CLEMSON ALGEBRA PROJECT
UNIT 16: SEQUENCES

PROBLEM 1: GOING INTO DEBT

In order to purchase a used car, you need to borrow $5,000. You decide to pay with a credit card, which, as many credit cards do, charges you 1.5% per month (18% per year) on the unpaid balance. You decide that you can afford to pay the credit card company $100 per month.

A. How much do you owe after 1 year? How much money have you paid?
B. How much do you owe after 3 years? How much money have you paid?
C. Answer questions A and B assuming you pay $200 per month.
D. Answer questions A and B assuming you are charged only .75% per month in interest.
E. Explore similar questions and reflect upon what you have found. Write a short paragraph discussing these results.

MATERIALS

Casio CFX-9850Ga PLUS or ALGEBRA FX2.0 Graphing Calculator

EXTENSIONS

Find out the interest rates charged by different credit card companies, the bank, or other institutions you might use to borrow money. Explore the effects these rates have on the total amount of money you have to pay back.

ONE SOLUTION TO PROBLEM 1: GOING INTO DEBT

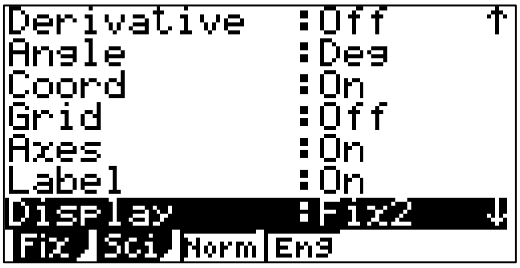
A. How much do you owe after 1 year? How much money have you paid?

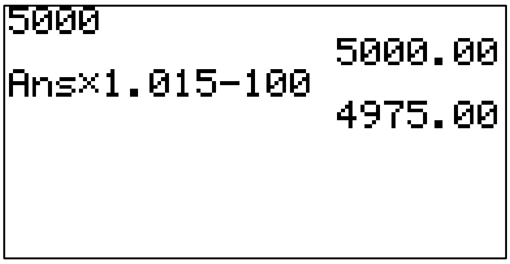
One way to begin this problem is by using the “RUN” mode on the calculator. From the MAIN MENU, choose “Run.” Then,

- Press SHIFT MENU for the SET UP.
- Scroll down to Display.
- Press F1 for Fix, and press F3 for two decimal places. Since we are dealing with money, this will automatically round our results to the nearest cent. See the screen below left.

Now we are ready to work on the problem. Press EXIT to return to the home screen. Then:

- “Seed” the calculator by typing in 5000 and pressing EXE .
- Next month we will owe 101.5% of this value minus the $100 we will pay. Using the automatic answer generated by the calculator, all we need do is multiply by 1.015 and subtract 100. See the screen below right.





This tells us that after one month, although we have paid $100 toward our $5,000 bill, we still owe $4,975. Pressing EXE 11 more times tells us that though we have paid $1,200 over the course of the year, we still owe $4,673.97. In other words, our bill is only $326.03 less than what it was when we began the year!

B. How much do you owe after 3 years? How much money have you paid?

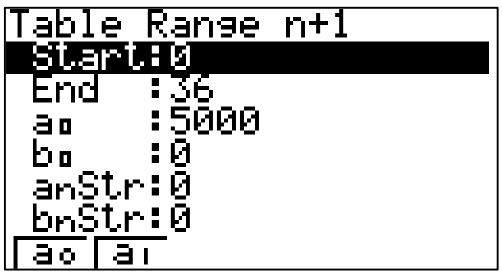
We could solve Part B using the same technique we used for Part A. However, this problem, one in which we use a result to generate a new result (a process called recursion), can be explored more easily with a different technique. To begin, from the MAIN MENU, choose “Recur.”

The problem we are working on is called a first order problem; in other words, we only need one previous result to determine the next value. From the primary “Recursion” screen, press [SHIFT] [MENU] to access the SET UP. Make sure that the first option, Σ Display, is Off. Then,

- Press [F3] for TYPE.
- Press [F2] for a_{n+1}, which tells us that we can generate the (n + 1)st value by knowing the nth value.
- Now press [F4] to access previous terms, [F2] for a_n, and then multiply this by 1.015 and subtract 100. Press [EXE] . The result is shown below left.

We now wish to explore this sequence. One way of doing so is with a table. To set up the range of the table:

- Press [F5] from the screen shown below left.
- We want to start with a_0, since at the beginning no months have gone by. Consequently, if we are looking for the first 36 months (3 years), we can set the Start value at 0 and the End value at 36. We also need a_0, the value of our loan, to be 5000. (Because we are working with only one recursion problem, we can ignore the b values. We are also not looking at convergence or divergence, so we can also ignore the anStr and bnStr values.) The table range values are shown below right.



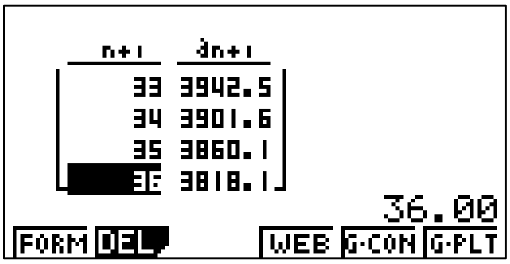
x Press EXIT and F6 to go to the table.

