Exploring The Rationale Behind Canning

Application: Problem Practice

Objective: **Confirm that canning is an efficient method of using a given amount of material to create containers that maximize volume.**

A prototype of the cans that we can see everywhere in our daily lives was first proposed in France and put into actual use by Napoleon in 1804. The tin cans of today were developed in England in 1810, and canning was introduced in Japan in 1871.

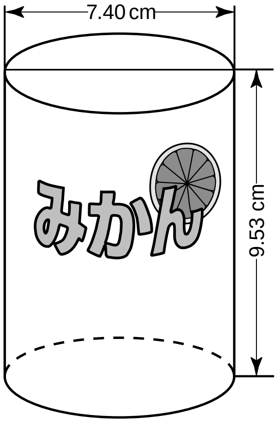
By examining the cylindrical shape that is the most common for today's cans, the student can understand that cans are manufactured under very rational concepts. Canning is superior in terms of efficiency of materials, ease of complete sealing and sterilization, strength, portability, ease of opening, and many other factors. This example deals with the "efficiency of materials" factor. The cylindrical shape was arrived at not only because it is suitable for canning any type of food, but rather as the shape that maximizes the volume that can be obtained from a given amount of steel or other material. Students confirm this fact by doing the applicable math on graphic calculators.

Example of a Classroom Session

(1) Determine the surface area and volume of an actual can.

Measurement of Can A (mandarin oranges, Company H) shown in Figure 1 yields a diameter of 7.40 centimeters, and cylinder height of 9.53 centimeters.
Based on these measured values, calculate the amount of steel used in Can A (surface area of can S_0) and the volume V_0 of the can.
The surface area of the can (a cylinder) can be obtained using the following formula: 2 x (area of top) + (area of side).

$$S_0 = 2\times 3.14\times\left(\frac{7.4}{2}\right)^2 + 2\times 3.14\left(\frac{7.4}{2}\right)^2\times 9.53$$
$$= 307.41228(cm^2)$$



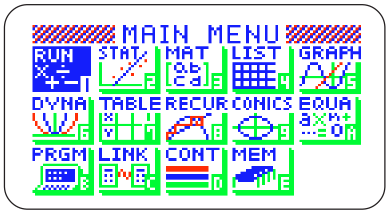
(Figure 1)

Likewise, the volume of cylinder V_0, is calculated using the following formula: (area of bottom) x (height).

$$V_0 = 3.14\times\left(\frac{7.4}{2}\right)^2\times 9.53$$
$$= 409.662298(cm^3)$$

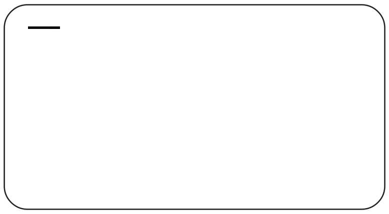
These calculations can be performed by using the RUN menu of the graphic calculator. Since the graphic calculator displays the calculation procedure (input formula) on the screen, there are fewer calculation errors. The following shows how to perform the calculation.

Press MENU. (Figure 2)



(Figure 2)

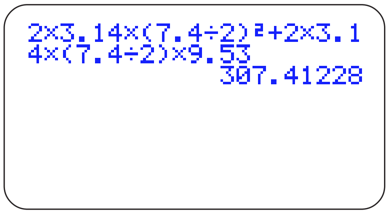
Use the cursor key (◈) to highlight RUN, and then press EXE.
This prepares the graphic calculator for the actual calculation. (Figure 3)



(Figure 3)

2 × 3.14 × (7.4 ÷ 2) x^2 + 2 × 3.14
× (7.4 ÷ 2) × 9.53 EXE

The above obtains the surface area of 307.41228 cm^2. (Figure 4)

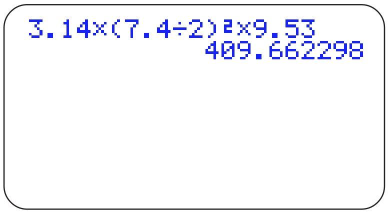


```
2×3.14×(7.4÷2)²+2×3.1
4×(7.4÷2)×9.53
             307.41228
```

(Figure 4)

3.14 × (7.4 ÷ 2) x^2 × 9.53

The above obtains the volume of 409.662298 cm^3. (Figure 5)



```
3.14×(7.4÷2)²×9.53
            409.662298
```

(Figure 5)

(2) Determine the maximum possible volume

Next, determine the maximum possible volume of the can (cylinder) when you have material with a surface area of 307.41228 cm^2, which was obtained from classroom example (1).

