# CLEMSON ALGEBRA PROJECT
# UNIT 6: SYSTEMS OF LINEAR INEQUALITIES

***PROBLEM 1: RAISING MONEY***

You and your classmates decide to sell sweatshirts and T-shirts to raise money for a school trip. You decide that you should sell at least thirty items, but do not want to exceed 120 items. Based on a small survey of students, you also decide that the number of T-shirts should be at least twice the number of sweatshirts.

A. Assign variables to the unknown quantities and write a system of inequalities that model the given restrictions.

B. Graph the system, indicating an appropriate window and scale and shading the feasible region.

C. Determine the vertices of the polygonal feasible region.

D. Assume the profit on each sweatshirt is \$5 and the profit on each T-shirt is \$2. What is the maximum profit you can obtain?

E. How many sweatshirts and how many T-shirts should you sell to maximize your profit?

***MATERIALS***

Casio CFX-9850Ga PLUS or ALGEBRA FX2.0 Graphing Calculator

***EXTENSIONS***

1. Use inequalities that do not involve “or equal to.”
2. Change the restrictions to ones determined by the class. Use numbers that do not result in integral solutions.
3. Assume different profit margins and determine the maximum profit.

SYSTEMS OF LINEAR INEQUALITIES

***ONE SOLUTION TO PROBLEM 1: RAISING MONEY***

**A. Assign variables to the unknown quantities and write a system of inequalities that model the given restrictions.**

Let $x$ represent the number of sweatshirts and let $y$ represent the number of T-shirts that you intend to sell. Since you want to sell at least thirty items, the sum of $x$ and $y$ must be 30 or greater. Algebraically, we write $x + y \geq 30$.

Similarly, if you do not want to exceed 120 items, the sum of $x$ and $y$ must be 120 or less. Algebraically, this becomes $x + y \leq 120$.

Our third restriction indicated that the number of T-shirts ($y$) must be greater than or equal to two times the number of sweatshirts. We can write $y \geq 2x$.

If we care to, we can include contextual, common sense inequalities stating that both $x$ and $y$ must be at least (greater than or equal to) 0.

**B. Graph the system, indicating an appropriate window and scale and shading the feasible region.**

From the MAIN MENU screen, call up the "Graph" menu.

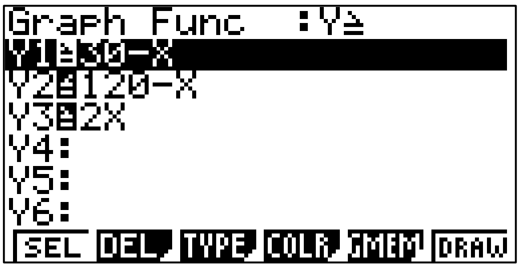
- Delete any functions by pressing [F2] for "Delete" and [F1] to confirm the deletion.

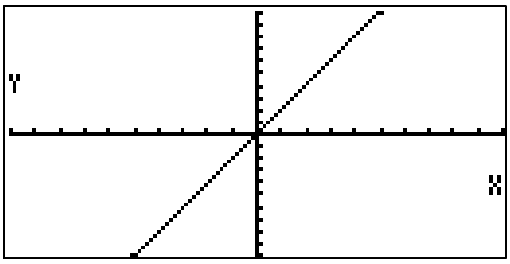
The first inequality we wish to enter is $x + y \geq 30$. First, however, we need to solve for $y$. Subtracting $x$ from both sides gives us $y \geq 30 - x$.

- To change the TYPE of relation, press [F3], [F6] for more options, and then [F3] to obtain the "greater than or equal to" relationship we are looking for.
- At this point, just type in $30 - x$ and press [EXE].
- To put in the second relation, which is a "less than or equal to" relationship, again solve for $y$ first. Then press [F3] for TYPE, [F6] for more options, [F4] for the "less than or equal to" relation, $120 - x$, and [EXE].

- Again change the type to enter the third relation, $y \geq 2x$. After pressing the up arrow a couple of times so you can see all three relations, your screen should look like the one shown below on the left.

