GRAPH TABLE

Third Degree

Roots of Equations
Graphing Cubics
Tables
Substitution

Standards: Problem Solving, Communication, Reasoning, Connections, Algebra, and Functions

Materials: *CFX-9850G or CFX-9850Ga PLUS*

Calculator Menus: GRAPH and TABLE

Sometimes graphing an equation holds the key to solving it. Sometimes, however, finding a solution by graphing will lead to a technological anomaly. The activity "A Sine of the Times" provides one example of this when sin(62X) is graphed.

Another situation can be investigated with $Y = X^3 - 2X + 1$. Enter the equation in the TABLE Menu. Set the range to go from -5 to 5 with increments of 1. Press F6 (TABL) to view the table of values shown in Figure 1. Use the arrow keys to scroll up and down the table.

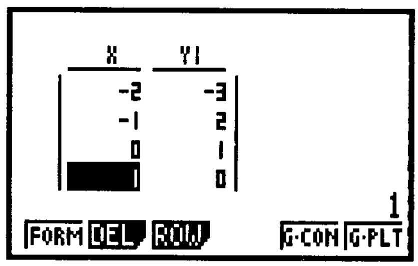

Figure 1

The information in Figure 1 shows there is a root between X = -2 and X = -1 because of the sign change for the Y values. Furthermore, it appears as if there is a second root when X = 1. Since this is a third degree equation, the possibility for 3 roots exists. Could it be that X = 1 is a double root? You could factor $X^3 - 2X + 1$ and determine the answer to the question but that would not be an easy thing to do since the factors are $(X-1)(X-\frac{\pm\sqrt{5}-1}{2})$.

Graphing might prove helpful since factoring would be difficult without technology.

Enter the GRAPH Menu. Before graphing the equation $Y = X^3 - 2X + 1$ set the V-Window to the STD setting (X and Y values range from -10 to 10). Graph the equation.

The graph appears to support the idea from the TABLE Menu that X = 1 is a double root as shown in Figure 2.

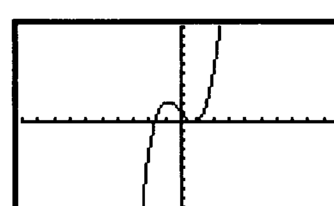

Figure 2

Entering an Equation in the GRAPH Menu

In the GRAPH Menu:
The equation must be solved for Y.
Enter the equation (Y is already provided)
Press EXE key to store equation.
For example:
To enter $Y = X^3 - 2X + 1$,
Enter the GRAPH Menu
Y1 = (is already there)
Press X
^ key
3
-
2
X
+
1
EXE key
Press F6 (DRAW) to graph the equation.

Deleting an Equation in the GRAPH Menu

Highlight the equation
Press F2 (DEL)
F1 (YES)

Setting the Range in the TABLE Menu

Press F5 (RANG)
Enter value for Start
Press EXE
Enter value for End
Press EXE
Enter value for Pitch
Press EXE
EXIT key
note: Pitch is increment

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Setting the V-Window to the Standard Setting (STD)

Press SHIFT key
F3 (V-Window)
F3 (STD)
EXIT key

Resizing the Graph Window

Press Shift key
F3 (V-WIN)
Highlight Xmin, Xmax , Ymin, or Ymax.
Enter desired value
EXE key
Repeat steps to alter min or max values for X or Y.

Zooming In Using a Box

With a graph on the screen
Press F2 (Zoom)
F1 (BOX)
Move cursor below and left of the region to be zoomed.
EXE key
Move cursor above and right of the region to be zoomed.
EXE key

Finding Intersections of Graphs

In the GRAPH Menu
With graphs on the screen:
Press F5 (G-Solv)
F5 (ISCT)
The cursor will appear on a graph. To select a graph:
Press EXE key
Use the arrow keys to move the cursor to a graph to select.
Press EXE key

Reset the V-Window to the INIT. Is X = 1 a double root? Why or why not? **A.**

Another *CFX-9850G* function can help here. Enter the Zoom mode of the calculator (F2). Create a box around the region in question. Do you still agree with your answer in **A**? Why or why not? **B.**

Trace the curve and determine the X value when Y is zero. Your trace may not yield a value where y is zero. You might get something like (0.61587301587, 1.85433938E-3). Moving one step to the right yields (0.62222222222, -3.5445816E-3) indicating the curve has passed over the X axis and thus, a root is present.

But, what is the X value that makes Y = 0? Is this close enough? **C.**

Suppose this is not close enough. The value for X can be substituted into the original equation $Y = X^3 - 2X + 1$ which will give a value close to Y=0. Doing the substitution yields Y = -3.5445816E-3. How can that be? **D.**

Continue the trace and list the other root. **E.**

The Casio *CFX-9850G* provides an alternate way to determine the roots. Return to the Graph Func window and enter a second equation, Y = 0. Graph again and use G-Solv (F5) to find the intersection (F5) between the horizontal line, Y=0, and the curve. You now get (0.61803398875, 0) and (1, 0). Resetting the V-Window to the INIT setting will allow the G-Solve to show all 3 roots, the third one being (-1.6180339887, 0).

List another third degree equation that appears to have only two roots when the table function is used but in reality has three. List the 3 roots to your equation and describe how you found, or created your example. **F.**

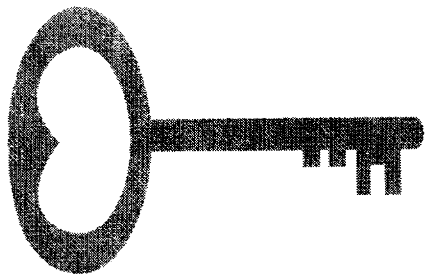
Another interesting question comes out of a third degree equation. Consider the general form of the equation $Y = RX^3 + SX^2 + TX + U$ where R, S, T, and U are real numbers. Without loss of generality, the equation can be divided through by R leaving $Y = X^3 + AX^2 + BX + C$ where A, B, and C are real numbers. Three distinctly different groups of curves will be generated by $Y = X^3 + AX^2 + BX + C$. There is a relation between A and B that tells which of the three curves will be generated. Substituting different values into the equation, graphing, and investigating the resultant tables will provide enough information to answer the question.

Set the V-Window at STD and graph the following 3 equations:

$$Y = X^3 + 2X^2 + 5X + 6$$
$$Y = X^3 + 4X^2 + 5X + 6$$
$$Y = X^3 + 5X^2 + 5X + 6$$

What is the relation between A and B that indicates whether the curve will have an inflection point, a "flat" spot (not really flat which can be seen with ZOOM), or a "backwards, sideways S"? **G.**

Solution Key



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A. Answers will vary. The calculation is an approximation limited to the setting used. In this case, both values are rounded.

B. Answers will vary. (0.99682539682, -3.1444008E-3)

C.- F. Answers will vary.

G. $A^2 < 3B$ gives inflection; $A^2 = 3B$ gives a "flat" spot (if A^2 is close to 3B it works); $A^2 > 3B$ gives a "backwards, sideways S".