Don't Block My View! Teacher Notes

Topic Areas: Ratios and Conics

NCTM Standards:

- Apply appropriate techniques, tools and formulas to determine measurement.
- Use visualization, spatial reasoning, and geometric modeling to solve problems.

Objective

Given a picture of a window and one measurement, the student will be able to use ratios to determine the dimensions of a new window design.

Getting Started

Using the Casio fx-CG series, have students work in pairs or small groups to investigate graphically the distance between two points and to use ratios to determine the actual dimensions from the scale dimensions. Emphasis will also be on changing a semicircle to a semi-ellipse.

Prior to using this activity:

- Students should be able to plot points and determine their coordinates.
- Students should have a basic understanding of ratios and proportions.
- Students should be able to determine distance between two points.
- Students should have a basic understanding of circles and ellipses.

Ways students can provide evidence of learning:

- If given a picture, students can plot points and find the distance between the two points
- If given a scale drawing, students can convert the scale dimensions of the drawing to actual dimensions.
- If given a semicircle and dimensions, students can determine a formula for an ellipse.

Common mistakes to be on the lookout for:

- Students may not have the correct points when finding distance.
- Students may not be using the correct formula for an ellipse.

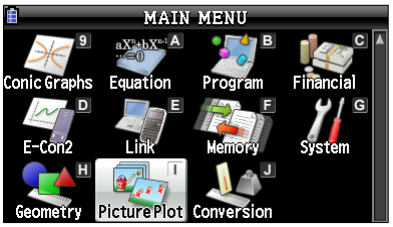
Definitions

- Center
- Circle
- Dimensions
- Distance
- Ellipse
- Major axis
- Minor axis
- Proportion
- Ratio
- Scale

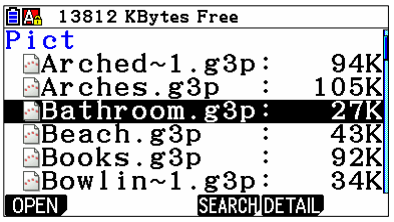
Don’t Block My View! “How To”

The following procedures will demonstrate how to retrieve an image, plot points on the picture, and use the equation solver to answer the desired questions.

To open a background image in Picture Plot:

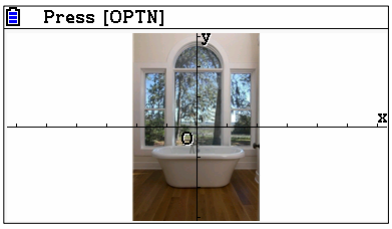


1. From the Main Menu, highlight the Picture Plot icon and press EXE or press (.



2. Press F1 (OPEN) to open the CASIO folder.

3. The g3p folder contains 47 background images. Press ▼ F1 (OPEN) to open the folder. Scroll down the list of images and highlight the desired image. You will be using the “Bathroom” image in this activity. Press F1 (OPEN).



To plot points on the image:

1. Press OPTN and choose F2 (PLOT).

2. Move the Arrow to the desired position using ◀ ▶ ▲ ▼ and press EXE. In this activity, move the arrow to the upper left corner of the left window and press EXE.

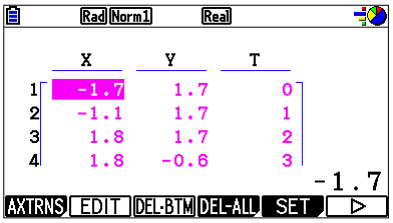
3. Continue to move the arrow and press EXE until you have all the points you want.

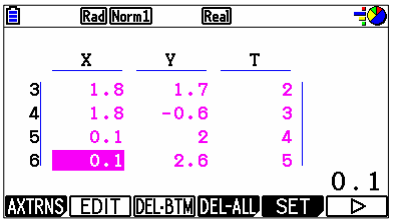
4. To stop plotting, press EXIT.