If the radius of the bottom of the can (cylinder) is *x* (centimeters), the height of the cylinder is *h* (centimeters), the surfaces area is S_0 (307.41228 cm^2, fixed), and the volume is *y* (cm^3):

$$S_0 = 2\pi x^2 + 2\pi x h \quad \text{..................} \ (1)$$
$$y = 2\pi x^2 h \quad \text{..................} \ (2)$$

This becomes:

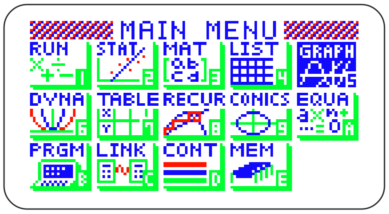
$$h = \frac{S_0 - 2\pi x^2}{2\pi x^2} \quad \text{..................} \ (1)'$$

Substituting (1)’ in equation (2) yields:

$$\begin{aligned} y &= \pi x^2 \times \frac{S_0 - 2\pi x^2}{2\pi x} \\ &= \frac{S_0}{2} x - \pi x^3 \\ y &= 153.70614x - 3.14x^2 \ \text{.....} \ (3) \end{aligned}$$

If the material (area) is fixed and the value of radiu *x* is known, the volume of the can *y* can be obtained using formula (3). This means the graph function of the graphic calculator can be used to obtain the maximum value of *y*. The procedure is shown below.

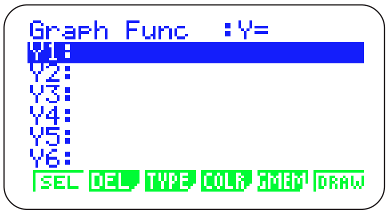
Press MENU.



(Figure 6)

Use the cursor keys (◇) to highlight GRAPH. (Figure 6)

Press EXE. (Figure 7)

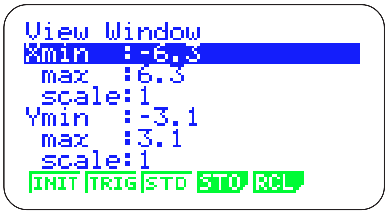


```
Graph Func   :Y=
Y1:
Y2:
Y3:
Y4:
Y5:
Y6:
```

(Figure 7)

Adjust the graph scale (x and y-ax-s range and scale values) before drawing the graph.

Press SHIFT F3 (V-WIN) to display the screen for setting rectangular coordinates. (Figure 8)



```
View Window
Xmin  :-6.3
 max  :6.3
 scale:1
Ymin  :-3.1
 max  :3.1
 scale:1
```

(Figure 8)

Use the cursor up and down keys (▲ ▼) to highlight items and change the values as necessary.

The following are the required settings for this example:

Highlight Xmin (minimum value of x-axis), and press 0 EXE.

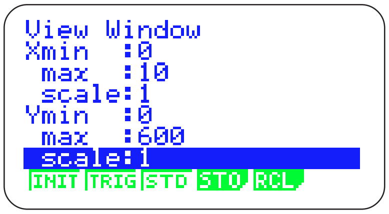
Highlight max (maximum value of x-axis), and press 10 EXE.

Highlight scale (scale of x-axis), and press 1 EXE.

Highlight Ymin (minimum value of y-axis), and press 0 EXE.

Highlight max (maximum value of y-axis), and press 600 EXE.

Highlight scale (scale of y-axis), and press 1 EXE. (Figure 9)

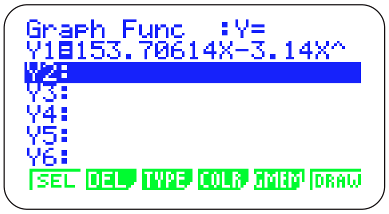


```
View Window
Xmin  :0
 max  :10
 scale:1
Ymin  :0
 max  :600
 scale:1
```

(Figure 9)

Press EXE again to return to the display shown in Figure 7.

