

TRANSFORMATIONS

PROBLEM 2: ABSOLUTE VALUE MOTION

One general form of the absolute value function is $y = A|x - B| + C$. This function can be explored with the MOTION program on your calculator. For this program, x represents the number of seconds since you pressed the trigger on the data collector, and y represents your distance from the motion detector. The purpose of this investigation is to use this program to discover the effects that A , B , and C have on the absolute value function.

A. Explore the effects that changes in C have on the vertex of the graph.

Type in 5 for B .

Type in .5 for A .

Then explore the function with C being first 1, then 2, and finally 3.

B. Explore the effects that changes in B have on the vertex of the graph.

First type in 3, then 4, and finally 5 for B . Each time, do the following:

Type in .5 for A .

Type in 2 for C .

C. Explore the effects that changes in A have on the graph.

Type in 5 for B .

Try A equal first to -.4, then .1, and finally .6.

Type in 3 for C .

D. Summarize the effects A , B , and C have on the graph

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ONE SOLUTION TO PROBLEM 2: ABSOLUTE VALUE MOTION

In this problem, we have shifted from a quadratic function to an absolute value function in the form $y = A|x - B| + C$. The goal of the problem is to help students recognize that A , B , and C have identical effects on this function as they do on the quadratic function.

We assume that students are familiar with the MOTION program, which is described in the unit on Linear Functions. Again for this activity, the class should break into groups of size 3 or 4. Each person should have a responsibility, and these responsibilities should rotate through the group as the problem is explored. As suggested in the unit on Linear Functions, for a group with four people, one person can be the “techie,” assigned to make sure the equipment is hooked up properly and the path for the walker is clear. A second person can be the “trigger person,” giving the commands to the data collector and the calculator. The third person can be the “commander,” telling the “walker” where and when to start, as well as the direction and speed to go; and the fourth person can be the “walker.”

Once the duties for each person have been established,

- x Clear a path about 6 meters long (for walking) and about 2 meters wide.
- x Link the calculator to the data collector.
- x Turn the data collector on.
- x Connect the motion detector to the data collector’s sonic port. Set the motion detector at the end of the walking path at a height that will “hit” the walker.
- x Turn on the calculator and select “Program” from the MAIN MENU.
- x Call up the MOTION program on the calculator.
- x Select option 4, WALK MY EQUATION and press EXE.
- x Then choose option 2, ABSOLUTE VALUE, and press EXE.
- x Although not always stated below, continue to press EXE after you have made a selection in the program.

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A. Explore the effects that changes in C have on the vertex of the graph.

- x WALK MY EQUATION asks that you put in values for A , B , and C , but not in this order. Follow the instructions on the screen to enter the values for B (put in 5), A (put in .5) and C (use 1 the first time). In other words, you are to first try to “walk” the function $y = .5|x - 5| + 1$.
- x After you have typed in the equation, the instructions on the calculator tell you to press **TRIGGER** on the EA-100 (the data collector) when you are ready to begin and **EXE** on the calculator when the data have been collected.

When you have completed this, you will see on the calculator the plot you have walked marked with Xs along with the graph that you were trying to walk. Discuss with your group how you could improve the walk.

- x Press **EXE** when finished to return to the program. Again select ABSOLUTE VALUE. Either try the first part again, or move on to the second, choosing 2 for C instead of 1.

Again after finishing, either try again or move on to the third problem, choosing 3 for C . When they have completed this part of the exercise, students should note that different values of C shift the vertex of the graph vertically. Because it shows vertical displacement, it is measured in the same units as y , in this case meters. As C moves from 1 to 2 and from 2 to 3, the vertex of the graph shifts from (5, 1) to (5, 2) and then from (5, 2) to (5, 3). In terms of the real world, the turn-around point in all cases happens five seconds after starting and at distances of 1, 2, and 3 meters, respectively, from the motion detector.

