

CLEMSON ALGEBRA PROJECT

UNIT 4: INEQUALITIES IN TWO VARIABLES

PROBLEM 1: GIFT GIVING

Gift-giving season is approaching and you wish to purchase gifts for four relatives and six friends. The amount you spend on each relative is usually, but not necessarily, different from the amount you spend on each friend. You have saved your money and can spend any amount up to \$200. How much should you spend, on average, for each relative, and how much, on average, should you spend on each friend?

MATERIALS

Casio CFX-9850Ga PLUS or ALGEBRA FX2.0 Graphing Calculator

EXTENSIONS

1. Change the amount of money from \$200 to \$100 and then to \$500. What happens to the graph as you make these changes? Why?
2. Leave the amount at \$200, but change the number of relatives. What happens to the graph as you make these changes? Why?
3. Leave the amount at \$200, but change the number of friends. What happens to the graph as you make these changes? Why?
4. Compose some general rules that show how the graph of $ax + by \leq c$ is affected by a , b , and c respectively.
5. Investigate the problem if you were to spend the money on 4 close relatives, 6 distant relatives, and 8 friends.

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ONE SOLUTION TO PROBLEM 1: GIFT GIVING

This problem can be addressed any number of ways. Two reasonable approaches include using a table and using a graph. In either case, however, some algebra simplifies the problem.

Let x represent the average amount spent on each relative and y represent the average amount spent on each friend. Because you have four relatives for whom you will buy something, $4x$ represents the amount you will spend on relatives. Similarly, $6y$ represents the amount you will spend on friends. You can spend any sum up to and including \$200, assuming any tax has already been included in the expressions listed above. Our relationship can thus be described by the inequality $4x + 6y \leq 200$.

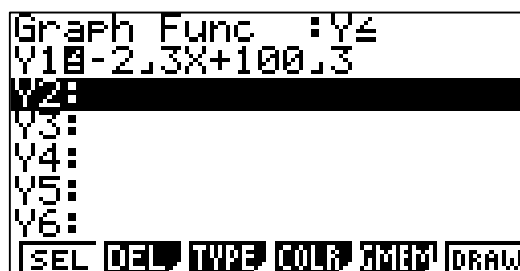
Let's begin with a graph. To enter the relation into the calculator, we must first solve for y . First, subtract $4x$ from both sides to obtain $6y \leq -4x + 200$. Then, divide both sides by 6, obtaining $y \leq \frac{-2}{3}x + \frac{100}{3}$. (NOTE: If we chose not to simplify, we could use $y \leq (-4x + 200) \div 6$.)

Now we'll graph it. From the MAIN MENU, call up the "Graph" menu.

- x If relations from previous work are entered, either delete them (highlight them and press **[F2]** followed by **[F1]**) or turn them off (highlight them and press **[F1]** so they are no longer selected). The following assumes that Y1 is available for the problem.
- x To enter our relation, we first want to enter the correct type of relation. With Y1 highlighted, press **[F3]** for TYPE, **[F6]** for more options, and **[F4]** for the less than or equal to relation. This returns you to Y1.
- x Simply type in the relation, using the **[a b/c]** key for the two fraction bars and press **[EXE]**. (NOTE: If the division symbol is entered instead, either enclose the $-2 \div 3$ in parentheses or put a multiplication symbol between the 3 and the x .
If not, the calculator will treat the variable term as $\frac{-2}{3x}$ instead of $\frac{-2}{3}x$.)

The entry screen is shown below.

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After entering the relation, press $\boxed{F6}$ to view the graph. Assuming a standard window, which uses a domain and range from -10 to 10 , the entire graph will be shaded in! This is due to an inappropriate window.

