

CLEMSON ALGEBRA PROJECT UNIT 13: TRANSFORMATIONS

PROBLEM 1: FREE-FALL TRANSFORMATIONS

Suppose several objects are dropped at two-second intervals from a height of 1,450 feet, the height of the top of the roof of the Sears Tower in Chicago.

- Let x represent the number of seconds elapsed since the first object is dropped.
- Let B represent the number of seconds from the start when a particular object is dropped. For successive objects, B will be 0, 2, 4, 6, and so on.
- Let y represent the height of an object at any moment in time.
- Then the height of any of the objects is given by the formula

$$y = -16(x - B)^2 + 1450 .$$

In the formula, -16 is the result of the pull of gravity, which causes the objects to accelerate in a downward direction at 32 feet per second each second, and 1450 shows the initial height, the height from which the objects are dropped.

- A. Explore the effects that the different values of B have on the graph. Interpret the changes in terms of the real world.
- B. Focus on the first object, the one for which B is 0. First determine and explain the meanings of the x and y intercepts. Then, explore the effects of changing the maximum height from 1,450 to other values in increments of 50 feet.
- C. Again focus on the first object. This time explore the effects on the graph if the gravitational constant is changed from -16 . Consider the changes that would occur in places where gravity would be both more and less powerful.

MATERIALS

Casio CFX-9850Ga PLUS or ALGEBRA FX2.0 Graphing Calculator

EXTENSIONS

Explore the effects of changes of A , B , and C on equations in the form $y = Ax^2 + Bx + C$.

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ONE SOLUTION TO PROBLEM 1: FREE-FALL TRANSFORMATIONS

A. Explore the effects that the different values of B have on the graph. Interpret the changes in terms of the real world.

This solution uses the dynamic graphing features of the calculators. To be in position to make changes to all of the parameters, we will use the form $y = A(x - B)^2 + C$. (NOTE: For students new to this, you may find it easier to use the form $y = -16(x - B)^2 + 1450$. The form used here was selected for greater flexibility.)

- x From the MAIN MENU, choose “Dynamic.”
- x Either de-select any functions that are there with **F1** or delete them with **F2** followed by **F1**.
- x We'll first set the window. Press **SHIFT** **F3** to access the window. Recall that x represents the number of seconds since the first object is dropped, and y represents the height of the object, which is released from 1450 feet. A possible window is shown below left. To return to the primary “Dynamic” screen, press **EXIT** or **EXE**.
- x If necessary, press **F3** for type and select **F1** for $Y =$. Type in the right side of the function and press **EXE**. Your screen should look like the one below right.

```
View Window
Xmin : 0
max : 20
scale: 2
Ymin : 0
max : 1500
scale: 100
INIT TRIG STD STO RCL
```

```
Dynamic Func: Y=
Y1: A(X-B)^2+C
Y2:
Y3:
Y4:
Y5:
Y6:
SEL DEL TYPE VAR B-IN RCL
```

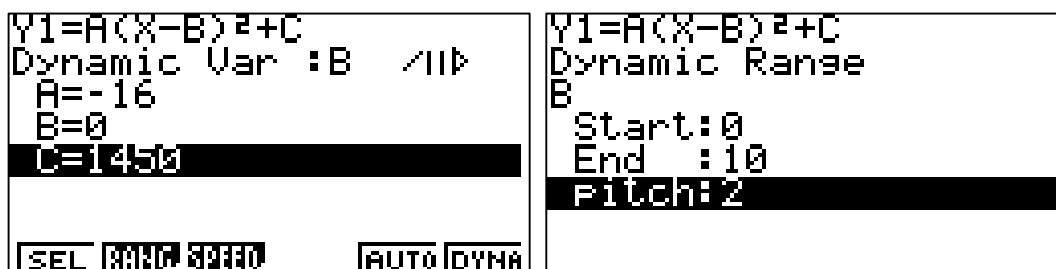
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We now need to tell the calculator values for the parameters we want fixed and a range of values for the variable we want to be dynamic.

