

Vectors Part III

This is week 3 of our eActivity coverage of vectors, and the vector grand finale. There are many useful things that can be done with vectors. We hope that you have learned a few interesting vector tricks over the past few weeks. Although we say that this is the grand finale, no one knows what the future holds!

This file includes eActivities on:

(p-a) (p-b)=0 Can you use vectors to find the equation of a circle, given two points?

(p-c) (a-c)=r^2 Can you use vectors to find a tangent line to a circle?

(p-c) (p-c)=r^2 Given the center and radius of a circle, use vectors to find its equation.

3D Distance Find the distance between the tips of two vectors in 3-dimensional space.

(p-p0) n=0 A simple way to find the equation of a line perpendicular to a vector.

p=(1-t)a+tb Learn a new way to find the equation of a line given two points.

p= p0+td A somewhat simple way to find the equation of a line parallel to a vector.

(p-a) (p-b)=0

Can you use vectors to find the equation of a circle, given two points?

File Edit Insert Action

Vector Circle-2
 $(p-a) \cdot (p-b) = 0$

<Example>
 Geometry window

When the point $a = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ and $b = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$ are the boundary of the redus, find the equation for circle.

■ Solution
 $\text{dotP}(\begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} 3 \\ 2 \end{bmatrix}, \begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} -1 \\ -2 \end{bmatrix}) = 0$
 $(x-3) \cdot (x+1) + (y-2) \cdot (y+2) = 0$

Alg Standard Real Rad

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■ Solution
 $\text{dotP}(\begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} 3 \\ 2 \end{bmatrix}, \begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} -1 \\ -2 \end{bmatrix}) = 0$
 $(x-3) \cdot (x+1) + (y-2) \cdot (y+2) = 0$
 $\text{expand}(\text{ans})$
 $x^2 - 2 \cdot x + y^2 - 7 = 0$

Alg Standard Real Rad

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$b = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$ are the boundary of the redus, find the equation for circle.

■ Solution
 $\text{dotP}(\begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} 3 \\ 2 \end{bmatrix}, \begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} -1 \\ -2 \end{bmatrix}) = 0$
 $(x-3) \cdot (x+1) + (y-2) \cdot (y+2) = 0$
 $\text{expand}(\text{ans})$
 $x^2 - 2 \cdot x + y^2 - 7 = 0$

Try your own.
 Geometry window
 Calculator

Alg Standard Real Rad

(p-c) (a-c)=r^2

Can you use vectors to find a tangent line to a circle?

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Vector Circle-3
 Tangent Line
 $(p-c) \cdot (a-c) = r^2$

<Example>
 Geometry window

When the center is $c = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$ and the radius $r=3$, find the tangent line that passes through $\begin{bmatrix} 0 \\ 2-\sqrt{5} \end{bmatrix}$.

■ Solution
 $\text{dotP}(\begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 2-\sqrt{5} \end{bmatrix} - \begin{bmatrix} 2 \\ 2 \end{bmatrix}) = 9$

Alg Standard Real Rad

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■ Solution
 $\text{dotP}(\begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 2-\sqrt{5} \end{bmatrix} - \begin{bmatrix} 2 \\ 2 \end{bmatrix}) = 9$
 $-2 \cdot (x-2) - \sqrt{5} \cdot (y-2) = 9$
 $\text{expand}(\text{ans})$
 $-2 \cdot x - \sqrt{5} \cdot y + 4 + 2 \cdot \sqrt{5} = 9$

Alg Standard Real Rad

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$\text{dotP}(\begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 2-\sqrt{5} \end{bmatrix} - \begin{bmatrix} 2 \\ 2 \end{bmatrix}) = 9$
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 $\text{expand}(\text{ans})$
 $-2 \cdot x - \sqrt{5} \cdot y + 4 + 2 \cdot \sqrt{5} = 9$
 $(\text{ans}-9) / (-\sqrt{5})$
 $\frac{\sqrt{5} \cdot (2 \cdot x + \sqrt{5} \cdot y + 5 - 2 \cdot \sqrt{5})}{5}$
 $\text{expand}(\text{ans})$
 $y + \frac{2 \cdot \sqrt{5} \cdot x}{5} - 2 + \sqrt{5} = 0$

Try your own.
 Geometry window

Alg Standard Real Rad

(p-c) · (p-c) = r²

Given the center and radius of a circle, use vectors to find its equation.

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Vector Circle-3
A Circle's Equation
 $(p-c) \cdot (p-c) = r^2$

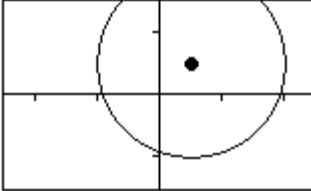
<Example>
 Geometry window
 Given the circle's center is $c = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and the radius is 3, the equation of the circle is
 $\text{dotP}\left(\begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) = 3^2$
 $(x-1)^2 + (y-1)^2 = 9$

Try your own.
 Geometry window
 Calculator

Alg Standard Real Rad

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Geometry window
 Given the circle's center is $c = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and the radius is 3, the equation of the circle is
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Alg Standard Real Rad

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<Example>
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 $(x-1)^2 + (y-1)^2 = 9$

Try your own.
 Geometry window
 Calculator

Alg Standard Real Rad

3D Distance

Find the distance between the tips of two vectors in 3-dimensional space.

