Loop the Loop Teacher Notes

Topic Area: Curve fitting; piecewise-defined functions; parametric regressions

NCTM Standards:

- Use Cartesian coordinates to analyze geometric situations.
- Identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships.
- Use a variety of symbolic representations, including recursive and parametric equations, for functions and relations
- Apply and adapt a variety of appropriate strategies to solve problems

Objective

Given a photo of a rollercoaster, students will be able to fit a piecewise-defined function onto a section of the coaster. Using their knowledge of the standard equation of a circle and how to restrict the domain of a function, students will create multiple equations based on plotting and analyzing a series of points. As an extension, students will learn how to perform a parametric regression.

Getting Started

Have students work in small groups to help determine how to separate the standard equation for a circle into two explicit functions.

Prior to using this activity:

- Students should understand how to solve literal equations.
- Students should be able to create the standard equation for a circle, given the center and radius.
- Students should be able to determine an appropriate regression given a scatter plot.

Ways students can provide evidence of learning:

- Students should display graphs that match the photograph.
- Students should select appropriate regressions given a scatter plot.

Common mistakes to be on the lookout for:

- Students may be careless in the placement of points.
- Students may have difficulty setting parameter values.

Definitions

Piecewise-defined function

Formulas

Standard Equation of a Circle $(x - h)^2 + (y - k)^2 = r^2$

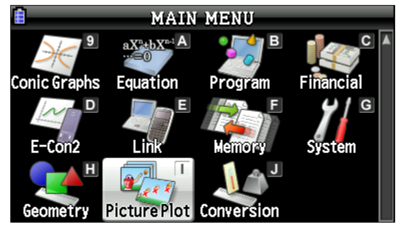
Explicit Equations for a Circle: $y = \pm\sqrt{r^2 - (x - h)^2} + k$

Loop the Loop "How To"

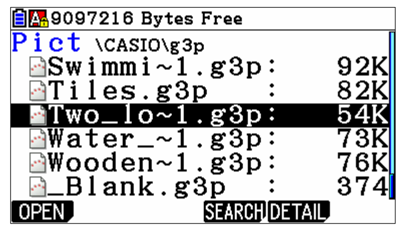
The following will walk you through the keystrokes and menus required to successfully complete the Loop the Loop activity.

To open a background image in Picture Plot:

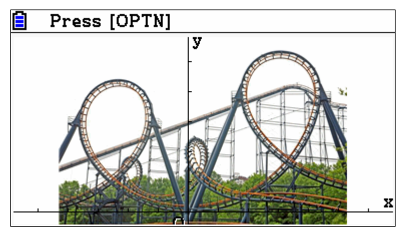
1. From the Main Menu, highlight the Picture Plot icon and press [EXE] or press [(].



2. Press [F1] (OPEN) to open the CASIO folder.



3. The g3p folder contains 47 background images. Press [▼] [F1] (OPEN) to open the folder. Scroll down the list of images and highlight the desired image. You will be using the "Two_lo~1" image in this activity. Press [F1] (OPEN).



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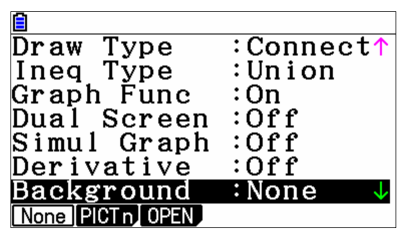
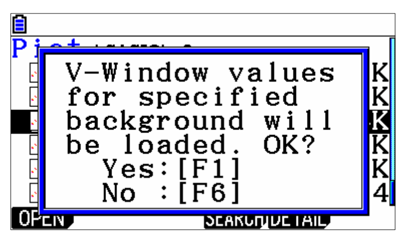
To add a background image to the GRAPH menu:

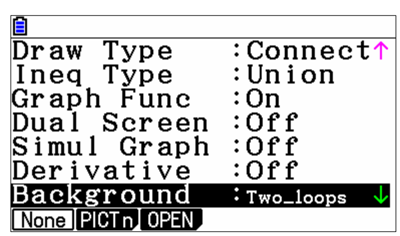
1. From within the GRAPH menu, press [SHIFT] [MENU] **(SET UP)** and [▼] until BACKGROUND is highlighted.



