

How Much Can I Save?

Teacher Notes

Topic Area: Quadratic Regression

NCTM Standard

- Students will be able to select and use appropriate statistical methods to analyze data.

Objective

Given a photo file, students will be able to analyze data to determine the quadratic regression equation for saving gold coins for retirement income.

Getting Started

Have students work in pairs or small groups to use statistical and algebraic representations to examine the results of savings.

Prior to using this activity:

- Students should be able to produce and manipulate graphs and tables of values manually and with the graphing calculator.
- Students should have a basic understanding of the regression equations.
- Students should be able to use regression equations to predict future outcomes.

Ways students can provide evidence of learning:

- Given a picture, students can plot points to be used to analyze data to find a graphical representation of a regression equation.
- Given an algebraic representation of a regression equation, the student can produce a table of values and determine the cumulative sum of a column of values.
- The student can state and explain how the results are interpreted to answer questions.

Common mistakes to be on the lookout for:

- Students may not follow the correct procedures to plot the points, determine the regression, constructing a table or finding a cumulative sum.
- Students may try to use a linear regression for the data instead of a quadratic regression.

Definitions

- | | |
|----------------------------|------------------------------|
| • Algebraic representation | • Quadratic regression |
| • Cumulative sum | • Ratios |
| • Graphical representation | • Regression |
| • Numerical representation | • Statistical representation |

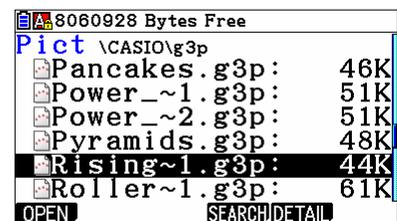
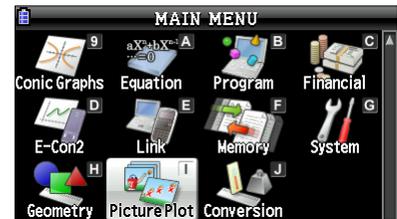
How Much Can I Save?

“How To”

The following procedures will demonstrate how to retrieve an image, plot points on the picture, determine the regression equation, enter results into a table, move the table data to a list, alter data and determine the cumulative sum to answer the desired questions.

To open a background image in Picture Plot:

- From the Main Menu, highlight the Picture Plot icon and press **EXE** or press **C**.
- Press **F1** (OPEN) to open the CASIO folder.
- The g3p folder contains 47 background images. Press **▼** **F1** (OPEN) to open folder. Scroll down the list of pictures and highlight the desired picture. You will be using “Rising~1” picture in this activity. Press **F1** (OPEN).



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To plot and move points on the image to desired x-values:

- Press **OPTN** and choose **F2** (PLOT).
- Move the Arrow to the desired position using **◀** **▶** **▲** **▼** and press **EXE**. In this activity, move the arrow to the top center of the first column of coins (to get the y-value) and then move the arrow directly to the right until it is above $x = 1$ and press **EXE**.

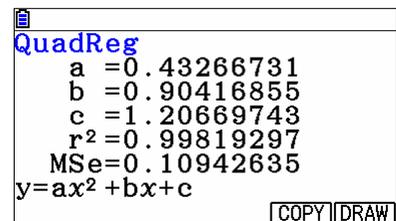
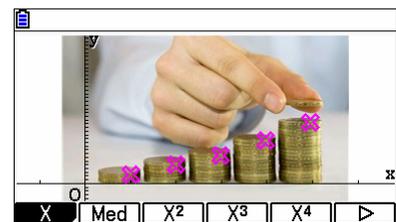


- Continue to move the arrow and press **EXE** until you have all the points you want (one point for each column of coins). To stop plotting, press **EXIT**.