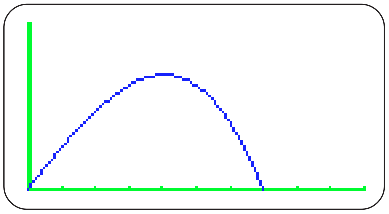
Press 153.70614 X,θ,T − 3.14 X,θ,T ^ 3 EXE to input the formula, $y = 153.70614x - 3.14x^3$. (Figure 10)



```
Graph Func  :Y=
Y1■153.70614X-3.14X^
Y2:
Y3:
Y4:
Y5:
Y6:
```

(Figure 10)

Press F6 (DRAW) or EXE to draw the graph of $y = 153.70614x - 3.14x^3$. (Figure 11)



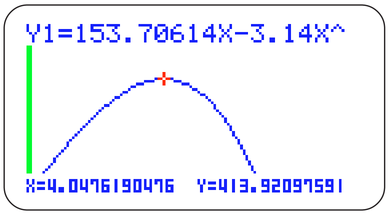
(Figure 11)

Determine the maximum value of y from Figure 11. The trace function helps to determine the approximate maximum value.

Press SHIFT F1 (TRACE), and use the cursor left and right keys (◀ ▶) to move the trace cursor (+) on the graph.

As you move the trace cursor, the coordinates at its current position are shown at the bottom of the screen.

The Screen shows the maximum value of $y = 413.92097591$ when $x = 4.0476190476$. (Figure 12)



```
Y1=153.70614X-3.14X^
X=4.0476190476  Y=413.92097591
```

(Figure 12)

The graph analysis function can also be used to instantly provide greater precision to the maximum value of y shown on the display in Figure 11.

Press SHIFT F5 (G-SLV) F2 (MAX) to obtain the maximum value of $y = 413.92352897$ when $x = 4.0394306932$.

(3) Compare the maximum possible volume to the actual volume of the can.

Compare actual can volume V_0 (409.662298 cm^3), and the maximum possible volume V_{max} (413.92352897 cm^3) that can be obtained from the fixed amount of material (surface area 307.41228 cm^2) derived from classroom examples 1 and 2.

$$h = \frac{V_0}{V_{max}} \times 100 = \frac{409.6622980}{413.92352897}$$

$$= 98.97052702\ (\%)$$

This indicates a material efficiency of 98.97052702%, which means that Can A is designed to use the given amount of material in a very efficient manner.

Examination of other cans also produces numerous other examples of efficient design. Drum B (Company S), for example, has a radius of 57.8 centimeters and material efficiency of 97%. Coffee can C (Company M) has a radius of 15.6 centimeters, a height of 17.3 centimeters, and material efficiency of 99.8%.

The above example uses the graphic calculator to perform the calculations required to fully confirm the rationale behind cans seen we use in the real world.

< Note 1 >

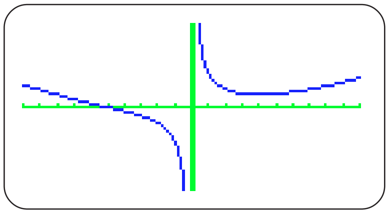
For example, let's consider the method for determining the minimum material (surface area) if Can A volume, V_0, which is fixed at 409.662298 cm^3.

When the surface area is y (cm^2) and the radius is x (cm):

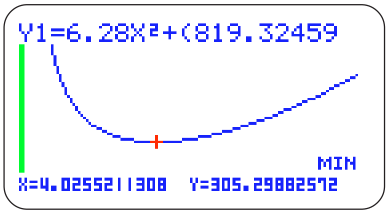
$$y = 2\pi x^2 \times \frac{2V_0}{x}$$

$$= 6.28x^2 + \frac{819.324596}{x}$$

Now we can use the graphic calculator's graph function to draw the graphs shown in Figures 13 and 14. The trace function is used again to obtain the minimum value of y (305.29882572 = Smin) when x = 4.0255211308. Actual Can A surface area S is 307.41228 cm^2. Using a minimum value (Smin) of 100, the actual material (surface area S_0) has a value of 100.6922576. This means that the design of Can A is very efficient.



(Figure 13)



Y1=6.28X²+(819.32459

MIN

X=4.0255211308 Y=305.29882572

(Figure 14)

Reference: Canning Handbook (1995), Canners Association of Japan
Hagoromo Foods Home Page (http://hagoromofoods.co.jp)