To find the coordinates of each point:

1. Press F3 (List).

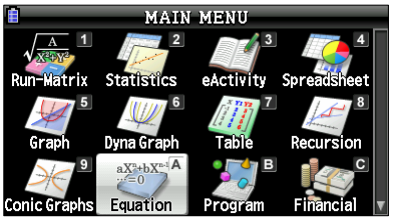
2. The X-column is the x-coordinate of the point, the Y-column is the y-coordinate of the point. The points in this list are in the order that they were plotted on the calculator. Scroll down to see the coordinates of all the points that were plotted.



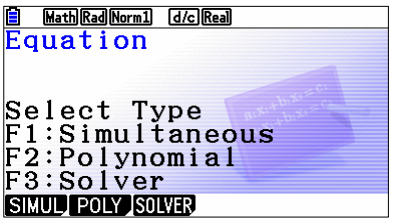


To find the distance between two points:

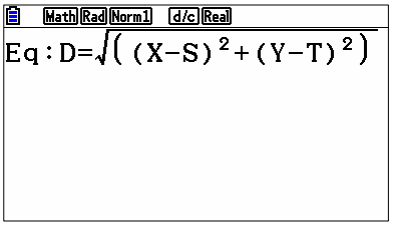
1. From the Main Menu, highlight the Equation icon and press EXE or press X,θ,T.



2. Choose F3 (SOLVER).

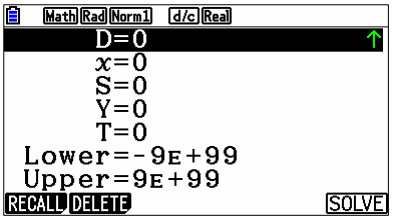


3. The distance formula between two points is $D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. However, you cannot enter subscripts into the calculator, therefore, enter the formula as $D = \sqrt{(X - S)^2 + (Y - T)^2}$; where D = distance between the two points; X = x-coordinate of the second point; S = x-coordinate of the first point; Y = y-coordinate of the second point; and T = y-coordinate of the first point.

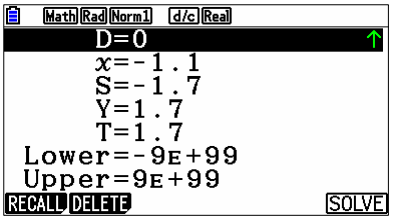


Press [ALPHA] [sin] [SHIFT] [•] [SHIFT] [x²] [(] [ALPHA] [+] [−] [ALPHA] [×] [)] [x²] [+] [(] [ALPHA] [−] [−] [ALPHA] [÷] [)] [x²] [EXE].

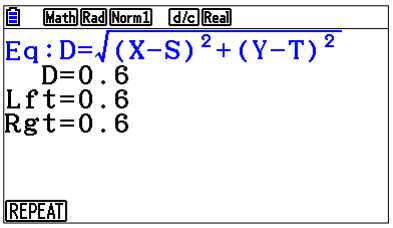
4. Fill in D= 0 (even though you are looking for this number). Use point A as having coordinates (S, T) and point B as having coordinates (X, Y). Press [EXE] after each entry.



5. When finished entering the data, highlight the variable you are finding, in this case, highlight D = 0 and press [F6] (SOLVE).



6. The distance between point A and point B is 0.6 units.



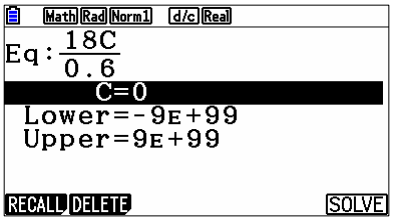
7. To determine the distance between point A and point C, press [F1] (REPEAT). This allows you to just enter the new coordinates and use the formula you previously entered. Repeat this procedure for all points.

To find the actual distance between two points:

1. Use ratios and proportions to convert the calculator distance to actual distance. Since the actual distance from point A to point B is 18 inches and the calculator distance is 0.6 units, the proportion you will use is:

$$\frac{0.6}{18} = \frac{calculator\ distance\,(C)}{actual\ distance\,(A)}$$

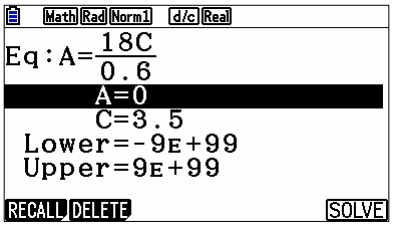
2. Use the Equation Solver as you did in the last Section, now solving for A in the proportion.