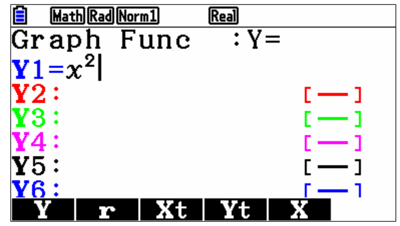
2. Press [F3] (OPEN), arrow down to the desired file and press [F1] (OPEN).



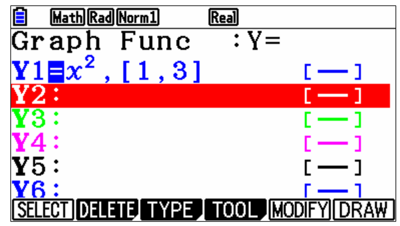
3. Press [F1] (YES) to accept the specified View Window.



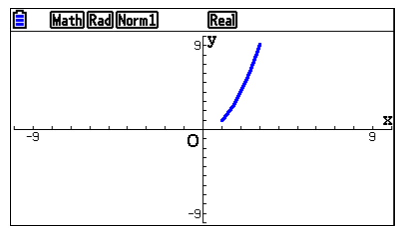
To restrict the domain of a function:



1. From within the GRAPH menu, enter the desired function, by pressing [X,θ,T] [x^2].

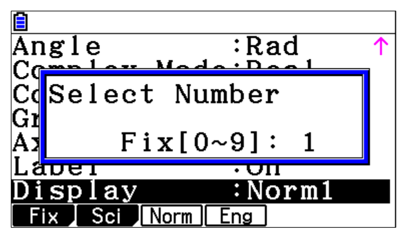


2. Domain restrictions are placed inside brackets. The restricted domain is [1, 3]. Immediately following the function, enter the following: [,] [SHIFT] [+] [1] [,] [3] [SHIFT] [−] [EXE].



3. To see the restricted graph, press [F6] (DRAW).

To set the number of decimal places from 0 to 9:

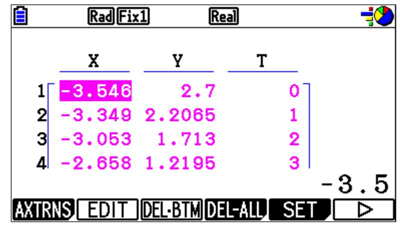


1. To set decimal places to 1, from within the GRAPH menu press [SHIFT] [MENU] **(SET UP)** [▲] [F1] (FIX) [1] [EXE].

To plot points on the picture and create a list of points:

1. To plot points, press [OPTN] [F2] (PLOT). A pink arrow will appear; use [◀] [▶] [▲] [▼] to move the arrow to where you would like for it to plot a point. (Any of the number keys can also be used to jump to different areas on the screen). Press [EXE] to plot the point.

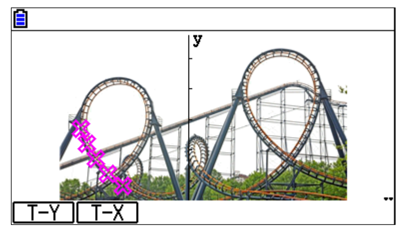
2. Continue moving the arrow and pressing EXE until you have all the desired points. Press EXIT to stop plotting.



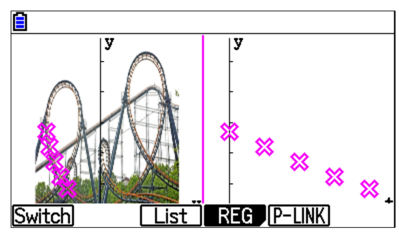
3. Press OPTN F3 (LIST) to view the list of points plotted. Press EXIT to go back to the picture and points or press F4 (DEL-ALL) to delete all points.