- x Choose **F4** for variables.
- x Type in -16 for A. Press **EXE**.
- x Press **F1** to select B as the dynamic variable. Type in 0 for it. Press **EXE**.
- x Type in 1450 for C and press **EXE**. See below left.

With these values set, we are now ready to set the RANGE for our dynamic variable.

- x Press **F2** for RANGE. Type in the appropriate values, pressing X(EXE) after each. See below right.

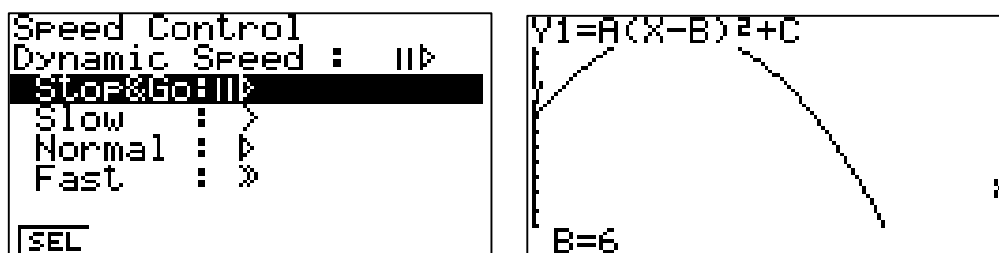


We also wish to control the speed of the graph. Pressing **EXE** after the RANGE has been set returns you to the VARIABLES screen shown above left. From this screen,

- x Press **F3** for SPEED.
- x Select the option you want by highlighting it and pressing **F1**. The “Stop&Go” option allows you to move the graph from one value to another by pressing **EXE**. This option is what has been chosen here. See below left.

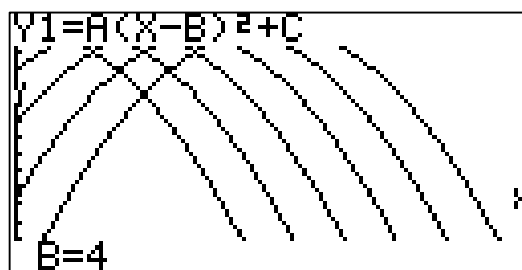
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- x After the Speed Control has been set, pressing **[EXE]** returns you to the screen shown above left. To begin viewing the dynamic graph, press **[F6]** for DYNAMIC. This will take a few seconds – your calculator is doing quite a bit of work! Pressing **[EXE]** from the screen will adjust the values of B according to the RANGE you have set. One such screen is shown below right.



Note a couple of features about the graph. First of all, you may wish to reset the window so that the maximum y -value is significantly higher. Also, since in the scenario the objects were being dropped, to interpret the graphs, we must recognize that the increasing component of the graph is not relevant here.

When you wish to stop looking at the graph, press **[AC/ON]**. The calculator also gives you the capability of viewing the graphs with the different values of B simultaneously. From the primary screen in the dynamic graphing mode, press **[SHIFT]** **[MENU]** for SET UP. Then set the LOCUS to ON. With this, the dynamic graph screen looks like the one below. Although it is not apparent from the picture, as you press **[EXE]** to move through the different values for B , the calculator highlights (in color on the 9850Ga Plus) the particular graph being displayed.



