

Vectors Part II

Vectors are very interesting, and yet we often overlook what makes them interesting. We can do much more than just find their sum or difference, dot or cross products. If you have a friend or neighbor who is a math fanatic, you may be able to impress them with these activities. If not, next weeks will.

This file includes eActivities on:

(na+mb)/(m+n) Can we divide a segment using vectors?

Center of Gravity A formula for the center of gravity of a triangle.

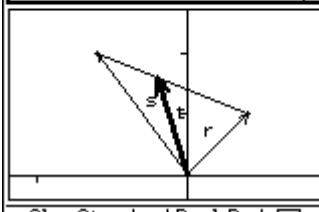

Midpoint Can we find a segments midpoint using vectors?

Perpendicular Transform two vectors into perpendicular vectors.

p=ma+nb Use vector addition to transform the sum of two vectors into a given vector.

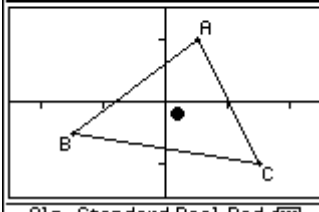
(na+mb)/(m+n)

Can we divide a segment using vectors?

<p>File Edit Insert Action</p> <p>Vector</p> <p>Internal Division</p> <p>External Division</p> $p = \frac{na+mb}{m+n}$ <p><Example></p> <p>Internal Division</p> <p>When $r = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$ and $s = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$, the internal division vector which divide RS 3:2 is</p> $2 \begin{bmatrix} 2 \\ 2 \end{bmatrix} + 3 \begin{bmatrix} -3 \\ 4 \end{bmatrix}$ $3+2$ <p>Alg Standard Real Rad</p>	<p>File Edit Insert Action</p> <p>When $r = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$ and $s = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$, the internal division vector which divide RS 3:2 is</p> $2 \begin{bmatrix} 2 \\ 2 \end{bmatrix} + 3 \begin{bmatrix} -3 \\ 4 \end{bmatrix}$ $3+2$  <p>Alg Standard Real Rad</p>	<p>File Edit Insert Action</p> <p>External Division</p> <p>The external division vector which divide RS as 2:3 is</p> $(-3) \begin{bmatrix} 2 \\ 2 \end{bmatrix} + 2 \begin{bmatrix} -3 \\ 4 \end{bmatrix}$ $2+(-3)$  <p>Alg Standard Real Rad</p>
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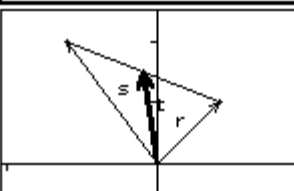
Center of Gravity

A formula for the center of gravity of a triangle.

<p>File Edit Insert Action</p> <p>Center of Gravity</p> $g = \frac{a+b+c}{3}$ <p><Example></p> <p>Geometry window</p> <p>$\triangle ABC$ has the point,</p> <p>A: $a = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$</p> <p>B: $b = \begin{bmatrix} x \\ y \end{bmatrix}$</p> <p>C: $c = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$</p> <p>and the center of gravity</p> <p>1</p> <p>Alg Standard Real Rad</p>	<p>File Edit Insert Action</p> <p>A: $a = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$</p> <p>B: $b = \begin{bmatrix} x \\ y \end{bmatrix}$</p> <p>C: $c = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$</p> <p>and the center of gravity</p>  <p>Alg Standard Real Rad</p>	<p>File Edit Insert Action</p> <p>Solution</p> $\frac{1}{3} \begin{bmatrix} 1 \\ 2 \end{bmatrix} + \frac{x}{3} \begin{bmatrix} 3 \\ -2 \end{bmatrix}$ $\begin{bmatrix} \frac{x+4}{3} \\ \frac{y}{3} \end{bmatrix}$ $\begin{cases} \frac{1}{3} = \frac{x+4}{3} \\ -\frac{1}{3} = \frac{y}{3} \end{cases} \quad x, y$ <p>$\{x=-3, y=-1\}$</p> <p>Then $b = \begin{bmatrix} -3 \\ -1 \end{bmatrix}$.</p> <p>Alg Standard Real Rad</p>
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Midpoint

Can we find a segments midpoint using vectors?