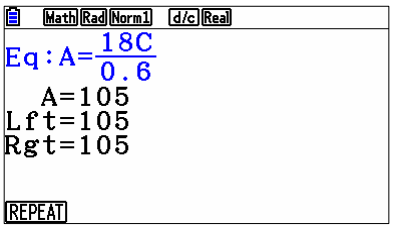
Enter the new formula, A = $\frac{18C}{0.6}$, by pressing

ALPHA X,θ,T SHIFT • 1 8 ALPHA ln $a\frac{b}{c}$ 0 • 6 EXE.

3. Fill in the calculator distance for each C, highlight A and press F6 (SOLVE).



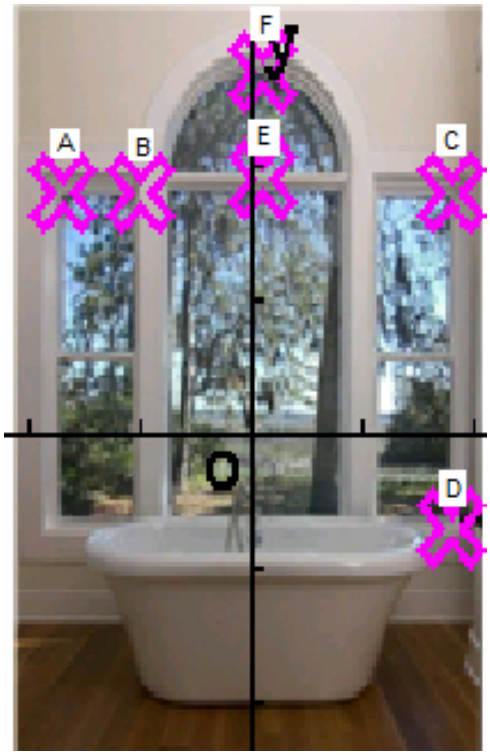
4. Press F1 (REPEAT) to solve additional problems.



Don’t Block My View! Activity

The owner of this house found a major fault to this window design for the bathroom. When relaxing in the tub, the view is blocked by the inside border of the window. The owner wants to remove the window on the left and install a window with the design below right. The only actual measurement is the distance from point A to point B which is 18 inches. Point A and point B are the top corners of the glass of the present window. All points are to mark the edge of the glass, not the frame. The radius of the semicircle, from point E to point F, will be the same as half the minor axis for the semi-ellipse in the new design. Point E is not collinear with points A and C. The distance from A to C on the present design will be the same as the distance from A to C on the new design. The distance from point C to point D is also the same on both designs.

In this activity, you will need to find the actual dimensions of the rectangular section of the new design and the area. You will also need to find the coordinates of point G which is the left end of the major axis of the semi-ellipse and point H which is the right end of the major axis. You will need to determine the standard form of the equation of the ellipse, where point E is the center. Also, you will need to find the area of both the ellipse that is used and the semi-ellipse that is in the design of the new window. Finally, the total area of the glass needed for the new window.



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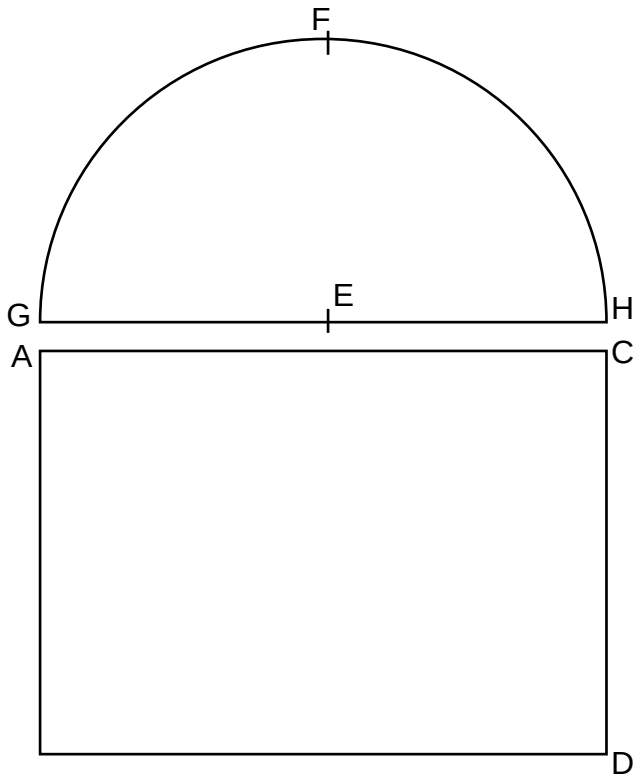