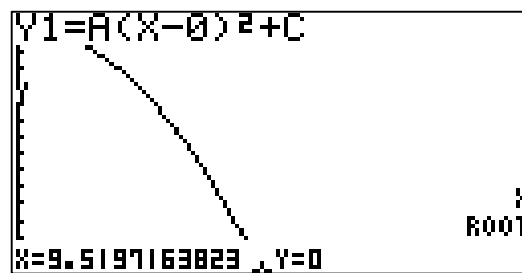
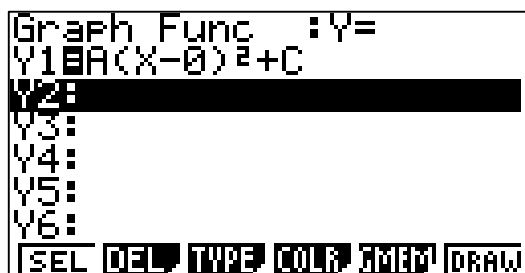
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The conclusion that students should draw from this part of the exploration is that as B increases, the graph shifts to the right. In more formal terms, the graph of $f(x - B)$ translates the graph of $f(x)$ to the right B units. In terms of the real world, if x represents time, then so too must B . Furthermore, whatever relationship exists between x and y remains the same except that it happens B units of time (in our case seconds) later. That is, as B is increased by 2 seconds for our function, the graph shifts 2 units (seconds) to the right. As B is decreased by 2 seconds, the graph shifts 2 units (seconds) to the left.

B. Focus on the first object, the one for which B is 0. First determine and explain the meanings of the x and y intercepts. Then, explore the effects of changing the maximum height from 1,450 to other values in increments of 50 feet.

To explore the x and y intercepts, we will use the GRAPH menu.

- x From the MAIN MENU, choose the “Graph.” Overtyping B with 0 on Y1. See below left.
- x Press **EXE** or **F6** to draw the graph.
- x Press **F5** to access the Graph Solver.
- x Press **F1** to find the root (x -intercept) of the function. This point, which has coordinates (9.52, 0), tells us that the object will hit the ground 9.52 seconds after it is released. See below right.
- x Press **F5** to access the Graph Solver again. This time, press **F4** for the y -intercept. This point, (0, 1450), tells us that at time 0, the instant the object was released, the object was 1450 feet above the ground.



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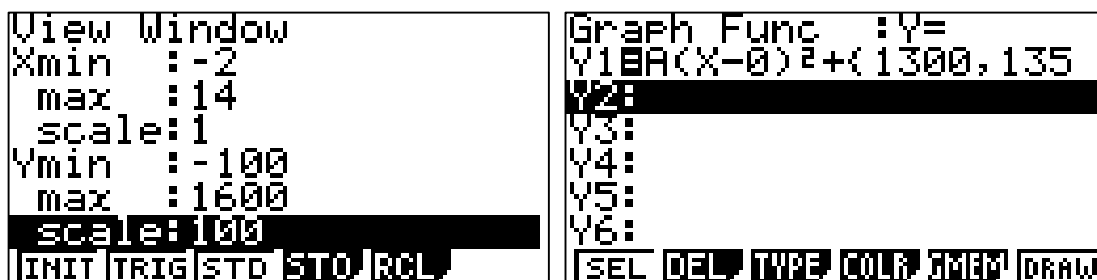
To explore the effects upon the graph of changing the maximum height, we could once again use the dynamic graphing features of the calculator. This time, however, we will take a different approach. Before doing so, we will change the window a little so that we can see higher values, have less blank screen on the right side of the graph, and can clearly see the axes.

- x Press **SHIFT** **F3** to access the window, and type in appropriate values.

See below left for one possible set of values. Press **EXE** when finished.

- x This time, overwrite C in the function with a range of values set in braces.

The function now reads $Y = A(X - 0)^2 + \{1300,1350,1400,1450\}$. Part of the screen is shown below right. Recall that A is still -16 .

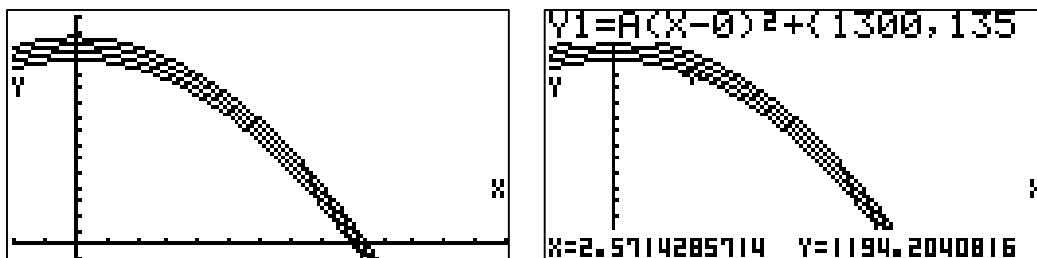


- x From the screen above right, press **F6** to draw the graph. The result is shown below left.