To perform a parametric regression of Y on T:

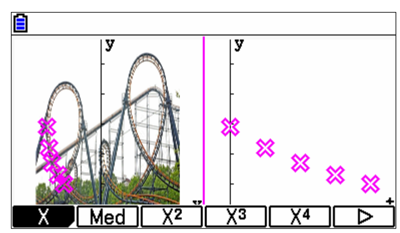
1. From within the PICTURE PLOT menu, press OPTN F6 (▷) F1 (AXTRNS) F1 (T-Y).

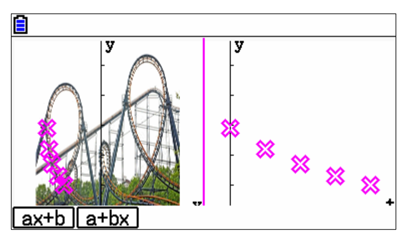


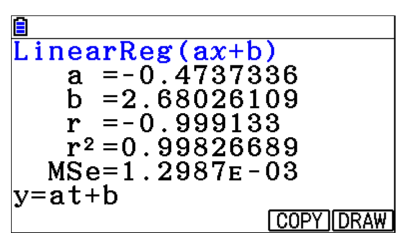
2. From the dual screen, press OPTN F4 (REG).



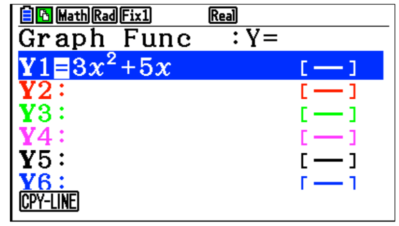
3. Select the appropriate regression. In this case, it is linear, so press F1 (X) F1 (ax+b).







To copy and paste a function to another location:



1. From within the GRAPH menu, press the arrow to the desired function. Press SHIFT 8 **(CLIP)** F1 (CPY-LINE) to copy.

2. Arrow to desired destination and press SHIFT 9 **(PASTE)** to paste.

Loop the Loop Activity

When most people first view a looping roller coaster (Figure 1), they think that the loop is a circle; this is a common misconception. If you were to look more closely, you might notice that the top of the loop may look like a half-circle, whereas the bottoms look different, having an increasing radius of curvature closer to the ground (Figure 2). These type of vertical loops have a tear drop shape and actually follow a special shape called a clothoidloop, which involves equations and mathematics that are beyond the scope of this activity.

In this activity, you will need to find a piecewise-defined function consisting of three different equations for a circle. You will accomplish this by determining the coordinates of the center of each circle and at least two points of the circle, that lie on the axes that has each respective center as the origin. You will need to be able to write an explicit function for either the top or bottom half of each circle. You will also need to determine an appropriate domain for each equation.

In the extension, you plot appropriate points along one loop of the rollercoaster and perform a parametric regression to model the loop. You will also become familiar with changing various settings and using features of the Casio fx-CG series.

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Figure 1

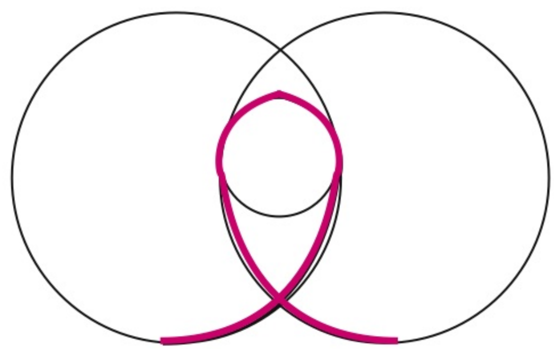


Figure 2

Questions

1. Using the leftmost loop of the rollercoaster, plot one point that best represents the center of the small circle at the top of the loop and two points on the small circle that lie directly horizontal and directly vertical from the center. Write the coordinates in the spaces provided. Round each coordinate to the nearest tenth.

   Center ______________________

   Point 1 ______________________

   Point 2 ______________________

2. Using the coordinates in Question 1, determine the radius of the small circle by averaging the distances between both points and the center of the small circle.