Questions

1. Using the Casio fx-CG series, copy image “Bathroom” to the calculator screen. All the points that you will be graphing should be placed at the edge of the glass of the window, not in the frame, as shown in the previous drawing. Plot point A as the top left corner of the small window. Point B is the top right corner of the small window. Point C is the top right corner of the other small window. Point D is the bottom right corner of the right window. Point E is the center of the semicircle and point F is the highest point of that window. Plot all these point on the fx-CG series.

2. List the coordinates of points A, B, C and D.

POINT	COORDINATE
A	
B	
C	
D	

3. Find the distance between the following points:

POINTS	DISTANCE
A to B	
A to C	
C to D	

4. Since the actual distance from points A to B is 18 inches, find the other actual dimensions of the rectangular section of the new window.

   Length AC = _____

   Height CD = _____

5. What is the area of the new rectangular window?

   _____

6. Determine the coordinates of the following points:

POINT	COORDINATE
E	
F	
G	
H	

7. Find the distance between the following points:

POINTS	DISTANCE
E to F	
E to G	
G to H	

8. What is the length of the minor axis of the ellipse that is used?

_____

9. What is the length of the major axis of the ellipse that is used?

_____

10. Using the standard form for the equation of the ellipse, where E is the center of the ellipse at coordinates (h, k). The distance from E to G is *a*. The distance from E to F is *b*. Standard form is:

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$

Find the standard form of this ellipse.

_____

11. What is the actual length of *a* for the new window?

_____

12. What is the actual length of *b* for the new window?

_____

13. Using the formula for the area of an ellipse: $A = \pi ab$, determine the area of the ellipse.

_____

14. What is the area of the semi-ellipse?

_____

15. What is the area of the total glass for both sections of the new window?

_____

Extensions

1. Why will your answers not match those of your partner or other students in your class?

_____

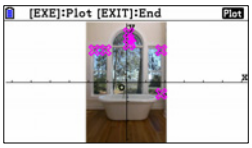
_____

_____

2. What would be the area of the new window if the upper window remains a semicircle, with the radius the length of EG?

_____

Solutions

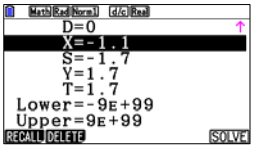
1. 

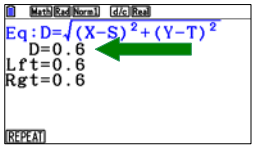
2.

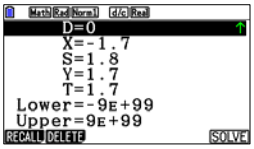
POINT	COORDINATE
A	(-1.7, 1.7)
B	(-1.1, 1.7)
C	(1.8, 1.7)
D	(1.8, -0.6)

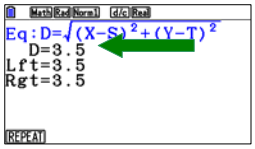
3.