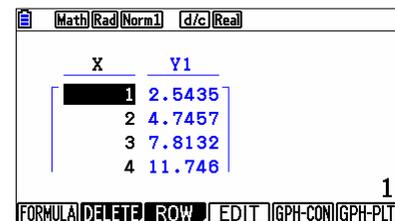
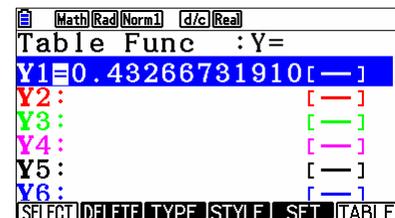
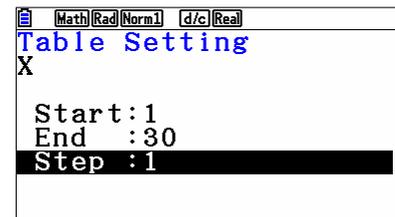
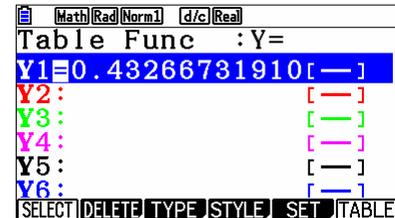
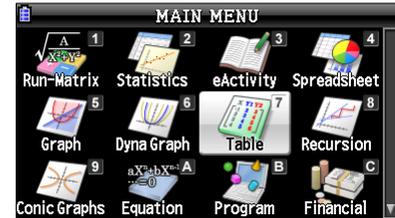
To plot the quadratic regression:

- Press **F6** (\triangleright).
- Press **F2** (REG).
- Press **F3** (x^2) for a quadratic regression.
- Press **F5** (COPY). This will copy the equation to other menu options such as Graph and Table, for future purposes. Press **EXE** to enter the equation for the **Y1**: equation. Keep in mind that the position of the arrow was estimated. Therefore, your equation will not be exactly the same as this equation or that of your partner.
- Press **F6** (DRAW). Notice that the quadratic regression comes very close to passing through all five plotted points.



To move the coordinates of the regression to a table:

- From the Main Menu, highlight the Table icon and press **EXE** or press **7**.
- Press **F1** (SELECT) to select the quadratic regression equation that was stored in the last section.
- Press **F5** (SET). Choose the start point, end point and step units. This activity will need to know the savings for at least 30 years of working. Enter **1** for the start value and press **EXE**. Enter **30** for the end value and press **EXE**. Enter **1** for the step value to get data for each year of working and press **EXE**. The SET screen should look like the screen on the right.
- Press **EXIT** to return to the previous choices. Press **F6** (TABLE) to display the table that represents the amount of coins purchased each year of working. You can scroll down to see the other values.



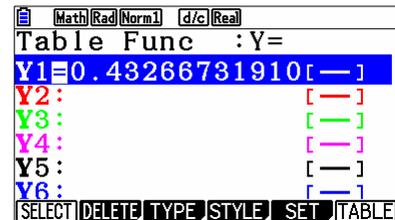
To convert y-values using ratios:

- To convert your y-values to the actual coin quantities, you will need to use the following ratio:

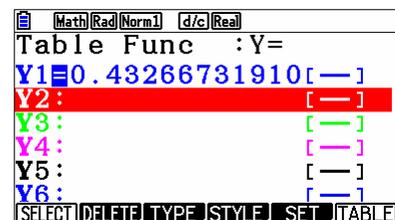
$$\frac{3 \text{ coins}}{\text{corresponding } y\text{-value}} = \frac{x \text{ coins}}{y_1 \text{ value}}$$

The value of the 3 coins from the table in this activity is 2.5435. Your value will probably differ.

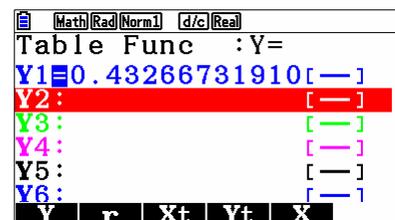
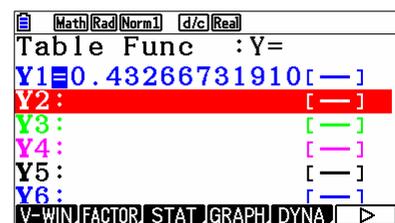
- Press **EXIT** to return to the equation screen.



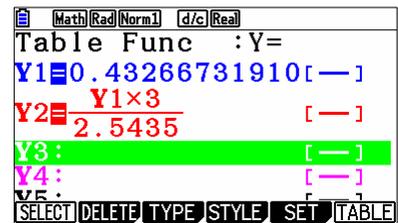
- Enter the ratio into the **Y2:** by pressing **VAR**, **F4** (GRAPH), **F1** (Y). Then, press **1** **X** **3** **÷** **2** **·** **5** **4** **3** **5** **EXE**.



