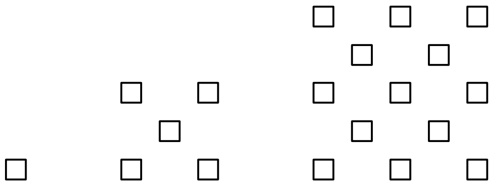
PROBLEM 2: GEOMETRIC NUMBER PATTERS

Consider the following sequence of geometric number patterns and the number represented by each term.



1 5 13

A. Use color tiles to build this pattern. Build the 4^{th} term in this sequence. How many tiles did you use?

B. How many tiles would you need to represent the 5^{th} term?

C. Create a table relating the figure number with the number of tiles.

D. Graph the information from the table. Describe the graph.

E. Find a pattern in the table of values. In order to help discover the pattern, investigate the first order differences of the function values.

F. Investigate the second order differences.

G. Using the general form for a quadratic function, set up a system of equations from your table of values.

H. Solve this system of equations to find the quadratic function that describes this geometric number pattern.

I. Find the number of tiles needed to build the tenth term of this sequence.

EXTENSIONS

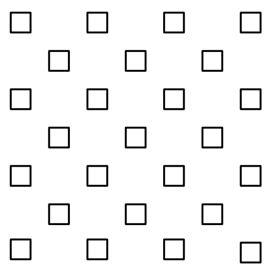
1. If two different colored tiles are to be alternated, determine the number of tiles of each color needed to cover the floor of a room.
2. Determine the number of tiles required for a pattern of this sort in a garden.

REFERENCE: *Mathematics for Elementary Teachers: An Activity Approach*
by Albert B. Bennett, Jr / L. Ted Nelson. WCB McGraw - Hill, 1998.

ONE SOLUTION TO PROBLEM 2: GEOMETRIC NUMBER PATTERNS

A. Use color tiles to build this pattern. Build the 4th term in this sequence. How many tiles did you use?

The pattern should look like the one below. It takes 25 tiles.



B. How many tiles would you need to represent the 5th term?

So far the sequence has been 1, 5, 13, and 25. If we build the 5th set, we find that it takes 41 tiles.

C. Create a table relating the figure number with the number of tiles.

Figure Number (*n*)	Number of Tiles (f (*n*))
1	1
2	5
3	13
4	25
5	41

D. Graph the information from the table. Describe the graph.

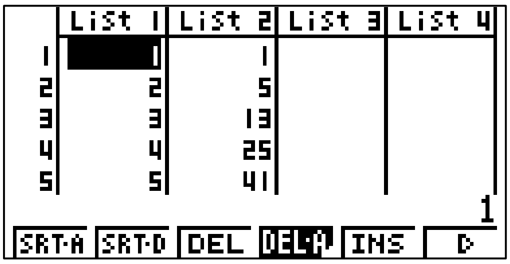
From the MAIN MENU, call up the "Statistics" menu. Then,

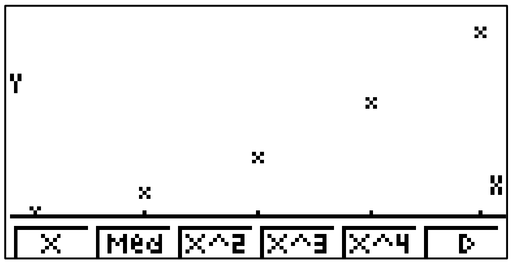
- To clear Lists 1 and 2: Highlight a value in list 1, press [F6] for more options, [F4] to delete all items, and [F1] to confirm the deletion. Use the right arrow to move into List 2 and repeat the process.
- Enter the figure number into List 1, pressing [EXE] after each entry.
- Enter the number of tiles into List 2, again pressing [EXE] after each entry. See below left.

Next, we'll make sure the window is set to "automatic."

- Press SHIFT MENU to get to the Set Up.
- If "Stat Window" is not on automatic, press F1 . Then press EXIT .

To view the scatterplot, press F1 twice. See below right.

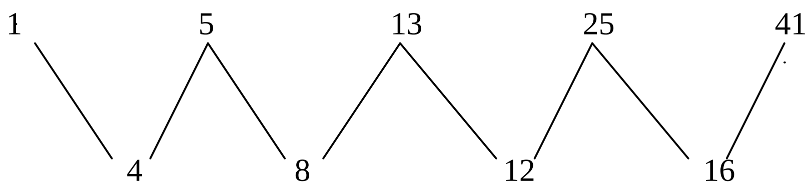




Note that the graph does not appear to be linear. It appears that the graph might be a portion of a polynomial, perhaps quadratic.