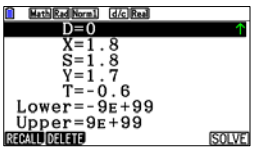
POINTS	DISTANCE
A to B	0.6
A to C	3.5
C to D	2.3

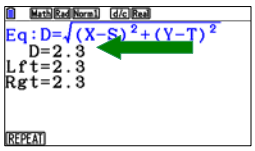




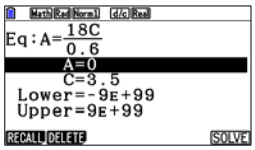


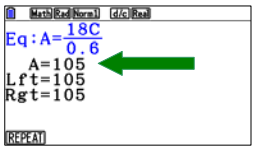


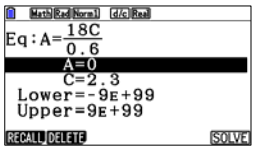


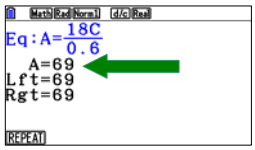


4. 105 inches; 69 inches

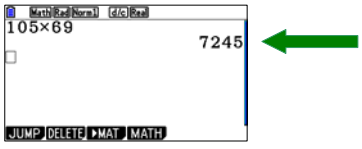








5. (105 in)(69 in) = 7,245 inches^2

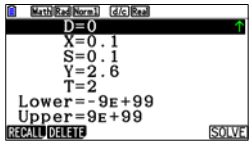


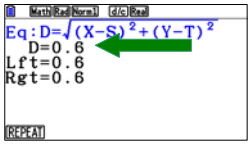
6.

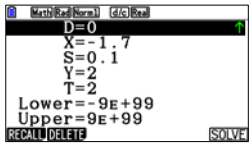
POINT	COORDINATE
E	(0.1, 2)
F	(0.1, 2.6)
G	(-1.7, 2)
H	(1.8, 2)

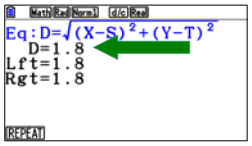
7.

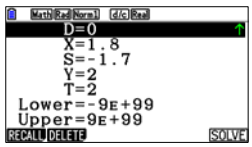
POINTS	DISTANCE
E to F	0.6
E to G	1.8
G to H	3.5

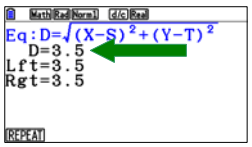




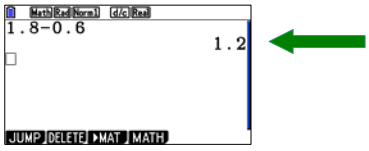








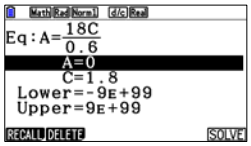
8. 1.8 – 0.6 = 1.2

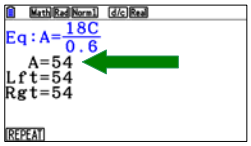


9. 3.5

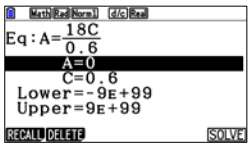
10. $\frac{(x-0.1)^2}{3.24}+\frac{(y-2)^2}{0.36}=1$

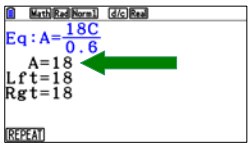
11. 54 inches



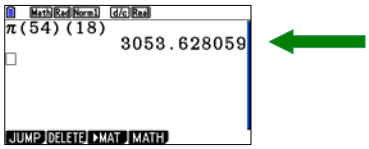


12. 18 inches

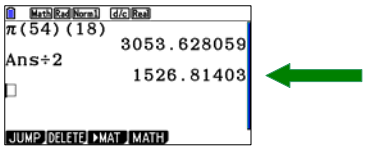




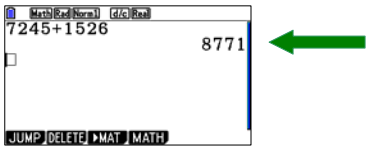
13. A = π(54)(18) = 3,054 inches^2



14. $\frac{3{,}054}{2}$ = 1526 inches^2



15. 7,245 + 1,526 = 8,771 inches^2



Extension Solutions

1. The calculator points are approximated.

2. 11,823 inches^2

Area of semicircle = $\frac{1}{2}\pi r^2 = \frac{1}{2}\pi(54)^2$ = 4,580 inches^2

Total area = 7,245 + 4,580 = 11, 825 inches ^{2}