Notice that the fraction was entered in natural display when you press **EXE**.



- Press **F6** (TABLE) to display the table of approximate number of coins saved each year of working. Again, you can scroll down to see how many coins were collected for each of the 30 years of working.

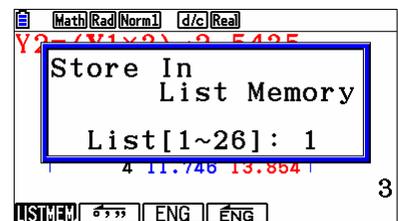
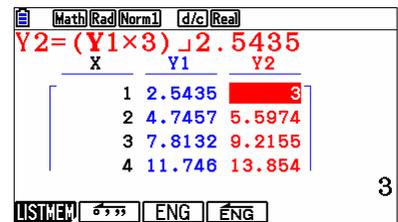


X	Y1	Y2
1	2.5435	3
2	4.7457	5.5974
3	7.8132	9.2155
4	11.746	13.854

[FORMULA] [DELETE] [ROW] [EDIT] [GPH-CON] [GPH-PLT]

To copy this table to a list:

- Since you will need to copy the **Y2:** column of the table, make sure that the calculator cursor is positioned anywhere in that column.
- Press **OPTN**. Press **F1** (LISTMEM).
- Input the number of the list you want to copy and press **EXE**. In this activity, use List 1.

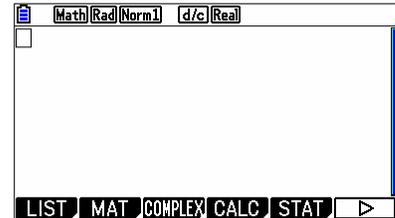


To create a cumulative list:

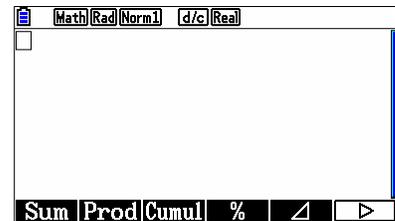
- From the Main Menu, highlight the Run-Mat icon and press **EXE** or press **1**.



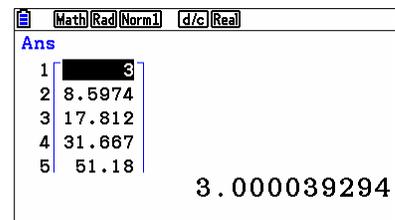
2. Press **OPTN** and **F1** (LIST).



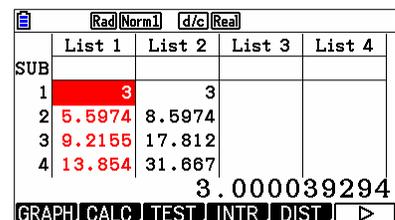
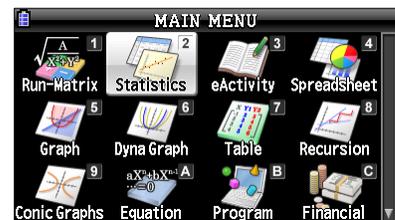
3. Press **F6** (>) twice.



4. Press **F3** (Cumul) **F6** (>) **F1** (LIST) and then press the number of list that you want to create the cumulative sum. Our data was stored in List 1, therefore press **1**. Press **→** (Store), **F1** (LIST), **2**. This is because you need to store the values in List 2. Press **EXE**.



5. From the Main Menu, highlight the Statistics icon. Press **EXE** or press **2** to display the list of cumulative sums of the coins for each year of working.

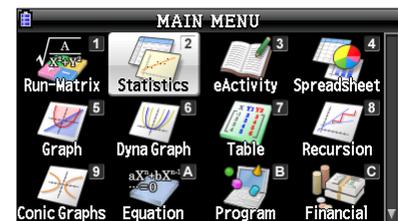
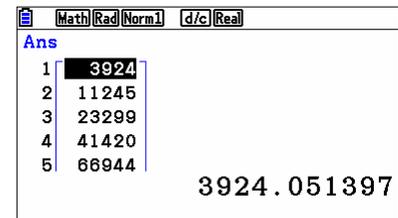
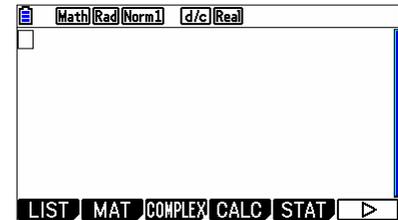
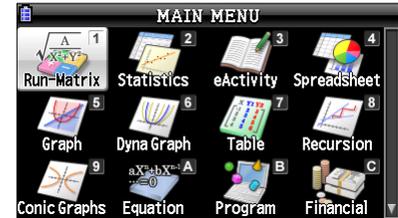