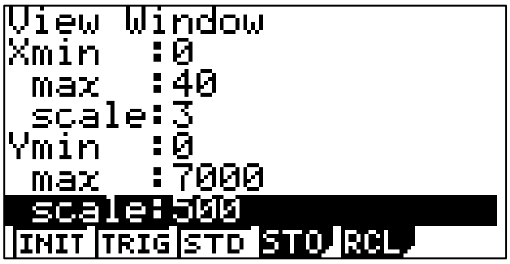
Using the down arrow key to scroll down through the table, you can determine that, although you have paid a total of $3,600 for the $5,000 you borrowed, you still owe $3,818.10 (see below left). In other words, you have only paid back $1,181.90 of the principal!

Another way to look at this is with a graph. Our horizontal axis represents n, the number of payments you have made. The vertical axis, a_n, represents the amount you still owe on the loan.

Before we look at the graph, we need to set our window. From the table,

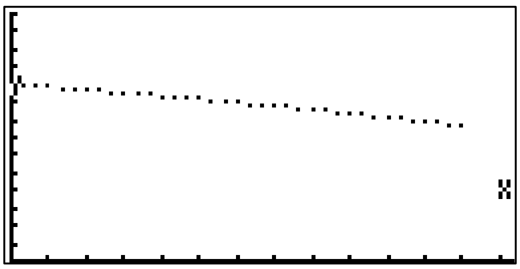
x Press SHIFT F3 to access the View Window. Put in appropriate values (some possible values are shown below right). When you finish entering the values, press EXIT to return to the primary “Recursion” screen.

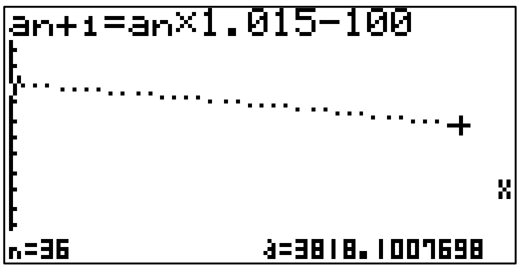




x Press F6 to reenter the table and F6 for a graph plot. See below left.

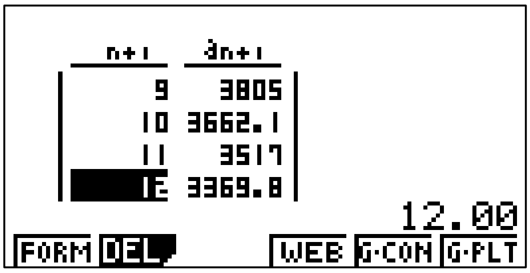
The TRACE feature, accessed by pressing F1 from the graph, can be used to move from one value to another. Below right shows the result when the cursor is moved to the value showing the amount owed after 36 months.

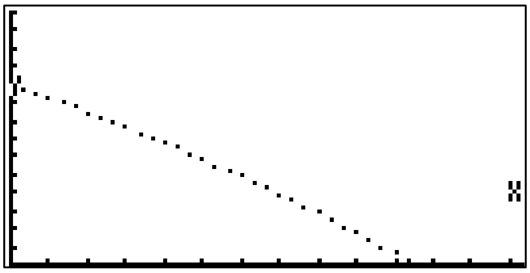




C. Answer questions A and B assuming you pay $200 per month.

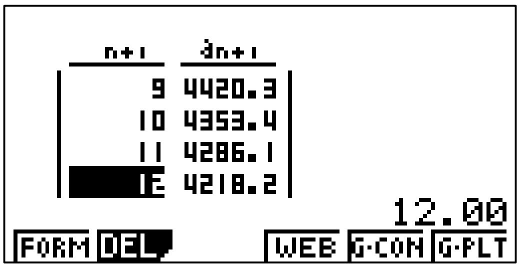
To investigate this problem, we need to change the formula slightly. Press EXIT twice to return to the formula. Instead of subtracting $100, now subtract $200. The table values or viewing window need not be changed. The results tell us that after 12 months of paying $200 per month, we would still owe $3,369.90 (having now paid off $1,630.10 of the principal). After 36 months we would owe -$909.40; in other words, we have more than paid back the loan. The table values around 12 months and the graph are shown below.

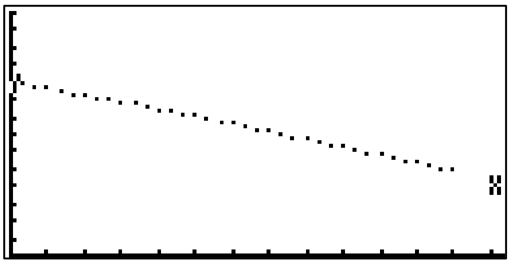




D. Answer questions A and B assuming you pay only .75% per month in interest.

To explore the change the interest rate has on the original problem, multiply a_n by 1.0075 and subtract $100. At 12 months we owe $4,218.20 and at 36 months $2,427.90. Note that after paying $3,600, we've done much better with this interest rate, having paid off $2,572.10 of the original loan as compared to the $1,181.90 we had paid when charged at the 1.5% rate.





E. Explore similar questions and reflect upon what you have found. Write a short paragraph discussing these results.

Results will vary. However, students should note how much difference paying more each month or getting a lower interest rate can make in terms of the balance that remains or in terms of how much is needed to pay off the entire debt.

One point of discussion that may be of interest is the effect that paying with a credit card may have upon the cost of the car. Because major credit card companies charge the sellers a fee for any transaction, car dealers who accept credit cards may not be as willing as others to negotiate on the price of the car.