   ____

3. Using the center and the radius of the small circle, write the standard equation of the circle.

   ____

4. Rewrite the standard equation for the small circle as an explicit function.

   ____

5. What is the domain of the explicit function for the small circle?

   ____

6. Using the leftmost loop of the rollercoaster, plot one point that best represents the center of the larger circle entering the loop and two points on the circle that lie directly horizontal and directly vertical from the center. Write the coordinates in the spaces provided. Round each coordinate to the nearest tenth.

   Center ______________________

   Point 1 ______________________

   Point 2 ______________________

7. Using the coordinates in Question 6, determine the radius of the entering circle by averaging the distances between both points and the center of the entering circle.

____

8. Using the center and the radius of the entering circle, write the standard equation of the circle.

____

9. Rewrite the standard equation for the entering circle as an explicit function.

____

10. What is the domain of the explicit function for the entering circle?

____

11. Using the leftmost loop of the rollercoaster, plot one point that best represents the center of the larger circle exiting the loop and two points on the circle that lie directly horizontal and directly vertical from the center. Write the coordinates in the spaces provided. Round each coordinate to the nearest tenth.

Center ______________________

Point 1 ______________________

Point 2 ______________________

12. Using the coordinates in Question 11, determine the radius of the exiting circle by averaging the distances between both points and the center of the exiting circle.

____

13. Using the center and the radius of the exiting circle, write the standard equation of the circle.

____

14. Rewrite the standard equation for the exiting circle as an explicit function.

____

15. What is the domain of the explicit function for the exiting circle?

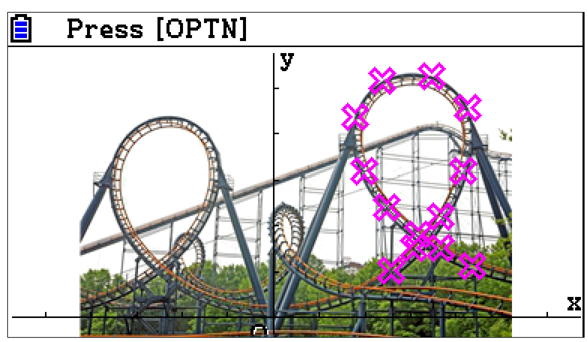
____

16. Graph each explicit function in the PICTURE PLOT menu on the Casio fx-CG series, with appropriate domains to create a piecewise-defined model of the leftmost rollercoaster loop. Show the results to your classmates and teacher. Sketch the piecewise-defined function over the image below.

Extension

1. Plot at least 10 points along the rightmost loop of the rollercoaster. Round each coordinate to the nearest tenth. Record the coordinates in the space provided.

Point 1	
Point 2	
Point 3	
Point 4	
Point 5	
Point 6	



Point 7	
Point 8	
Point 9	
Point 10	
Point 11	
Point 12	
Point 13	
Point 14	

2. Use the Casio fx-CG series to plot the Y (vertical) coordinates with respect to T (time) and perform the appropriate Y-T regression. Copy the full regression equation into the GRAPH menu of the Casio Fx-CG series. Round all coefficients and constants to the nearest hundredth and record the regression equation below.

____

3. Use the Casio fx-CG series to plot the X (horizontal) coordinates with respect to T (time) and perform the appropriate X-T regression. Copy the full regression equation into the GRAPH menu of the Casio fx-CG series. Round all coefficients and constants to the nearest hundredth and record the regression equation below.

____

4. Open the GRAPH menu and copy both regression equations into the appropriate line of a parameterized equation. [Note: The PICTURE PLOT menu can only graph Y = f(x), therefore we must alter each regression equation to include the parameter T when graphing parametric equations in the GRAPH menu.] Write the parametric equation below. Round each coefficient and constants to the nearest hundredth.

____

____

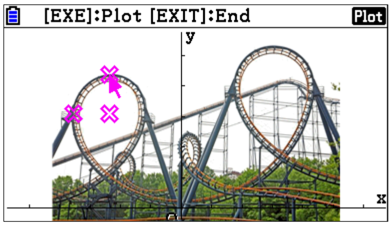
5. Graph the parametric function on the Fx-CG series in the GRAPH mode, with the appropriate values for the parameter, to create a model of the rightmost rollercoaster loop. Show the results to your classmates and teacher. Write the values selected for Tθmin/Tθmax below.