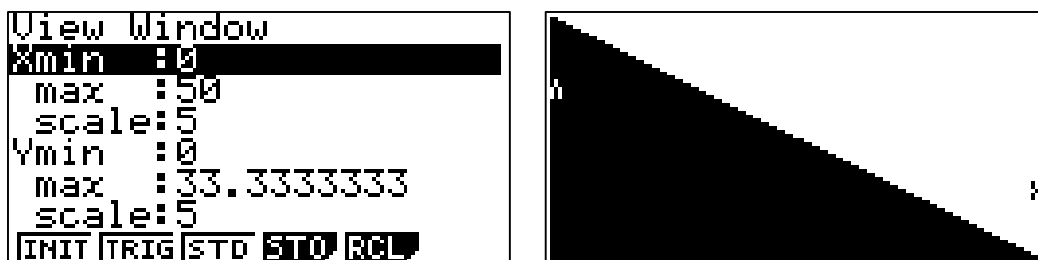
We'll fix the problem by determining a reasonable domain and range. First, discuss with students a reasonable domain for x . Keep in mind that x represents the average price of each gift for relatives. Obviously, it doesn't make sense to make the average negative, so a reasonable minimum value for x is 0 . Because there are four family members and a maximum of $\$200$ to spend, if the person decides to spend all of the money on relatives, the average still cannot exceed $\$50$, so 50 is a reasonable maximum. Setting the scale at 5 may be an agreeable number of tick marks on the x -axis.

When thinking about the range for y , students should also realize that 0 is a minimum value to use. Because there are six friends to buy gifts for, the maximum the average could be is $200/6$ or $\$33.33$, rounded to the penny. If students wish, they can use $200/6$ as the maximum value for y , or they may wish to round to perhaps 35 . A scale of 5 again is appropriate, although students may prefer a different value.

To enter these values, when looking at the graph,

- x Press $\boxed{\text{SHIFT}}$ $\boxed{F3}$ to access the viewing window.
- x Type in the numbers discussed above.
- x Press $\boxed{\text{EXE}}$ after each entry. After the last entry, press $\boxed{\text{EXIT}}$ to return to the input screen.
- x Pressing $\boxed{F6}$ will draw the graph with the new window. The window and graph are shown below.

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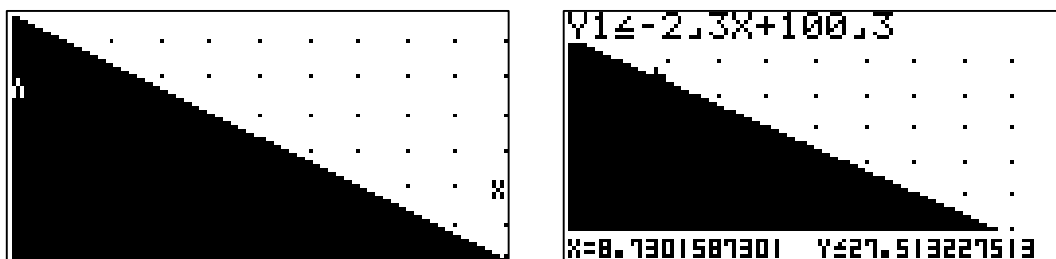
An alternate method of setting up an appropriate viewing window is to input the desired values for x as explained above and drawing the graph. Then,

- x Press **F2** to obtain the ZOOM window.
- x Press **F5** for an "AUTOMATIC" window determined by the calculator.

This ZOOM AUTO feature can be very helpful when you have determined a reasonable domain, but wish help in determining a reasonable range.

Although it does not transfer well to paper as shown below, turning on the grid lines may make it easier for students to read and interpret the graph. Because we set both scales at 5, both horizontal and vertical moves of one dot represent a change of \$5. To turn on the gridlines, with the graph displayed,

- x Press **SHIFT** **MENU** to reach the graph set-up screen.
- x Then, use the down arrow until GRID is highlighted.
- x Press **F1** to turn the grid on and **EXIT** followed by **F6** to return to the graph.
(See below left.)
- x The Trace feature, accessed by pressing **F1**, can help students recognize the vast number of possible solutions. See below right.



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Depending on their maturity and background, students may have expected only one specific answer when you first presented the problem. Obviously, that is not the case; several values are possible for the mean cost of a relative's present and the mean cost of a friend's present. After they have studied the graph and traced through the boundary line, ask them to identify points in the shaded region, reminding them that the scale has been set at five. As they create their list, they should keep in mind what the point means. For example, the point (20, 15) is in the shaded region. This means that to keep within the \$200 limit, it is okay to average \$20 per gift for each relative and \$15 per gift for each friend.

Then ask other questions, such as the following:

- What is the significance of moving to the right within the shaded region? (Because the x -value is increasing, this means they are spending more on average for each relative gift.)
- What is the significance of moving up within the shaded region? (This represents increasing the average cost of a friend's gift.)
- Are there values that indicate spending exactly \$200? (With this scale, four points, (5, 30), (20, 20), (35, 10), and (50, 0), are on the line that defines the region. Because they are on this boundary, they indicate that the maximum amount available, \$200, was spent.)
- Based on your graph, what would you do? Support your answer. (Answers will vary.)