The change in values of C in increments of 50 feet shifts the graph vertically 50 feet. Because the graphs are not linear, the graphs may appear closer together as x increases; this is because as time increases the graphs that started higher are now “moving” faster. However, the vertical differences between the graphs remain 50 feet. To confirm this,

- x Press **F1** to trace.
- x Use the right (and after you start, the left) arrow key to move to any point on the graph. Note the y -value of the point. See below right.
- x Use the up and down arrow keys to move among the curves, maintaining the same x -value. Note how the y -values change in increments of 50.

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- C. Again focus on the first object. This time explore the effects on the graph if the gravitational constant is changed from -16 . Consider the changes that would occur in places where gravity would be both more and less powerful**

The function we are now interested in investigating is $y = Ax^2 + 1450$, since B is 0 and C is 1450. This time we want A to vary. This could be done several ways, including using the dynamic graphing features of the calculator or setting different values for A in braces, as done in part B above. The dynamic feature is the method shown here.

- x From the MAIN MENU, choose “Dynamic.”
- x Press **[SHIFT]** **[MENU]** for SET UP, and turn the Locus Off, if necessary.
- x Type the function in for Y1 as shown below left.
- x Choose **[F4]** for VARIABLE and then **[F2]** for RANGE. (Since A is the only parameter in the equation, it is automatically selected as the dynamic variable.)
- x Select a desired range for A . One possible set of values is shown below right.

```
Dynamic Func:Y=
Y1:AX^2+1450
Y2:
Y3:
Y4:
Y5:
Y6:
[SEL] [DEL] [TYPE] [VAR] [BIN] [RCL]
```

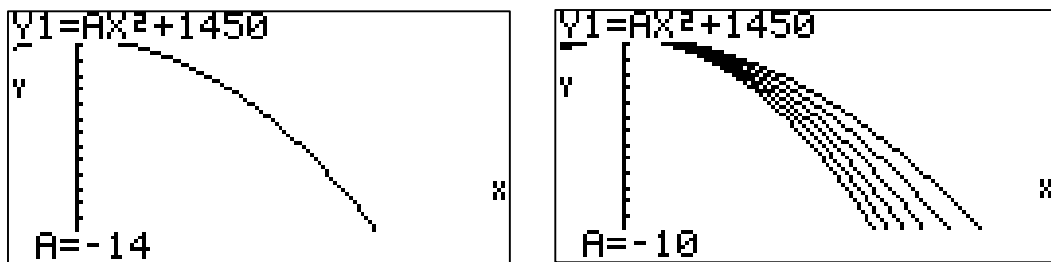
```
Y1=AX^2+1450
Dynamic Range
A
Start: -10
End : -20
Pitch: -2
```

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- x Press **EXIT** after you have entered the values and **F6** for the DYNAMIC graph. Again, be patient; your calculator is doing quite a bit of work!
- x With the graph displayed, press **EXE** to move to the different values for A .

One screen is shown below left. If your window is different, your screen will look different. If desired, all of the graphs can be viewed simultaneously. From the graph:

- x Press **EXIT** three times to return to the primary “Dynamic” screen.
- x Press **SHIFT** **MENU** for SET UP.
- x Turn the Locus on.
- x Press **EXIT** and **F6** to redisplay the graphs. See below right.



As the absolute value of A increases, the object picks up speed more quickly, causing the graph to turn down more quickly. Visually, the greater the absolute value of A , the “skinnier” the graph. Conversely, the smaller the absolute value of A , the “fatter” the graph. In context, as the effect of gravity becomes stronger, the absolute value of A increases and the curve becomes steeper more quickly.