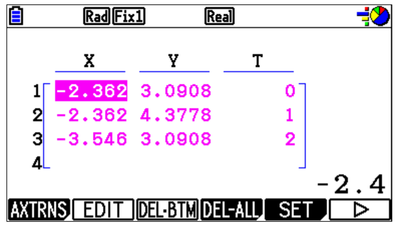
____

6. Sketch the parametric function over the image below.

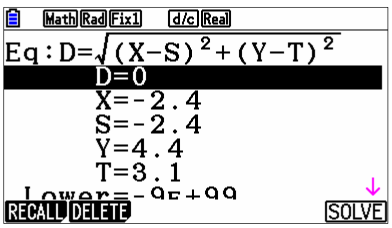
Solutions
Answers will vary, depending on the plotted points.

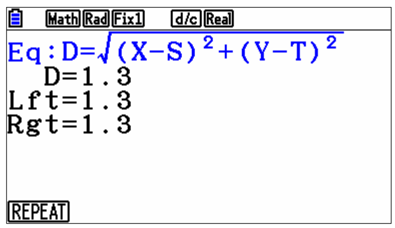
1. Center (-2.4, 3.1) Point 1 (-2.4, 4.4) Point 2 (-3.5, 3.1)

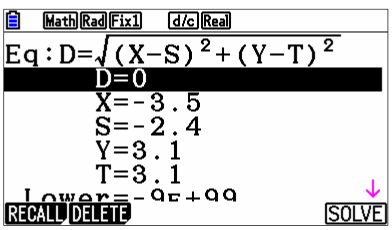


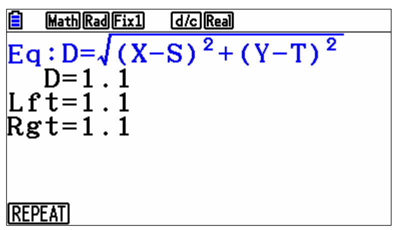


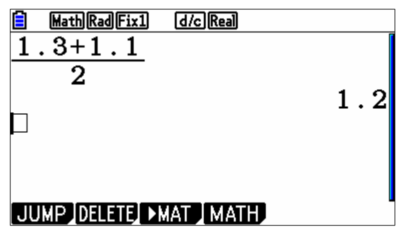
2. Radius = 1.2











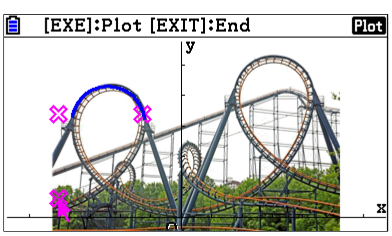
3. Standard equation: $(x+2.4)^2+(y-3.1)^2=1.2^2$

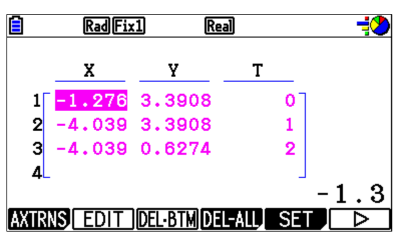
4. Explicit functions: $y=\pm\sqrt{1.2^2-(x+2.4)^2}+3.1$

5. The domain is not necessary, however it is [-3.5, -1.2].

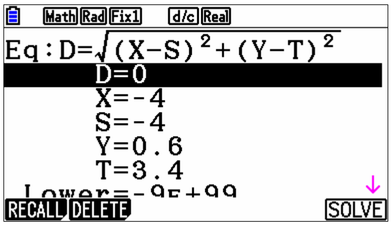
6. Center (-4.0, 3.4) Point 1 (-4.0, 0.6) Point 2 (-1.3, 3.4)

