

PROBLEM 3: COMPUTER AIDED TOMOGRAPHY (CAT SCANS)

Computer Aided Tomography (CAT) scanners compute a picture of the inside of the body by making use of a coordinate grid and evaluating the mass of objects that can only be measured indirectly with X-rays. The basic principle behind CAT technology is that the intensity of an X-ray passing through a body is decreased by an amount proportional to the mass of the tissue and the distance the X-ray travels. As X-rays travel through the body, the change in intensity is measured and masses with different densities are located.

CAT scanners report measurements in units called *linear attenuation units* (LAUs). These units are logarithms of ratios of input energy to output energy. As an X-ray passes through grid cell A, the X-ray is weakened by a LAU. As the X-ray passes through grid cell B, it is weakened by b LAU. The total attenuation when the X-ray passes through both grids is $a + b$.

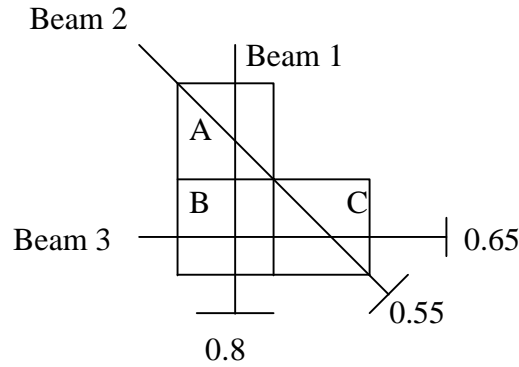
Ranges of LAU values have been established for healthy tissue, tumorous tissue, and bone. Note that there is overlap in Linear Attenuation Units between substance categories. This overlap is an indication that some CAT scan results may be inconclusive.

CAT Scanner Ranges

| Substance | Linear Attenuation Values |
|-----------------|---------------------------|
| Healthy Tissue | 0.16225 — 0.2977 |
| Tumorous Tissue | 0.2679 — 0.3930 |
| Bone | 0.3857 — 0.5108 |
| Metal | 1.54 — ∞ |

Today, CAT scan machines use 512×512 grids which result in extremely complicated systems of equations. For the sake of simplicity, this activity deals with much smaller grids.

Consider the following example:



Suppose that as Beam 1 passes through grid cells A and B the total attenuation is 0.80. A linear equation that describes this relation is $a + b = 0.80$

As Beam 2 passes through grids A and C, the total attenuation is 0.55. The equation that describes this relation is $a + c = 0.55$. As Beam 3 passes through grid cells B and C, the total attenuation is 0.65 and $b + c = 0.65$ describes this relation.

$$\begin{aligned} a + b &= 0.80 \\ a + c &= 0.55 \\ b + c &= 0.65 \end{aligned}$$

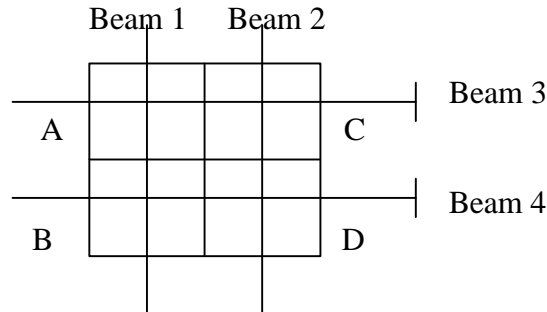
Solving this system, we find that $a = 0.35$, $b = .45$, $c = 0.2$. Comparing these values to the table, one might conclude that cell A contains tumorous tissue, cell B contains bone, and cell C contains healthy tissue.

- A. Consider the following data from several patients. Using the three-cell grid above, what can you determine about each patient?

Beam Decrease in Linear Attenuation Units

| Patient | Beam 1 | Beam 2 | Beam 3 |
|---------|--------|--------|--------|
| W | 0.54 | 0.40 | 0.52 |
| X | 0.65 | 0.80 | 0.75 |
| Y | 0.51 | 0.49 | 0.44 |
| Z | 0.44 | 2.21 | 2.23 |

B. Consider the following 4 grid cells.



Observations showed that Beam 1 is decreased by 0.60 units, Beam 2 is decreased by 0.75 units, Beam 3 is decreased by 0.65 units, and Beam 4 is decreased by 0.70 units. Is this enough information to determine the values of a, b, c, and d?

C. Using the following values for a, find b, c, and d for each value of a.

- 1) $a = 0.23$
- 2) $a = 0.33$
- 3) $a = 0.43$

D. Are there sufficient X-rays in to make a determination about the patient?

EXTENSIONS

1. Add two additional X-ray beams in the four- cell grid drawing through the diagonals. Label the diagonal through cells B and C Beam 5. Label the beam through cells A and D Beam 6. Suppose that Beam 5 is decreased by 0.85 units and Beam 6 is decreased by 0.50 units. What can be determined if this information is combined with the information from part B?
2. Can you find an arrangement of exactly 4 beams that will produce a solution?

REFERENCE: Jabon, Nord, Wilson, Coffman and Nord, *Medical Applications of Systems of Linear Equations*, Mathematics Teacher, May, 1996.