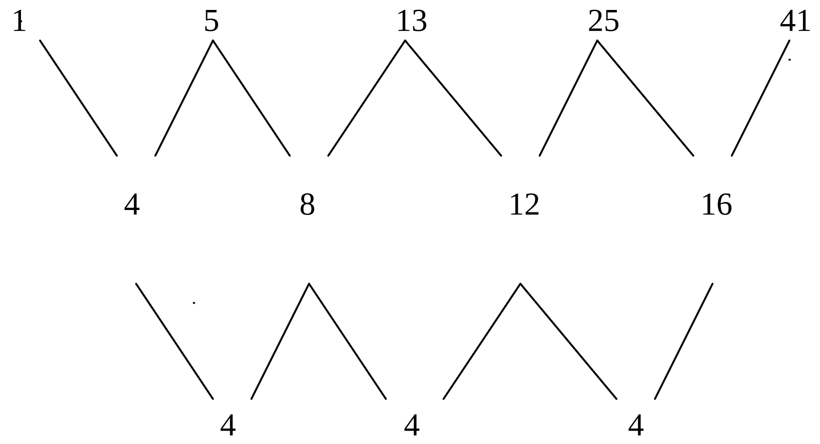
E. Find a pattern in the table of values. In order to help discover the pattern, investigate the first order differences of the function values.

Upon initial inspection, no pattern may be apparent, other than that the values are increasing. However if we look at the differences in the function values in List 2, we do find a pattern. The differences are {4, 8, 12, 16}.



F. Investigate the second order differences.

The second order differences refer to the differences of the first order differences. We can readily see that each value is increasing by 4.



Because the second order differences are constant, the function that describes this sequence is a quadratic polynomial.

G. Using the general form of a quadratic function, set up a system of equations from your table of values.

The general form for quadratic functions is $y = Ax^2 + Bx + C$. Substituting *x* and *y* values from our first three points, we obtain the following system:

$$A + B + C = 1$$
$$4A + 2B + C = 5$$
$$9A + 3B + C = 13$$

H. Solve this system of equations to find the quadratic function that describes this geometric number pattern.

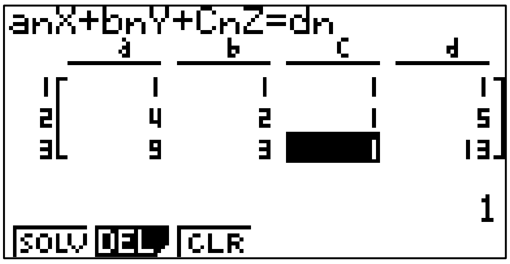
See the following page for two methods of solution. In either case, we find that *A* = 2, *B* = -2, and *C* = 1. Our function, therefore, is $y = 2x^2 - 2x + 1$.

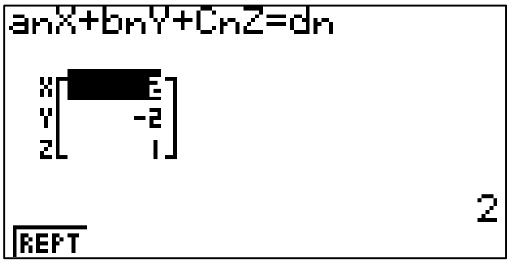
I. Find the number of tiles needed to build the tenth term of this sequence.

Substituting 10 for *x* tells us we'll need 181 tiles to build the 10^{th} term.

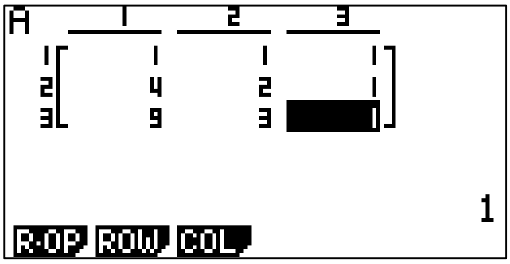
TWO SOLUTION METHODS FOR PART G

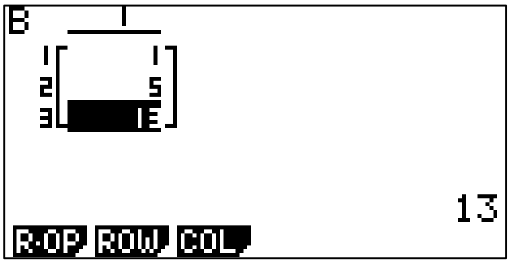
To solve the system of equations presented in part G, we could choose any of several methods. Two are shown here. Choose the “Equation” function from the MAIN MENU and select “Simultaneous.” The set up and solution look like the one below.





Alternately, we might solve with inverse matrices. First, choose the “Matrix” option from the MAIN MENU to create Matrices A and B. Set up Matrix A as the coefficient matrix (see below left) and Matrix B as the solution matrix (see below right).





We now want to find the product of the inverse of Matrix A and Matrix B.

- From the MAIN MENU, choose “Run.”
- Press OPTN F2 for MATRIX, F1 for Matrix and type in A
- Obtain the inverse by pressing SHIFT and the right parenthesis.
- Then press F1 for Matrix followed by B. See below left.
- Press EXE for the solution. See below right.