

## CLEMSON ALGEBRA PROJECT UNIT 14: CONIC SECTIONS

### ***PROBLEM 1: LORAN - LONG-DISTANCE RADIO NAVIGATION***

LORAN, long-distance radio navigation for aircraft and ships, uses synchronized pulses transmitted by widely separated transmitting stations. These pulses travel at the speed of light (186,000 miles per second). The difference in the times of arrival of these pulses at an aircraft or ship is constant on a hyperbola which has transmitting stations located at the foci. Assume that two stations, 300 miles apart, are positioned on a rectangular coordinate system at points  $(-150, 0)$  and  $(150, 0)$  and that a ship is traveling on a path with coordinates  $(x, 75)$ .

- A. Find the  $x$ -coordinate of the position of the ship if the time difference between the pulses from the transmitting stations is 1000 microseconds (0.001 second).
- B. Write the equation of the hyperbola on which the ship is located.

**REFERENCE:** *Precalculus, Third Edition*, by Larson and Hostetler, D.C. Heath and Company, 1993.

### ***MATERIALS***

Casio CFX-9850Ga Plus or ALGEBRA FX2.0 Graphing Calculator

## CONICS

### ONE SOLUTION TO PROBLEM 1: LORAN

- A. Find the  $x$ -coordinate of the position of the ship if the time difference between the pulses from the transmitting stations is 1000 microseconds (0.001 second).**

The difference in the distance from a point on the hyperbola to the foci is constant. In this instance, use the time difference in receipt of the pulses to determine that constant difference.

Time travels at 186,000 miles per second. Because the time difference is 0.001 second, the constant difference must be .001 seconds times 186,000 miles per second, or 186 miles.

Next, we'll use the distance formula to find an expression that describes the distance from each of the foci to the location of the ship. The equation is:

$$\sqrt{(x+150)^2 + 75^2} - \sqrt{(x-150)^2 + 75^2} = 186$$

To solve, this equation, we will use the "Equation" mode on the calculator. From the MAIN MENU, choose "Equation," and then:

- x Press **F3** for the "Solver."
- x Type in the equation, being careful with parentheses – make sure you include parentheses around the entire expressions under the radicals. See below left for the beginning of the equation. (NOTE: Your calculator may show something different for  $X$  at this point.)
- x Press **F6** to solve the equation. See below right.

```
Eq:√((X+150)²+75²)-√
  X=110.2788597
Lft=186
Rgt=186
|REPT
```

```
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  X=110.2788597
Lft=186
Rgt=186
|REPT
```

Our solution is  $X = 110.28$ . This tells us that the ship is located at the point which has coordinates  $(110.28, 75)$ .

## CONICS

### B. Write the equation of the hyperbola on which the ship is located.

The general form for our hyperbola is  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ . The relationship between

$a$  (the distance from the center to a vertex),  $b$  (the distance from the center to the conjugate axis), and  $c$  (the distance from the center to a focus) is  $a^2 + b^2 = c^2$ .

Consequently,  $b^2 = c^2 - a^2$ .

We already know that the foci are at  $(-150, 0)$  and  $(150, 0)$ , giving us a value of 150 for  $c$ . Substituting, we have that  $b^2 = 150^2 - a^2$ .

Making another substitution, the equation for our hyperbola becomes

$\frac{x^2}{a^2} - \frac{y^2}{150^2 - a^2} = 1$ . We can now substitute the  $x$  and  $y$  values of the point we found

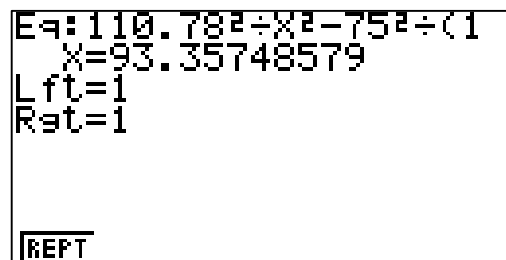
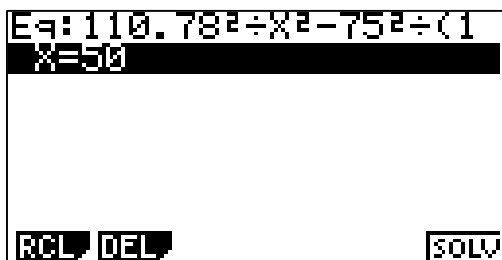
in part A into this equation. This gives us an equation with  $a$  as the only variable,

$$\frac{110.78^2}{a^2} - \frac{75^2}{150^2 - a^2} = 1.$$

Once again, we will use the equation solving capabilities of the calculator.

From the MAIN MENU, call up "Equations." Then,

- x Press **F3** for "Solver."
- x Type in the equation. Be careful with parentheses, and use  $x$  instead of  $a$ .  
Press **EXE**. See below left for part of this.
- x Press **F6** to solve the equation.



## CONICS

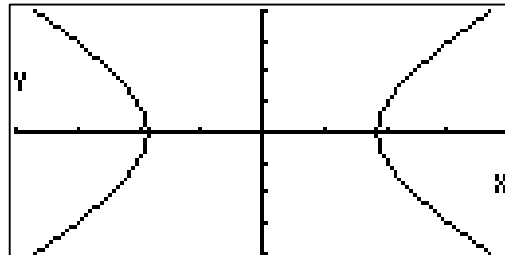
We now have  $a = 93.36$  and  $c = 150$ . From the MAIN MENU, choose “Run,” and find  $\sqrt{150^2 - 93.36^2}$  to find  $b$ . You should get  $b = 117.40$ . Thus our equation is

$$\frac{x^2}{93.36^2} - \frac{y^2}{117.4^2} = 1.$$

To graph this hyperbola, from the MAIN MENU, call up “Conics.” Then,

- x Set the viewing window by pressing **SHIFT** **F3** . Remember to press **EXE** after each entry. One possible window is shown below left. Press **EXIT** when finished.
- x Then, use the down arrow to highlight the horizontal hyperbola and press **EXE** . Type in 93.36 for A, 117.4 for B, 0 for H, and 0 for K, pressing **EXE** after each entry.
- x Press **F6** to draw the graph. See below right.

```
View Window
Xmin  :-200
max   :200
scale:50
Ymin  :-200
max   :200
scale:50
INIT TRIG STD STO RCL
```



- x You can trace points on the hyperbola by pressing **F1** and using the arrow keys.
- x If you press **F5** you will access the “Graph Solver.” Note you can find the foci, the intercepts, and the vertices with this.
- x To draw asymptotes, press **F5** twice.