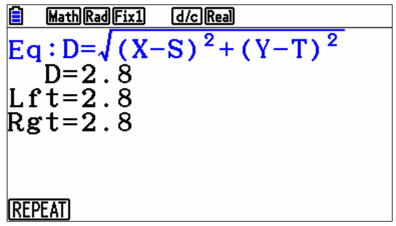
   [Hint: To more accurately place the entering circle, draw the top circle for reference points]

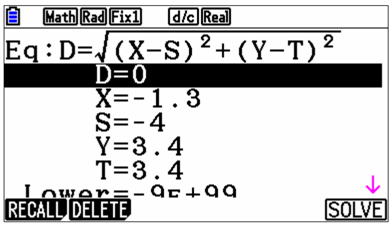


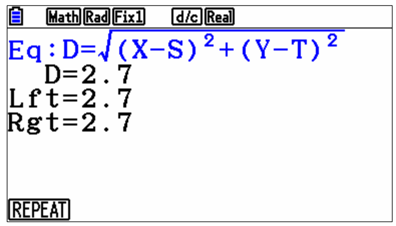


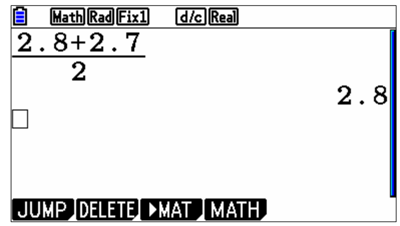
7. Radius = 2.8











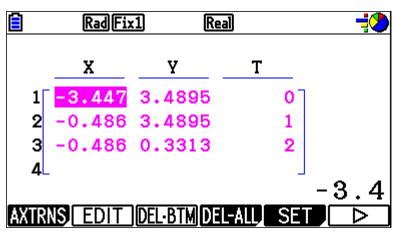
8. Standard equation: $(x+4.0)^2 + (y-3.4)^2 = 2.8^2$

9. Explicit functions: $y = \pm\sqrt{2.8^2 - (x+4.0)^2} + 3.4$

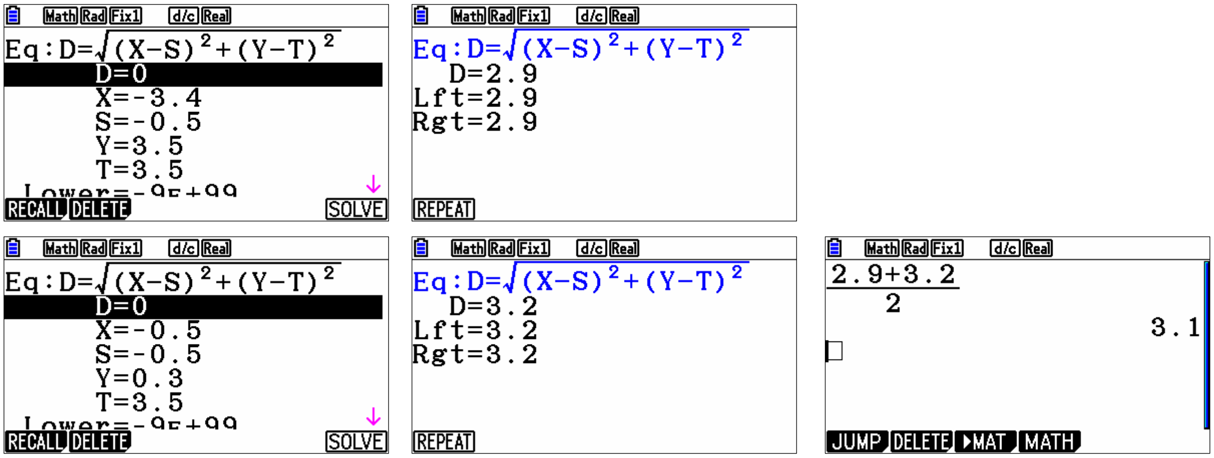
10. The domain used to get a better fit was [-3.2, -1.2].

11. Center (-0.5, 3.5)   Point 1 (-3.4, 3.5)   Point 2 (-0.5, 0.3)

[Hint: To more accurately place the exiting circle, draw the top circle for reference points.]



