

## SEQUENCES

### ***PROBLEM 2: THE FIBONACCI SEQUENCE***

The Fibonacci Sequence is one that connects mathematics to nature and to art. Students may be interested in researching this and making a class presentation on it. The sequence itself is a second order sequence; in other words, it uses two previous values to generate the next value. To begin, we need two initial values, which we now identify as 1 and 1. To generate the next value in the sequence, simply add the two previous values. The sequence thus can be represented as  $\{1, 1, 2, 3, 5, 8, \dots\}$ .

- A. Generate the first 25 values for the sequence and graph them.
- B. Find the ratio of the 25<sup>th</sup> term to the 24<sup>th</sup> term, and compare this to earlier ratios in the sequence.
- C. Instead of seeding the sequence with initial values both 1, try different seeds. Then generate the first 25 values for the sequence and graph them.
- D. Again find the ratio of the 25<sup>th</sup> term to the 24<sup>th</sup> term, and compare this with the ratio you found in part B.
- E. The Golden Ratio, the ratio often considered most pleasing to the eye and one that has been used in a great deal of art, is  $\frac{1+\sqrt{5}}{2}$ . Compare your results from parts B and D to this value.

### ***EXTENSIONS***

A wealth of information is available on the Fibonacci Sequence, the Golden Ratio, and the Golden Rectangle. Students interested in art and/or nature may wish to explore these subjects in greater detail and prepare a presentation for the class. In addition to traditional references, the Internet has many sites with an abundance of information and pictures on this topic.

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### **ONE SOLUTION TO PROBLEM 2: THE FIBONACCI SEQUENCE**

#### **A. Generate the first 25 values for the sequence and graph them.**

From the MAIN MENU, choose “Recursion.” Then,

- x Press **F3** for TYPE.
- x Press **F3** for second order recursion.
- x Before entering the formula, press **F4** to be able to access previous terms in the sequence.
- x Put in the formula  $a_{n+2}=a_n + a_{n+1}$ , using the appropriate function keys. Press **EXE**. The result is shown below left.

To set the range,

- x Press **F5** for RANGE.
- x Set the Start at 0, the End value at 24 (the 1<sup>st</sup> term is  $a_0$ ), and both  $a_0$  and  $a_1$  at 1. The values are shown below right. Press **EXIT** when finished.

```

Recursion
an+2=an+an+1
an+2:
-----
SEL: DEL TYPE NAME RANGE TABL
    
```

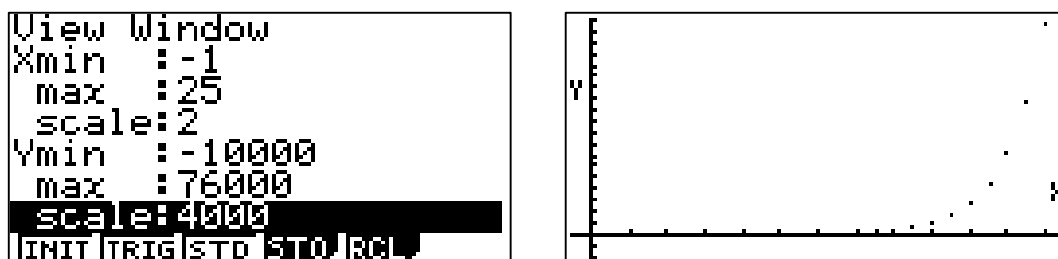
```

Table Range n+2
Start: 0
End : 24
a0 : 1
a1 : 1
b0 : 0
b1 : 0
-----
a0 a1
    
```

After performing the commands above, you should be back at the primary “Recursion” screen. Pressing **F6** will show the table. Scrolling down through the table reveals that  $a_{24}$ , the 25<sup>th</sup> term of the sequence, is 75,025. Before graphing the sequence, we should use this information to set the window.

- x Press **SHIFT F3** to access the window.
- x One set of possible values is shown below left. Press **EXIT** when finished.
- x Press **F6** to reenter the table and then **F5** to see a graph with the points connected, selecting the  $a_n$  option. The graph is shown below right. Notice how steep the graph becomes.

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From the graph, pressing  $\boxed{\text{F1}}$  followed by the right arrow key allows you to TRACE through the values.

**B. Find the ratio of the 25<sup>th</sup> term to the 24<sup>th</sup> term, and compare this to earlier ratios in the sequence.**

The table shows the first 25 numbers in the sequence: {1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46358, 75025}. Although initially students have no indication of why we might be interested in studying the ratio of successive terms, the connections will be made later for them. The ratios of  $a_{n+1}:a_n$  for these terms, rounded to three decimal places, are listed below. (The values were obtained using the “Run” menu.)

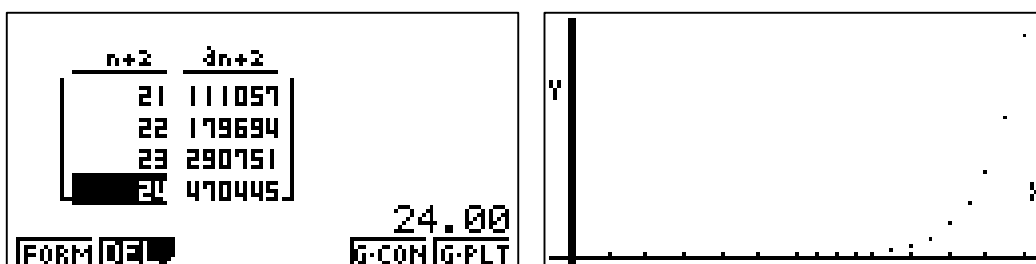
{1, 2, 1.5, 1.667, 1.600, 1.625, 1.615, 1.619, 1.618, 1.618, 1.618, 1.618, ...}

Clearly, the ratios are converging at 1.618.

**C. Instead of seeding the sequence with initial values both 1, try different seeds. Then generate the first 25 values for the sequence and graph them.**

Students will likely choose different seed values. To set different seeds, from the primary “Recursion” screen, call up the Table Range and change the values for  $a_0$  and  $a_1$ . For the example here,  $a_0$  and  $a_1$  were set at  $-3$  and  $12$ , respectively. The last few values are quite different from the original (see below left), but the graph looks very much like the original (see below right). The window for the graph was changed so that the maximum y-value is 500,000.

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- D. Again find the ratio of the 25<sup>th</sup> term to the 24<sup>th</sup> term, and compare this with the ratio you found in part B.**

The ratio of the 25<sup>th</sup> to 24<sup>th</sup> terms, 470445:290751 is, once again, 1.618. Despite the different seeds and the different values for the terms, the end ratio remains the same. Students may wish to explore how quickly the ratio of the terms converges towards 1.618.

- E. The Golden Ratio, the ratio often considered most pleasing to the eye and one that has been used in a great deal of art, is  $\frac{1+\sqrt{5}}{2}$ . Compare your results from parts B and D to this value.**

From the “Run” menu, if necessary change the SET UP so that the display is normal. The values of  $\frac{1+\sqrt{5}}{2}$ , 75025:46368 (the ratio of the 25<sup>th</sup> to 24<sup>th</sup> terms with seeds of 1 and 1), and 470445:290751 (the ratio of the 25<sup>th</sup> to 24<sup>th</sup> terms with seeds of -3 and 12), are shown on the screen below. Note that the values are identical to 9 significant digits. The ratio of consecutive terms of the Fibonacci Sequence converges to  $\frac{1+\sqrt{5}}{2}$ . This value is called the Golden Ratio.

$(1+\sqrt{5})\div 2$	1.618033989
75025÷46368	1.618033989
470445÷290751	1.618033988