PROBLEM 2: MAKING THE GRADE

Your teacher tells you that your course grade will be determined by quizzes and tests, with your quiz average accounting for 40% of your grade and your test average for the remaining 60% of your grade. Explore the averages you need on quizzes and tests to ensure that you will have at least 80% for your course grade.

EXTENSION

Investigate the same problem, but make sure your quiz and test averages do not exceed 100%.

ONE SOLUTION PROBLEM 2: MAKING THE GRADE.

Our first order of business is identifying variables to represent the unknown quantities we wish to investigate.

- Let x represent the quiz average for the course.
- Let y represent the test average for the course.

Because of the weights put on these and our wish to have an average of 80 or higher, we can write an inequality as follows:

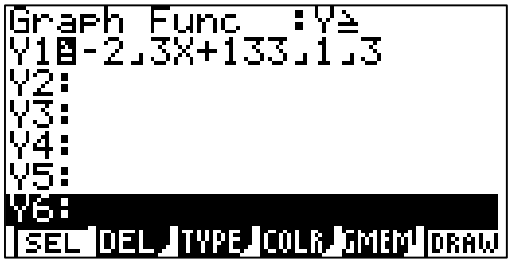
$$.40x + .60y \geq 80$$

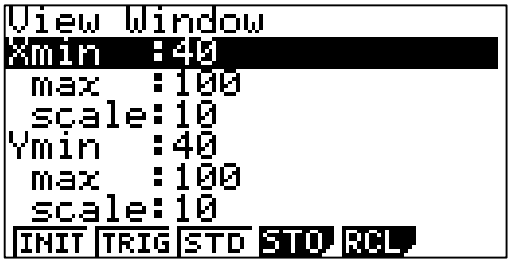
In the inequality above, we could have chosen to use .80 instead of 80, with x and y being expressed as decimals or percents. We have chosen, instead, to use scores such as 90 instead of 90%. This is an arbitrary decision, but one that should be made consciously.

We now wish to explore the relationship between x and y. To isolate y, we need to subtract $.40x$ and then divide by .60. If we do not worry about simplifying, we have $y \geq (80 - .40x) \div .60$.

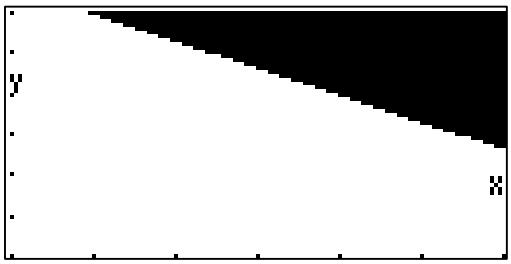
We will first explore this with a graph.

- From the MAIN MENU, choose "Graph."
- Delete or de-select any functions already shown (F2 followed by F1 to delete a highlighted function or just F1 to de-select a highlighted function that is already selected.)
- Choose TYPE Y≥ by pressing F3 , F6 , and F3 .
- Type in $\frac{-2}{3}x + 133\frac{1}{3}$ after Y1. Use the fraction key, a b/c , for the fraction and mixed number. See the screen below left.
- To set an appropriate window, press SHIFT F3 and type in appropriate values, pressing EXE after each entry. One possibility is shown below right.





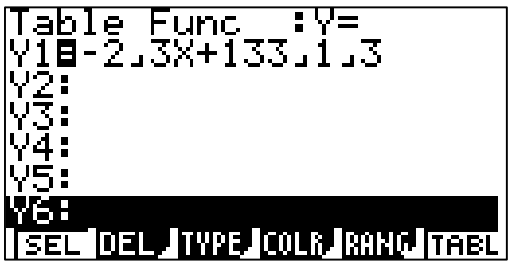
- After the window has been entered, press [EXIT] and [F6] to view the graph. See below left. If you desire, press [SHIFT] [MENU] to access the set-up to display the grid (or to turn it off). Shown below is the graph with the grid off.
- If desired, you can trace along the border by pressing [F1] and then using the right and left arrows as appropriate. Any point on or above this border will produce the desired result, an average of 80 or above. One such point, which can be rounded to (57.1, 95.2), is shown below right. This indicates that if we have a quiz average of 57.1 and a test average of 95.2, we will have an overall average of approximately 80. Higher values in either or both will result in a higher overall average.