To create a list of the value of the coins:

- From the Main Menu, highlight the Run-Mat icon and press **EXE** or press **1**.
- Press **OPTN**, **F1**(LIST), **F1**(LIST) again, **2** **X** **1** **3** **0** **8** **→** (Store) **F1**(LIST) **3** **EXE**.

This is because the value of the coin at the time this activity was written was \$1,308, and you are storing these values into List 3.

- From the Main Menu, choose the Statistics icon. Press **EXE** to display the amount that was cumulatively saved each year of working.



	List 1	List 2	List 3	List 4
SUB				
1	3	3	3924	
2	5.5974	8.5974	11245	
3	9.2155	17.812	23299	
4	13.854	31.667	41420	
			3.000039294	

How Long Do We Save?

Activity

Mr. and Mrs. Brinkley are hoping their nest egg that they have saved over the years of hard work will sustain their life style as they enter their golden years of retirement. When Brian and Amy first entered the workforce, they decided to save 1 oz. gold coins to reach this goal. They also decided that they would continue working until either their savings reached \$4,000,000 or they worked 30 years. Finally, they have reached their goal. The price of gold is now \$1,308 an ounce.

In this activity, you will be given a picture of five stacks of gold coins, shown below. The first stack represents the gold coins that were saved by the Brinkley's their first year of work. The second column represents the coins saved their second year, the third represents their third year, and so on. Brian and Amy continued to save at the same rate until they retired. You will need to find the regression to determine their future yearly and total savings. Did the Brinkley's reach their goal of saving four million dollars and retire early or did they retire after working 30 years? You will need to determine how many years they worked and how much they saved toward their goal.



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Questions

- Using the calculator, copy image "Rising~1" to the calculator screen. Plot a point at $x = 1$ to represent the number of coins saved the first year. Plot the second point at $x = 2$ to represent the number of coins saved the second year. Continue this pattern for all five stacks of coins.
- Determine and draw the quadratic regression equation on the calculator. What was the quadratic regression equation?

3. Use a table to list the y-value of coins saved each year for 30 years. Enter the values in the table, below:

Year	Y-value	Year	Y-value	Year	Y-value
1		11		21	
2		12		22	
3		13		23	
4		14		24	
5		15		25	
6		16		26	
7		17		27	
8		18		28	
9		19		29	
10		20		30	

4. Convert the y-values to the number of coins saved each year by using ratios. What is the Y_2 equation used to represent this ratio conversion?

5. Enter the number of coins saved each year in the table below:

Year	Coins Saved	Year	Coins Saved	Year	Coins Saved
1		11		21	
2		12		22	
3		13		23	
4		14		24	
5		15		25	
6		16		26	
7		17		27	
8		18		28	
9		19		29	
10		20		30	

6. Copy the Y_2 column of the table to a list. Use list 1. Create a cumulative list of the number of coins saved in list 2. Write your answers for list 2.

Year	Total Coins Saved	Year	Total Coins Saved	Year	Total Coins Saved
1		11		21	
2		12		22	
3		13		23	
4		14		24	
5		15		25	
6		16		26	
7		17		27	
8		18		28	
9		19		29	
10		20		30	

7. Create a list of the cumulative value of the coins to list 3. What equation did you use to determine the value of the coins for list 3?
-

8. Enter the values for list 3 into the following table.

Year	Total Savings	Year	Total Savings	Year	Total Savings
1		11		21	
2		12		22	
3		13		23	
4		14		24	
5		15		25	
6		16		26	
7		17		27	
8		18		28	
9		19		29	
10		20		30	

9. Did the Brinkley's earn 4 million dollars? If so, what year did they reach that goal?
-

10. How much do the Brinkley's earn in 30 years?
-

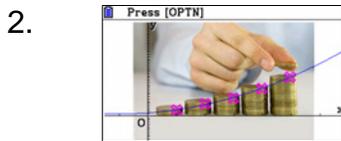
11. How many years do the Brinkley's work before retiring? Did they get to retire early?
-

Extensions

1. Explain why your answers do not exactly match the answers on this activity or the answers of other students in your class.