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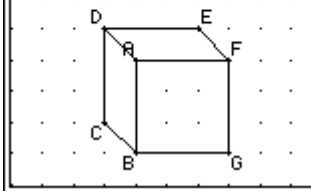
3D Distance
norm(p-q)

<Example>
 Cube 3x3x3
 When the point C of cube stays at the origin, the point F is $\begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}$.
 Then the distance between C and F is
 $\text{norm}\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}\right)$
 $3 \cdot \sqrt{3}$

Alg Standard Real Rad

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<Example>
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Alg Standard Real Rad

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Try your own.
 Calculator

Alg Standard Real Rad

(p-p0) · n = 0

A simple way to find the equation of a line perpendicular to a vector.

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Vector (p-p0) · n = 0

<Example>
 Geometry window
 When the line passes the point $\begin{bmatrix} 3 \\ 5 \end{bmatrix}$ and is perpendicular to the vector $r = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$, find the linear equation.

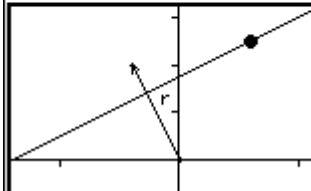
■ Solution
 $\text{dotP}\left(\begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} 3 \\ 5 \end{bmatrix}, \begin{bmatrix} -2 \\ 4 \end{bmatrix}\right)$
 $-2 \cdot (x-3) + 4 \cdot (y-5)$

Alg Standard Cplx Rad

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Vector (p-p0) · n = 0

<Example>
 Geometry window
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Alg Standard Cplx Rad

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■ Solution
 $\text{dotP}\left(\begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} 3 \\ 5 \end{bmatrix}, \begin{bmatrix} -2 \\ 4 \end{bmatrix}\right)$
 $-2 \cdot (x-3) + 4 \cdot (y-5)$
 $\text{ans} = 0$
 $-2 \cdot (x-3) + 4 \cdot (y-5) = 0$
 $\text{expand}(\text{ans}/2)$
 $-x + 2 \cdot y - 7 = 0$

Try your own.
 Geometry window
 Calculator

Alg Standard Cplx Rad

$p=(1-t)a+tb$

Learn a new way to find the equation of a line given two points.

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Vector and Lines
 $p=(1-t)a+tb$

<Example>
Geometry window
Given a line passes through points $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ & $\begin{bmatrix} -3 \\ 2 \end{bmatrix}$, find the equation of the line.

■ Solution
 $(1-t)\begin{bmatrix} 2 \\ 3 \end{bmatrix} + t\begin{bmatrix} -3 \\ 2 \end{bmatrix}$
 $\begin{bmatrix} -2\cdot(t-1)-3\cdot t \\ -3\cdot(t-1)+2\cdot t \end{bmatrix}$

Alg Standard Cplx Rad

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Vector and Lines
 $p=(1-t)a+tb$

<Example>
Geometry window
Given a line passes through points $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ & $\begin{bmatrix} -3 \\ 2 \end{bmatrix}$,

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expand(ans)
 $\begin{bmatrix} -5\cdot t+2 \\ -t+3 \end{bmatrix}$

Then we have:
 $x=-5\cdot t+2$
 $y=-t+3$
Using substitution, eliminate t.
You will get: $y=\frac{x}{5}+\frac{13}{5}$

Calculator

Try your own.
Geometry window
Calculator

Alg Standard Cplx Rad

$p= p_0+td$

A somewhat simple way to find the equation of a line parallel to a vector.

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Vector and || Lines
 $p= p_0+td$

<Example>
Geometry window
Given a line passes through point $\begin{bmatrix} 3 \\ 5 \end{bmatrix}$ and is parallel to vector $r=\begin{bmatrix} -2 \\ 4 \end{bmatrix}$, find the equation of the line.

■ Solution
 $\begin{bmatrix} 3 \\ 5 \end{bmatrix} + t\begin{bmatrix} -2 \\ 4 \end{bmatrix}$

Alg Standard Cplx Rad

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Vector and || Lines
 $p= p_0+td$

<Example>
Geometry window
Given a line passes through point $\begin{bmatrix} 3 \\ 5 \end{bmatrix}$ and is parallel to vector $r=\begin{bmatrix} -2 \\ 4 \end{bmatrix}$,

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Using substitution, eliminate t.
 $x=-2\cdot t+3$
ans-3 $x=-2\cdot t+3$
ans-3 $x-3=-2\cdot t$
ans/-2 $\frac{-(x-3)}{2}=t$
 $y=4\cdot\left(\frac{-(x-3)}{2}\right)+5$
expand(ans) $y=-2\cdot(x-3)+5$
 $y=-2\cdot x+11$

Try our own.
Geometry window
Calculator

Alg Standard Cplx Rad