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ANOTHER TECHNIQUE USING A TABLE

Another way to explore the problem is with a table, looking for the maximum values for x and y . Once again the calculator can provide an excellent means for investigating. From the MAIN MENU, call up the “Table” function. To look at the table, we need an equation, not an inequality.

- x To change the type of relation, press **F3** for type and **F1** for $Y=$.
- x With Y1 highlighted, type in $\frac{-2}{3}x + \frac{100}{3}$, again using the **a b/c** to enter the fraction bar.
- x The next step is to set the range for our x -values. Pressing **F5** allows us to choose the values of x we would like to investigate. Because x can vary from 0 to 50, we should set 0 as our Start value and 50 as our End value.
- x The pitch is up to us. If we want, we could look at every possible monetary value for x by entering a pitch of .01. This seems a little extreme, but some students may wish to find a solution that is this precise. Alternatively, we can check each dollar by entering a pitch of 1, or perhaps we could check fewer values by entering a pitch of 2 or even 5.
- x Below left shows a pitch of 2 and below right shows the beginning of the table, obtained by pressing **EXIT** and **F6** after the range was set.

Table Range	
X	
Start:	0
End :	50
Pitch:	2

X	Y1
0	33.333
2	32
4	30.666
6	29.333

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Use the down and up arrows to scroll through the table, noting the possible maximum values for x and y . These are average values for relatives' and friends' gifts in which all \$200 is spent; any values less than these would, of course, spend less than the allotted \$200.

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“BUSTING BARRIERS” WITH THE ALGEBRA FX 2.0

The “Algebra” menu on the ALGEBRA FX2.0 can be used to help students solve equations and inequalities for y or any other variable.

- x From the MAIN MENU, highlight “Algebra” and press **EXE** .
- x Press **F1** for the TRNS menu.
- x Press the appropriate number for “Solve.”
- x Type in the equation or inequality you wish to solve, a comma, the variable you wish to solve for, close the parentheses, and press **EXE** . To obtain a greater than or less than sign while typing, press the appropriate function key for “EQUATION,” the number for “INEQUALITIES,” and the number of the desired sign. The top of the screen might look like this:
solve ($4X + 6Y \leq 200, Y$) .

Using the Tutorial Menu

In addition to solving inequalities for specific variables, the FX 2.0 can also assist students in learning the process for themselves. From the MAIN MENU, access the “Tutorial” menu and follow the steps below.

- x To clear other entries, press the function key for “Clear,” the number for “ALL EQUATIONS,” and **EXE** for yes.
- x At the cursor, type in the equation or inequality you want to work on and press **EXE** . To obtain a greater than or less than sign, press the function for “EQUATION,” the number for “INEQUALITIES,” and the number of the desired sign. For example you might type in $4X + 6Y \leq 200$. This inequality is labeled as equation 1.
- x Suppose we wanted to solve this inequality for Y . All we need do to begin is simply tell the calculator to subtract $4X$. The calculator assumes you are referring to the equation in its memory. Press **EXE** . This becomes equation 2, and should appear as $4x + 6Y - 4X \leq 200 - 4X$.

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- x To simplify equation 2, press $\boxed{\text{F1}}$ for the TRNS menu, the number for simplify, and $\boxed{\text{EXE}}$. The result has isolated $6Y$, showing us that $6Y \leq -4X + 200$. This is labeled as equation 3. One of the many exciting features of the calculator is that, even if the student tells the calculator to do something wrong, the calculator does it, but the student can recognize that it does not help lead to a solution. The student can then return to the previous equation and try something else.
- x The next step, of course, is to divide by 6. Simply press the division sign, 6 and press $\boxed{\text{EXE}}$. The calculator shows $\frac{6Y}{6} \leq \frac{-4X + 200}{6}$.
- x This needs to be simplified, so again press $\boxed{\text{F1}}$ for the TRNS menu, the number for simplify, and $\boxed{\text{EXE}}$. The calculator shows that $Y \leq \frac{-2X}{3} + \frac{100}{3}$, using true fraction notation.

This TUTORIAL can be a very effective tool in helping students master the skills they need to solve inequalities. By “busting this barrier” that impedes the progress of so many students, the Algebra FX 2.0 can then allow the study of higher order and, perhaps, more significant mathematics.