2. What is the regression equation if the Brinkley's would have saved 3 coins the first year, 5 coins the second, 7 coins the third, 10 coins the fourth and 16 coins the fifth year?

Solutions



```

QuadReg
a = 0.43266731
b = 0.90416855
c = 1.20669743
r² = 0.99819297
MSe = 0.10942635
y = ax² + bx + c
    
```

Answers may vary,
 $y = .43266731x^2 + 0.90416855x + 1.20669743$

3. Answers may vary.

X	Y1
1	2.5435
2	4.7457
3	7.8132
4	11.746

X	Y1
5	16.544
6	22.207
7	28.736
8	36.13

X	Y1
9	44.39
10	53.515
11	63.505
12	74.36

X	Y1
13	86.081
14	98.667
15	112.11
16	126.43

X	Y1
17	141.61
18	157.66
19	174.57
20	192.36

X	Y1
21	211
22	230.5
23	250.88
24	272.12

X	Y1
25	294.22
26	317.19
27	341.03
28	365.73

X	Y1
27	341.03
28	365.73
29	391.3
30	417.73

4. Answers may vary.

```

Table Func : Y=
Y1 0.43266731910(-)
Y2 Y1x3 (-)
Y3 2.5435 (-)
Y4 (-)
    
```

5. Answers may vary.

X	Y2
1	3
2	6.5974
3	9.2155
4	13.854

X	Y2
5	19.513
6	26.193
7	33.894
8	42.615

X	Y2
9	52.357
10	63.119
11	74.903
12	87.706

X	Y2
13	101.53
14	116.37
15	132.24
16	149.12

X	Y2
17	187.03
18	195.98
19	205.91
20	226.88

X	Y2
21	248.87
22	271.88
23	295.91
24	320.96

X	Y2
25	347.03
26	374.12
27	402.24
28	431.37

X	Y2
27	402.24
28	431.37
29	461.53
30	492.7

6. Answers may vary.

Rad(Norm)	d(C) Real		
List 1	List 2	List 3	List 4
SUB			
1	3	3	
2	5.5974	8.5974	
3	9.2155	17.812	
4	13.854	31.667	
		3.000039294	
TOOL FDI DELETE DEL-ALL INSR >			

Rad(Norm)	d(C) Real		
List 1	List 2	List 3	List 4
SUB			
5	19.513	51.18	
6	26.193	77.374	
7	33.894	111.26	
8	42.615	153.85	
		42.61539729	
TOOL FDI DELETE DEL-ALL INSR >			

7. Answers may vary. List 2 x 1308 is stored to List 3.

Rad(Norm)	d(C) Real		
List 1	List 2	List 3	List 4
SUB			
1	3	3924	
2	5.5974	8.5974	11245
3	9.2155	17.812	23299
4	13.854	31.667	41420
		3.000039294	
GRAPH CALC TEST INTR DIST >			

Rad(Norm)	d(C) Real		
List 1	List 2	List 3	List 4
SUB			
17	167.03	1088.2	1.43e6
18	185.96	1284.2	1.87e6
19	205.91	1490.1	1.94e6
20	226.85	1717	2.24e6
		226.8806718	
GRAPH CALC TEST INTR DIST >			

9. Yes. In the 24th year.

10. Answers may vary. \$7,015,696.70

Rad(Norm)	d(C) Real		
List 1	List 2	List 3	List 4
SUB			
28	431.37	4409.4	5.76e6
29	461.53	4870.9	6.37e6
30	492.7	5363.6	7.01e6
31			
		7015696.698	
GRAPH CALC TEST INTR DIST >			

11. 24 years. Yes.

Extension Solutions

1. Because when plotting the points at the beginning of the activity, the points were approximated visually.

2. $y = 0.64285714x^2 - 0.7571428x + 3.4$

Rad(Norm)	d(C) Real
QuadReg	
a	= 0.64285714
b	= -0.7571428
c	= 3.4
r ²	= 0.99110617
MSe	= 0.45714285
y	= ax ² + bx + c
COPY	