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B. Explore the effects that changes in B have on the vertex of the graph.

- x The first time you attempt this, type in 3 for B . On the second and third trials, type in 4 and 5 respectively. Each time, do the following.
- x Type in .5 for A .
- x Type in 2 for C .

The first time you are trying to walk $y = .5|x - 3| + 2$. Again follow the directions on the calculator, which tell you to press **TRIGGER** on the data collector to begin collecting data and **EXE** on the calculator when finished. Compare your walk, marked with Xs, to the actual graph. Press **EXE** to continue. Either try again, or move on to the next part, inputting 4 for B and eventually 5 for B .

The goal is to note what happens to the vertex of the absolute function. When B is 3, the turn-around is at (3, 2). In other words, the direction changes 3 seconds after beginning and at a distance of 2 meters away from the motion detector. When B is 4, the turn-around is at (4, 2). In other words it is now 4 seconds after starting and again at a distance of 2 meters away from the motion detector. Finally, when B is 5, the turn-around is at (5, 2), five seconds after starting and still at a distance of 2 meters away from the motion detector.

Students may also note the changes in y -intercepts created by the changes in the values for B . This too is an excellent realization, but not one that is quite so easily interpreted. The y -intercepts of the three functions, using 3, 4, and 5 for B , are, respectively, at (0, 2), (0, 2.5), and (0, 3). These tell the students that their starting points differ according to what B is. They may also note that the speed and direction with which they are to walk does not change, but the length of time in which they walk in a particular direction is also affected.

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C. Explore the effects that changes in A have on the graph.

- x Type in 5 for B .
- x Try A equal first to $-.4$, then $.1$, and finally $.6$.
- x Type in 3 for C .

Again using WALK MY EQUATION on the MOTION program, you are exploring the differences among the equations $y = -.4|x - 5| + 3$, $y = .1|x - 5| + 3$, and $y = .6|x - 5| + 3$. After attempting each walk, press **EXE** to return to the program. Either try again or move on to the next part of the problem.

The first thing students will probably notice is that the negative value for A turns the absolute value function upside down. When A is positive, the graph looks like the “V” shape they may have grown to associate with the absolute value function.

Another item they should discover is that the vertex for all of the graphs is at $(5, 3)$. This tells us that five seconds after they start they should be three meters away from the motion detector, and it is at that point that they should change directions.

The third important point for students to recognize is that A affects the slope of the graph, which tells them the speed at which they need to walk. They may discover this by first calculating the y -intercept, the starting point for their walk. This is an excellent realization, but one which the teacher may have to give help on so that they make the connection to the speed. Because speed is distance covered per time, and y represents distance measured in meters and x represents time measured in seconds, the slope of the graph represents change in distance over change in time. The teacher should lead students toward the connections among A , the slope of the graph, and the speed (as well as direction) at which they need to walk. For the three values of A used here, the walks need to be at 0.4 meters (40 centimeters) per second, starting toward the detector; at .1 meters (10 centimeters) per second starting away from the detector; and at .6 meters (60 centimeters), starting away from the detector, respectively.

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D. Summarize the effects A , B , and C have on the graph of $y = A|x - B| + C$.

From this work, students should discover that the effects that A , B , and C have on the absolute value function are analogous to their effects on the quadratic function. A affects the slope of the graph, which represents the speed and direction of the walk. As the absolute value of A increases, the graph becomes “skinnier.” Positive values for A cause the graph to open upward, and negative values cause it to open downward.

B is measured in the same units as x , in this case seconds. As B increases, the graph shifts to the right, and as B decreases, the graph shifts to the left. For example, changing B from 0 to 2 shifts the graph 2 units (seconds) to the right.

C is measured in the same units as y , in this case meters (or feet, if the units have been changed on the OPTIONS component of the program). As C increases, the graph shifts upward, and as C decreases, the graph shifts downward. For example, changing C from 0 to 2 moves the graph 2 units up.