<p>Vector Mid Point</p> $p = \frac{a+b}{2}$ <p><Example> Geometry window <input type="button" value="GPA"/></p> <p>When $r = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$ and $s = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$, the mid division vector which divide RS as 1:1 is</p> $\frac{\begin{bmatrix} 2 \\ 2 \end{bmatrix} + \begin{bmatrix} -3 \\ 4 \end{bmatrix}}{2}$ <p>Try your own.</p> <p>Geometry window <input type="button" value="GPA"/></p>	<p>When $r = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$ and $s = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$, the mid division vector which divide RS as 1:1 is</p> $\frac{\begin{bmatrix} 2 \\ 2 \end{bmatrix} + \begin{bmatrix} -3 \\ 4 \end{bmatrix}}{2}$  <p>Alg Standard Real Rad <input type="button" value="GPA"/></p>	<p><Example> Geometry window <input type="button" value="GPA"/></p> <p>When $r = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$ and $s = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$, the mid division vector which divide RS as 1:1 is</p> $\frac{\begin{bmatrix} 2 \\ 2 \end{bmatrix} + \begin{bmatrix} -3 \\ 4 \end{bmatrix}}{2}$ <p>Try your own.</p> <p>Geometry window <input type="button" value="GPA"/></p> <p>Calculator <input type="button" value="GPA"/></p> <p>Alg Standard Real Rad <input type="button" value="GPA"/></p>
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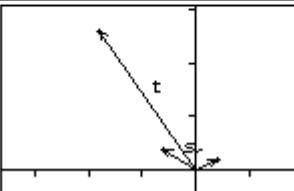
Perpendicular

Transform two vectors into perpendicular vectors.

<p>Vector Perpendicular</p> $a \cdot b \Leftrightarrow a \cdot b = 0$ <p><Example> When $r = \begin{bmatrix} 3 \\ -4 \end{bmatrix}$ and $s = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$, decide scalar t to make $t \times r + s$ is perpendicular to $t \times r - s$.</p> <p>$t \times r + s$ is $t \times \begin{bmatrix} 3 \\ -4 \end{bmatrix} + \begin{bmatrix} -2 \\ 1 \end{bmatrix}$</p> <p>$\begin{bmatrix} 3 \cdot t - 2 \\ -4 \cdot t + 1 \end{bmatrix}$</p> <p>Alg Standard Real Rad <input type="button" value="GPA"/></p>	<p>The perpendicular rule is $\text{dotP}(t \times r + s, t \times r - s) = 0$.</p> <p>$\text{dotP}(\begin{bmatrix} 3 \cdot t - 2 \\ -4 \cdot t + 1 \end{bmatrix}, \begin{bmatrix} 3 \cdot t + 2 \\ -4 \cdot t - 1 \end{bmatrix}) \rightarrow$ $(4 \cdot t - 1) \cdot (4 \cdot t + 1) + (3 \cdot t - 2) \cdot (3 \cdot t + 2)$ expand(ans)</p> $25 \cdot t^2 - 5 = 0$ <p>ans+5</p> $25 \cdot t^2 = 5$ <p>ans/25</p> $t^2 = \frac{1}{5}$ <p>$\sqrt{\text{ans}}$</p> <p>$t = \frac{\sqrt{5}}{5}$</p> <p>Alg Standard Real Rad <input type="button" value="GPA"/></p>
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$p = ma + nb$

Use vector addition to transform the sum of two vectors into a given vector.

<p>Vector $p = ma + nb$</p> <p><Example> When $r = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, $s = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$, rewrite the vector $t = \begin{bmatrix} -9 \\ 13 \end{bmatrix}$ with r and s.</p> <p>Geometry window <input type="button" value="GPA"/></p> <p>Let $t = m \times r + n \times s$, $m \times \begin{bmatrix} 2 \\ 1 \end{bmatrix} + n \times \begin{bmatrix} -3 \\ 2 \end{bmatrix}$</p> <p>$\begin{bmatrix} 2 \cdot m - 3 \cdot n \\ m + 2 \cdot n \end{bmatrix}$</p> <p>then</p> <p>Alg Standard Cplx Rad <input type="button" value="GPA"/></p>	<p>Let $t = m \times r + n \times s$, $m \times \begin{bmatrix} 2 \\ 1 \end{bmatrix} + n \times \begin{bmatrix} -3 \\ 2 \end{bmatrix}$</p> <p>$\begin{bmatrix} 2 \cdot m - 3 \cdot n \\ m + 2 \cdot n \end{bmatrix}$</p> <p>then</p>  <p>Alg Standard Cplx Rad <input type="button" value="GPA"/></p>	<p>Let $t = m \times r + n \times s$, $m \times \begin{bmatrix} 2 \\ 1 \end{bmatrix} + n \times \begin{bmatrix} -3 \\ 2 \end{bmatrix}$</p> <p>$\begin{bmatrix} 2 \cdot m - 3 \cdot n \\ m + 2 \cdot n \end{bmatrix}$</p> <p>then</p> $\begin{cases} -9 = 2 \cdot m - 3 \cdot n \\ 13 = m + 2 \cdot n \end{cases} \Bigg _{m, n}$ <p>$\{m=3, n=5\}$</p> <p>Vector t is $t = 3r + 5s$.</p> <p>Try your own.</p> <p>Calculator <input type="button" value="GPA"/></p> <p>Alg Standard Cplx Rad <input type="button" value="GPA"/></p>
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