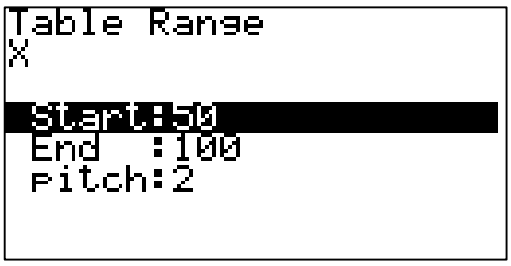


We may also explore our relation algebraically or numerically. Simplifying our inequality, we obtain $y \geq \frac{-2}{3}x + 133\frac{1}{3}$. One point to consider is the *y*-intercept. If *x* is 0, then *y* needs to be more than $133\frac{1}{3}$. Consequently, we cannot have an average of 0 on quizzes and expect to achieve an overall average of 80, assuming that 100 is the maximum test average. We can explore different points, perhaps substituting 100 for *x* (our quiz average) and determining what our test average needs to be. Similarly we can plug in 100 for *y* (our test average) to determine what *x*, our quiz average, needs to be.

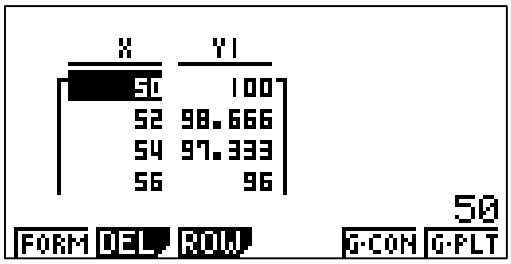
If we investigate the equation instead of the inequality, we can investigate our minimum requirements via a table. To do so, from the MAIN MENU, choose "Table." Then,

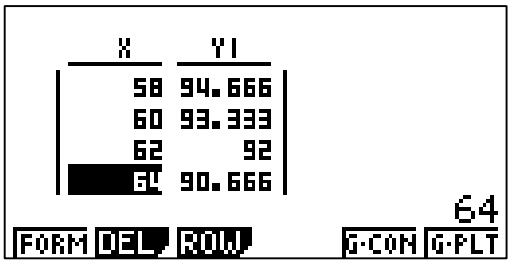
- Delete or de-select any functions already shown (F2 followed by F1 to delete a highlighted function or just F1 to de-select a highlighted function that is already selected.)
- Choose TYPE Y= by pressing F3 and F1 .
- Type in $\frac{-2}{3}x+133\frac{1}{3}$ after Y1. Use the fraction key, a b/c , for the fractions, including separating the 133 from the fraction. See the screen below left.
- Press F4 to set the range. Shown below right is a range of *x* from 50 to 100, with increments of 2 points each time.





- Press F6 to view the table. Use the down and up arrows as desired to scroll through the table. The first few values are shown below.





This tells us a great deal. For example, the first point tells us that to achieve an 80, if our quiz average, *x,* is 50, we need a test average of 100. For a quiz average of 64, we need a test average of almost 91. We might note that for every increase in 2 points in our

quiz average, our test average can decrease by one-and-one-third points. Half of one-and-one-third is two-thirds. Note that the slope of our equation is $\frac{-2}{3}$. In other words, for every increase of one point in quiz average, our test average can drop two-thirds of a point. Similarly, for every three-point increase in quiz average, our test average can drop two points.

Perhaps we may wish to think of the slope in another way. We can associate the negative sign with either the numerator or denominator. For example, we may choose to think of the slope as $\frac{4}{-6}$, which, of course, is equivalent to $\frac{-2}{3}$. If so, we can interpret our slope as saying that if we increase our test average by 4 points, we can let our quiz average drop 6 points. Using the table may be very helpful in discovering such relationships.