If the standard viewing window is used, you will not see much of the graph. What you do see is shown below right.

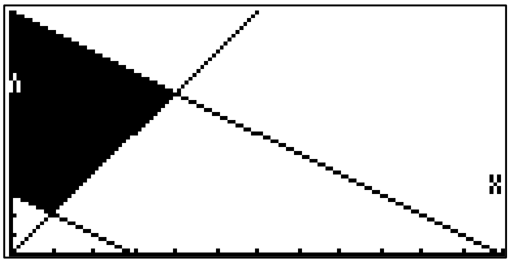



The problem, of course, is with the window. The standard window only shows values between –10 and 10 for both the *x* and *y* variables. To begin a search for a window, we may decide to look at values between 0 and 120 for both *x* and *y*. The logic behind this choice is that we cannot sell a negative number of sweatshirts or T-shirts, and certainly if the total is restricted to 120, neither value alone could exceed 120. A scale of 10 might be appropriate for both axes to avoid an over-abundance of tick marks.

- When looking at the graph, press SHIFT F3 for the viewing window. Type in the desired values, pressing EXE after each entry to obtain the screen shown below left.
- Pressing EXIT and F6 will redraw the graph with the new viewing window.
- If you wish, the window may be further refined, perhaps by setting the maximum *x*-value to 50. To do so, press SHIFT F3, make the change and press EXE, EXIT, and F6. The result is shown below right.

**C. Determine the vertices of the polygonal feasible region.**

The vertices of the shaded feasible region can be found numerically (by using table values on the calculator if desired), algebraically (by solving for the intersections of pairs of lines), or graphically. An approach that takes advantage of the calculator's capabilities is described below.

Assuming we are restricting ourselves to non-negative values (using $x \geq 0$, the *y*-axis) as the left boundary, our region is framed by a quadrilateral. We wish to find the vertices of this figure. Two of the points are on the *y*-axis; by definition, these are the y-intercepts of two of our boundary lines. Our three lines are $x + y = 30$, $x + y = 120$, and $y = 2x$. Before we find the y-intercepts graphically, let us note that it can be easily determined that the *y*-intercepts are 30, 120, and 0 respectively. Since (0, 0) is not a boundary point, two of the four vertices are (0, 30) and (0, 120).

The calculator also gives us these points quickly. While looking at the graph:

- Press [F5] to access the graph solver
- [F4] for Y-ICPT (the *y*-intercept), the up or down arrow key until the desired line is shown, and [EXE] .
- Repeat this process for each of these three lines. You should quickly identify the two desired points. Again, (0, 0) is not desired.

To find the other two vertices:

- Again access the graph solver by pressing [F5] , but then press [F5] to find the intersection points.

- Use the up or down arrows to find a desired function and press EXE when a function you want is listed.

Note that when you try to find the intersection of Y1 and Y2, you receive a "NOT FOUND" message. This is because the two lines are parallel and have no intersection. Repeat the process for other pairs of lines. The intersection of Y1 and Y3 is (10, 20) and the intersection of Y2 and Y3 is (40, 80). We now have our four vertices. Again, they are (0, 30), (0, 120), (10, 20), and (40, 80).

**D. Assume the profit on each sweatshirt is $5 and the profit on each T-shirt is $2. What is the maximum profit you can obtain?**

To maximize (or minimize) a value in linear programming, one need only check the vertices of the polygonal feasible region. Since *x* represents the number of sweatshirts and y the number of T-shirts, we need only check the value of the expression $5x+2y$ at each of the four points found above. Several techniques can be used, but for values such as these, simply plugging into the expression may be the simplest. This is shown below:

| | | |
|---|---|---|
| (0, 30) | - | 5(0) + 2(30) = $60 |
| (0, 120) | - | 5(0) + 2(120) = $240 |
| (10, 20) | - | 5(10) + 2(20) = $90 |
| (40, 80) | - | 5(40) + 2(80) = $360 |

Clearly, the maximum profit that can be achieved is $360. Students may wish to check other points shaded in the feasible region to evaluate the profit; this may help convince them that maximum and minimum values can occur only at vertices.