12. Radius = 3.1

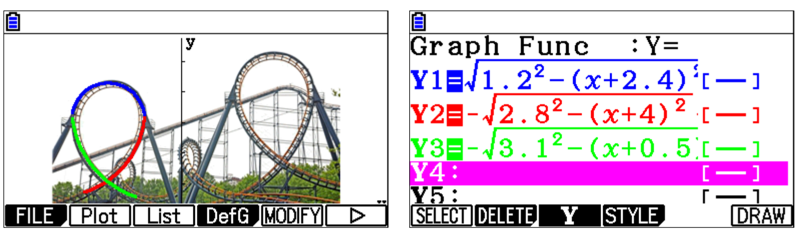


13. Standard equation: $(x+0.5)^2+(y-3.5)^2=3.1^2$

14. Explicit functions: $y=\pm\sqrt{3.1^2-(x+0.5)^2}+3.5$

15. The domain used to get a better fit was [-3.9, -1.5]

16.



$$Y1=\pm\sqrt{1.2^2-(x+2.4)^2}+3.1$$

$$Y2=\pm\sqrt{2.8^2-(x+4.0)^2}+3.4,\ [-3.2,-1.2]$$

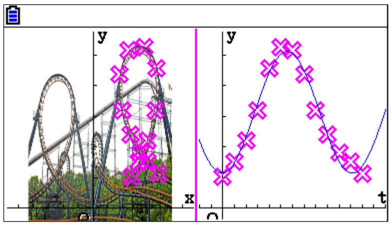
$$Y3=\pm\sqrt{3.1^2-(x+0.5)^2}+3.5,\ [-3.9,-1.5]$$

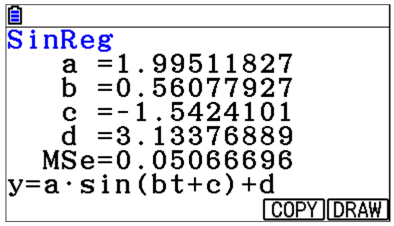
Extension Solutions

Answers will vary, depending on the plotted points.

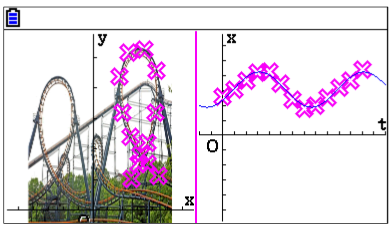
1. Answers may vary, depending on the plotted points.

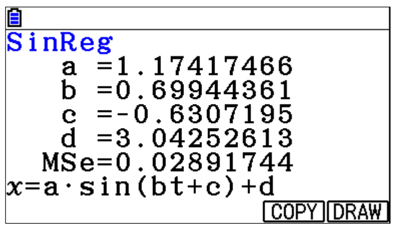
2. One possible solution: $\mathrm{Y} - \mathrm{T} = 2\sin(0.56\mathrm{T} - 1.54) + 3.13$



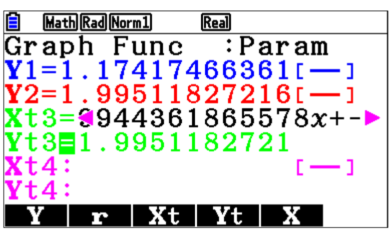


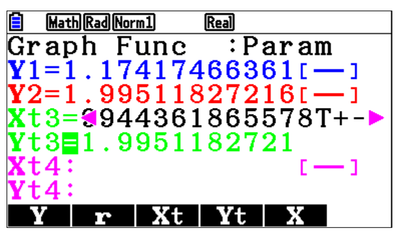
3. One possible solution: $\mathrm{X} - \mathrm{T} = 1.17\sin(0.7\mathrm{T} - 0.63) + 3.04$





4. One possible solution:
$$\mathrm{Xt} = 1.17\sin(0.7\mathrm{T} - 0.63) + 3.04$$
$$\mathrm{Yt} = 2\sin(0.56\mathrm{T} - 1.54) + 3.13$$





5. One possible response:
$$\mathrm{T}\theta\min = 0$$
$$\mathrm{T}\theta\max = 11$$

6.