CLEMSON ALGEBRA PROJECT
UNIT 10: RATIONAL EXPRESSIONS AND FUNCTIONS

PROBLEM 1: LIGHT INTENSITY

Light intensity is important in many aspects of our lives. For example, stairwells in public buildings, dark city streets, and classrooms must be properly illuminated. Sailors rely on the illumination from lighthouses in coastal regions. This investigation simulates determining the level of light intensity from a single point source such as a lighthouse or a lamp in a stairwell.

A. Using a light source, a light probe, and a data collector, gather data on the intensity of the light emanating from the light source at several distances, using distance as the independent variable.

B. Construct a scatterplot of the data. Describe the shape of the graph.

C. Are there any asymptotes? If so, estimate the value of these asymptotes. Explain this asymptotic behavior.

D. Calculate a mathematical model that describes the data. How well does the model fit the data?

E. Use your model to predict the light intensity at 6 feet from the light source and at 10 feet from the light source.

F. In theory, light intensity conforms to the model $I = \frac{k}{d^2}$, where I is the intensity of the light, k is a constant, and d is the distance from the light source. How closely does your mathematical model match this model? Explain any differences.

MATERIALS

Light Source (Flashlight without lens, Trouble Light, or Lamp without shade.)

Tape measure

Meter Stick (Optional)

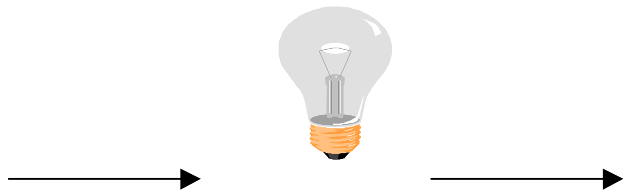
Casio EA-100 Data Collector

Light Probe

Casio CFX-9850Ga Plus or ALGEBRA FX2.0 Graphing Calculator

EXTENSION

Suppose the point source emits light equally in all directions and is placed at the midpoint of the tape measure that you placed on the floor. Describe the graph that you predict would be generated by the data collected if measures were taken from both sides of the light source. See diagram below. If possible, conduct the experiment to verify your answer.



ONE SOLUTION TO PROBLEM 1: LIGHT INTENSITY

A. Using a light source, a light probe, and a data collector, gather data on the intensity of the light emanating from the light source at several distances, using distance as the independent variable.

The following procedures can be used to gather the data.

1. One student will hold the light source, one student will collect data using the EA-100, and a third student will record the distance form the light source and the light intensity measurements from the data collector.
2. Open the tape measure and place it on the floor.
3. Insert the light probe into the channel 1 port on the data collector.
4. Turn on the data collector. "Rec Time" should be blinking and DONE should be displayed on the left side of the viewing screen.
5. Press [MODE] . This puts the EA-100 in multimeter mode, allowing you to read data when you wish.
6. Collect your data. To do so, point the front end of the light probe at the light source. Get close to the light source until -Hi- is visible in the viewing screen. Move slowly away from the light source until a numerical reading is visible in the viewing screen. Record this number and the distance from the light source. Move slowly away from the light source, **keeping the light probe at the same height**. It may be necessary to stabilize the light probe by holding it tightly against a meter stick in order to maintain the same height. You may actually use masking tape to affix the light probe to the meter stick to maintain the light probe at the same height. Make several recordings of the distance from the light source and the light intensity.

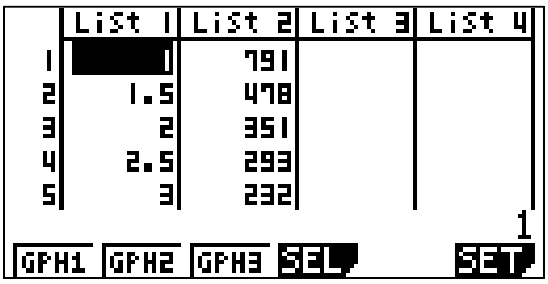
The solution presented here uses data from a previous experiment. The data, and thus the actual student answers, will vary from those shown below.

Distance from Light Source in feet	Light Intensity
1	791
1.5	478
2	351
2.5	293
3	232
3.5	192
4	157

B. Construct a scatterplot of the data. Describe the shape of the graph.

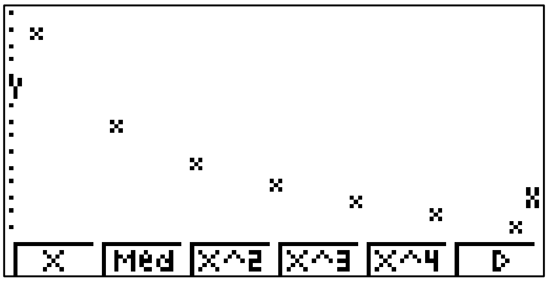
From the MAIN MENU, call up “Statistics.” Then,

- Clear the data in List 1 if necessary by pressing F6 , F4 , and F1 . Use the right cursor arrow to move the cursor to List 2. To clear the data in List 2, again press F4 and F1 .
- Press SHIFT MENU to check the set up. “Stat Wind “should be set on Auto. If it isn't, press F1 . Return to the lists by pressing EXIT .
- Enter the data from the table into Lists 1 and 2 pressing EXE after each entry. See below for the beginning of the lists.



