geometry

When $r = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$, $s = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$, the projection of r onto s is

$$\text{proj}_{R,S} = \left(\frac{r \cdot s}{|s|^2} \right) s$$

$$= \left(\frac{\text{dotP}(r,s)}{\text{norm}(s)^2} \right) s$$

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$$= \left(\frac{\text{dotP}(r,s)}{\text{norm}(s)^2} \right) s$$

$$= \left(\frac{\text{dotP}\left(\begin{bmatrix} 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 4 \\ -2 \end{bmatrix}\right)}{\text{norm}\left(\begin{bmatrix} 4 \\ -2 \end{bmatrix}\right)^2} \right) \begin{bmatrix} 4 \\ -2 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{4}{5} \\ -\frac{2}{5} \end{bmatrix}$$

Try your own.

Geometry

Calculator

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Vector Projection

$$\text{Proj}_{r,s} = \left(\frac{r \cdot s}{|s|^2} \right) s$$

<Example>

Geometry

Alg Standard Cplx Deg

Vector Scalar X

Simple with and without your ClassPad.

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Vector Scalar x

$$a[r_x \ r_y] = [axr_x \ axr_y]$$

<Example>

Geometry

When vector $r = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$, multiplication by a scalar is

$$3 \times \begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 1 \times 3 \\ -2 \times 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ -6 \end{bmatrix}$$

Try your own.

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Alg Standard Cplx Deg

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$$3 \times \begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 1 \times 3 \\ -2 \times 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ -6 \end{bmatrix}$$

Try your own.

Calculator

$$\times \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Alg Standard Cplx Deg

Vector Unit

How to find the unit vector.

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Vector Unit

$$\text{Unit vector} = \frac{r}{|r|}$$

<Example>

When $r = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$, unit vector is

$$\frac{r}{|r|} = \frac{\begin{bmatrix} 2 \\ -3 \end{bmatrix}}{\text{norm}\left(\begin{bmatrix} 2 \\ -3 \end{bmatrix}\right)}$$

$$= \frac{\begin{bmatrix} 2 \\ -3 \end{bmatrix}}{\sqrt{13}}$$

Alg Standard Cplx Rad

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$$\frac{r}{|r|} = \frac{\begin{bmatrix} 2 \\ -3 \end{bmatrix}}{\text{norm}\left(\begin{bmatrix} 2 \\ -3 \end{bmatrix}\right)}$$

$$= \frac{\begin{bmatrix} 2 \\ -3 \end{bmatrix}}{\sqrt{13}}$$

$$= \begin{bmatrix} \frac{2 \cdot \sqrt{13}}{13} \\ \frac{-3 \cdot \sqrt{13}}{13} \end{bmatrix}$$

Try your own.

Calculator

Alg Standard Cplx Rad

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$$\frac{\begin{bmatrix} 2 \\ -3 \end{bmatrix}}{\text{norm}\left(\begin{bmatrix} 2 \\ -3 \end{bmatrix}\right)}$$

$$= \begin{bmatrix} \frac{2 \cdot \sqrt{13}}{13} \\ \frac{-3 \cdot \sqrt{13}}{13} \end{bmatrix}$$

Alg Standard Cplx Rad