**E. How many sweatshirts and how many T-shirts should you sell to maximize your profit?**

This question has actually been answered within the work for question D above. The point that produced the maximum profit of $360 was (40, 80). Consequently, you should sell 40 sweatshirts and 80 T-shirts.

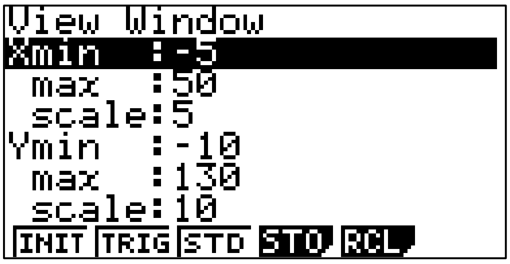
## *ANOTHER TECHNIQUE WITH LINEAR PROGRAMMING*

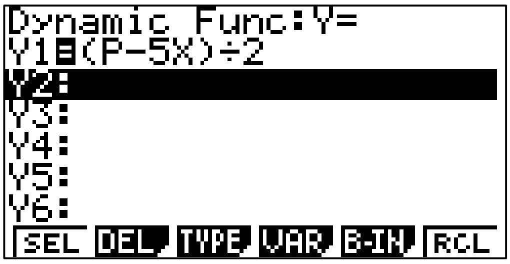
Another technique that can be effective in investigating linear programming problems, such as ***RAISING MONEY***, involves the DYNAMIC capabilities of the calculator. The authors of this material have come to call this technique the “Nina Technique,” named after a participant in the pilot sessions who shared her ideas with us.

Begin as normal, using the GRAPH menu to establish the feasible region. The window for this problem has been changed to the one shown below left. Then,

- With the graph displayed, press OPTN , F1 for picture, F1 to Store the picture, and F1 or any other function key to store the graph as a picture in any desired memory location.
- Press MENU for the MAIN MENU. Call up the “Dynamic Function” screen.
- Press SHIFT MENU for SET UP, scroll down to “Background,” press F2 for Picture, and F1 or whatever function key is needed for the location in which you stored the picture. Then press EXE to return to the “Dynamic Function” screen. (Remember to set the Background to None later.)
- Highlight each of the functions and delete them by pressing F2 and F1 .

Our profit function can be represented algebraically as $P = 5x + 2y$. We want to see what the maximum profit is that remains within our feasible region. Solving this function for y, we have $y = \dfrac{P - 5x}{2}$. Enter this function as shown below right.

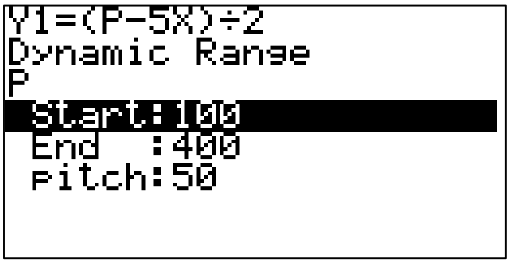


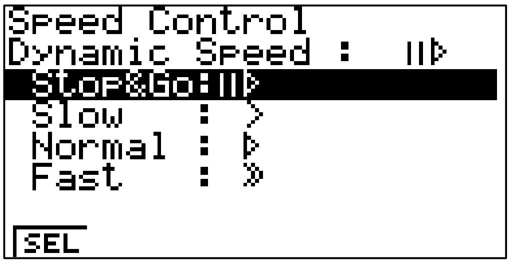




We now wish to set P up as a dynamic variable, that is one that can vary according to conditions we set. To do so:

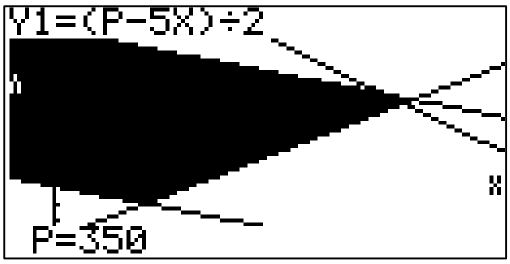
 

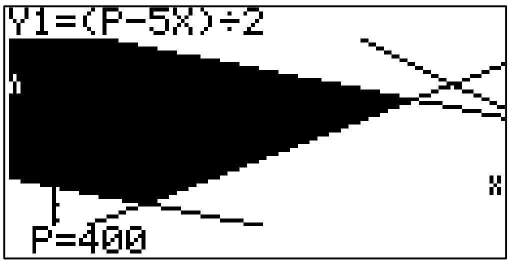
- Press [F4] for Variable. P will be automatically listed as the dynamic variable.
- Press [F2] for Range. Select values that you think might describe the possible profits. If you are unhappy with the values you have chosen, you can simply try again. One possible set is shown below left. Press [EXE] after entering each value.
- When you are finished, press [EXIT] .
- To select the speed, press [F3] . Highlight the speed you want and press [F1] to select it. "Stop&Go" has been selected, as shown below right.




- Press [EXIT] to return to the primary "Dynamic" screen and then [F6] . This will take your calculator a few moments to set up. If you get a MEMORY error, try setting a different range of values for P.
- Press [EXE] to move the profit function through the feasible region. You may notice that when P equals $350, the profit line is still within the feasible region (shown below left), but when P is $400, the profit line has left the feasible region (shown below right).




Changing the range so that the pitch is smaller allows you to be more accurate. This technique can provide an excellent visual for the students.

## "BUSTING BARRIERS" WITH THE ALGEBRA FX 2.0

Although it should not be necessary for this problem, the "Algebra" menu on the ALGEBRA FX2.0 can be used to help students solve equations and inequalities for *y*.

- From the MAIN MENU, highlight "Algebra" and press [EXE] .
- Press [F1] for the TRNS menu.
- Press the appropriate number for "Solve."
- Type in the equation or inequality you wish to solve, a comma, the variable you wish to solve for, close the parentheses, and press [EXE] . To obtain a greater than or less than sign while typing, press the function key for "Equation," the number for "Inequalities," and the number of the desired sign. The top of the screen might look like this: solve$\left(X+Y\geq 30, Y\right)$.

### Using the Tutorial Menu

In addition to solving inequalities for specific variables, the FX 2.0 can also assist students in learning the process for themselves. From the MAIN MENU, access the "Tutorial" menu and follow the steps below.

- To clear other entries, press the function key for "Clear," the number for "ALL EQUATIONS," and [EXE] for yes.
- At the cursor, type in the equation or inequality you want to work on and press [EXE] . To obtain a greater than or less than sign, press the function key for "Equation," the number for "Inequalities," and the number of the desired sign. For example you might type in $5x+2y\geq 100$ . This inequality is labeled as equation 1.
- Suppose we wanted to solve this inequality for *y*. All we need do to begin is simply tell the calculator to subtract 5*x*. The calculator assumes you are referring to the equation in its memory. Press [EXE] . This becomes equation 2, and should appear as $5x+2y-5x\geq 100-5x$ .

- To simplify equation 2, press [F1] for the TRNS menu, the number for simplify, and [EXE] . The result has isolated the Y term, showing us that $2y \geq -5x + 100$. This is labeled as equation 3. One of the many exciting features of the calculator is that, even if the student tells the calculator to do something wrong, the calculator does it, but the student can recognize that it does not help. The student can then return to the previous equation and try something else.
- The next step, of course, is to divide by 2. Simply press the division sign, 2, and press [EXE] . The calculator shows $\frac{2y}{2} \geq \frac{-5x+100}{2}$.
- This needs to be simplified, so again press [F1] for the TRNS menu, the number for simplify, and [EXE] . The calculator shows that $y \geq \frac{-5x}{2} + 50$, using true fraction notation.

This TUTORIAL can be a very effective tool in helping students master the skills they need to solve inequalities for particular variables. By "busting this barrier" that impedes the progress of so many students, the ALGEBRA FX 2.0 can then allow the study of higher order and, perhaps, more significant mathematics.