We're now ready to construct the scatterplot. If necessary, press F6 so that "GRAPH" is listed as a function option on the screen.

- Press F1 to access the graph option.
- Check the graph SET UP by pressing F6 .
- Make sure "Statgraph1" is listed. Then use the down arrow and highlight "Graph Type." Press F1 for scatter plot. Make sure that the Xlist is List 1 the Ylist is List 2, and the frequency is 1. Choose other options as you prefer. Press EXIT to return to the lists.
- To graph the scatterplot, press F1 for graph 1. See below.



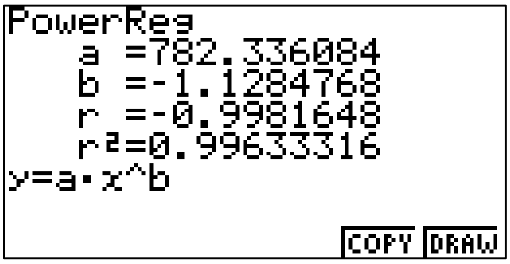
The graph shows the light intensity decreasing, but the plot is not linear. It appears that the light intensity is decreasing, but not at a consistent rate. It decreases at a slower rate as the distance from the light source becomes greater.

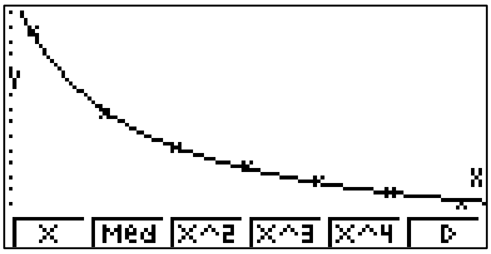
C. Are there any asymptotes? If so, estimate the value of these asymptotes. Explain this asymptotic behavior.

There do appear to be asymptotes, including a vertical asymptote at $x = 0$ and a horizontal asymptote at $y = 0$. The vertical asymptote occurs because the light intensity increases at ever increasing rates as we get closer and closer to the source. The horizontal asymptote occurs because as we get farther and farther away from the light source, the intensity of the light approaches, but theoretically at least, does not quite reach 0 at ever decreasing rates.

D. Calculate a mathematical model that describes the data. How well does the model fit the data?

- From the screen displaying the scatter plot, press [F6] for more options.
- Because light intensity is modeled by an inverse square law, press [F3] for power regression. See below left.
- Copy this function rule into Y1 so that you can access the function later. To do so, press [F5] . To store the equation in the high lighted position press [EXE] .
- To superimpose the graph of the regression equation over the scatterplot press [F6] . See below right.





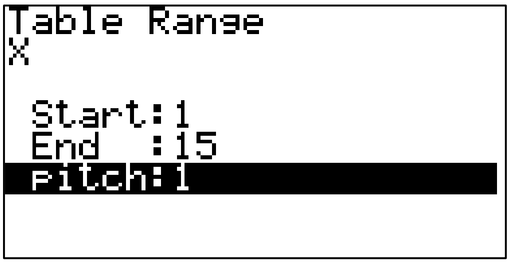
The regression equation obtained is $y = 782.3 * x^{-1.128}$. If the model from the data had matched the theory, the value for b would have been –2. Students may experiment with other regression models to find a regression equation that fits better. By saving the equations, students may change color and superimpose several models over the scatterplot simultaneously to help determine which model best fits the data. However, caution them about searching for models for which they do not have some rationale. Without some underlying theory, complex models may fit the sample data, but probably cannot be generalized. Theoretically, an inverse squared function should best fit the data.

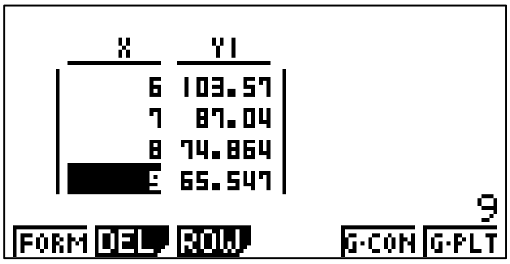
E. Use your model to predict the light intensity at 6 feet from the light source and at 10 feet from the light source.

From the MAIN MENU, select "TABLE."

- If you have completed the steps listed above, the regression equation is already stored in Y1. Delete or de-select any other functions.
- To set the Range for the table, press F5 . Start at 1 and end at 15 with a pitch of 1, remembering to press EXE after each entry. See below left.
- Press EXIT to return to the previous screen and F6 to see the table.

Use the arrow keys to find the intensity at distances of 6 and 10 feet. Your answers should be 103.57 and 58.199, respectively. See below right for the solution when the distance is 6.





F. In theory, light intensity conforms to the model $I = \frac{k}{d^2}$**, where *I* is the intensity of the light, *k* is a constant and *d* is the distance from the light source. How closely does your mathematical model match this model? Explain any differences.**

The experimental model is different from the theoretical model as mentioned above. Several sources of error in the experimental model are possible. Students may make errors in determining the distance from the light source. More importantly, background lighting will likely cause errors. Also, limitations of the measuring devices themselves can affect the measurements.