PROBLEM 2: THE SNACK PROBLEM

Assume you like snacks and insist on having at least one serving of dry roasted peanuts and one serving of potato chips each day. Each serving of the peanuts contains 15% of the recommended daily allowance of saturated fat; each serving of potato chips contains 10%. Each serving of the peanuts contains 12% of the recommended amount of dietary fiber; each serving of potato chips contains 5%. You determine that you want to consume no more than 60% of the recommended allowance of dietary fat from these two snacks, but you want to get at least 30% of the recommended allowance of fiber from them.

A. Sketch the feasible region relating the number of servings of potato chips you might have with the number of servings of dry roasted peanuts.

B. The peanuts cost 12.4 cents per serving and the potato chips cost 23.2 cents per serving. If you keep within the constraints mentioned, how many servings of each should you have each day to minimize your cost?

ONE SOLUTION TO PROBLEM 2: THE SNACK PROBLEM

First we need to identify our variables.

- Let x represent the number of servings of dry roasted peanuts we will have per day.
- Let y represent the number of servings of potato chips we will have per day.

Next we need to write our constraints mathematically. First we want both x and y to be at least one. Second, we want the amount of saturated fat to total no more than 60% of the recommended allowance. Finally, we want the amount of dietary fiber to be at least 30%. Mathematically we can write:

1) $x \geq 1$

2) $y \geq 1$

3) $.15x + .10y \leq .60$

4) $.12x + .05y \geq .30$

To address part A, we will graph the system. We will use the graphing window to set minimum values for x at 1. We will enter the second inequality directly in the calculator. For the third and fourth inequalities, we need to first solve for y. We could use the "Algebra" mode on the ALGEBRA FX2.0 for assistance, but we would hope this is not necessary, especially if we are not concerned with simplification. Solving inequalities 3) and 4) for y, we obtain:

3) $y \leq (.60 - .15x) \div .10$

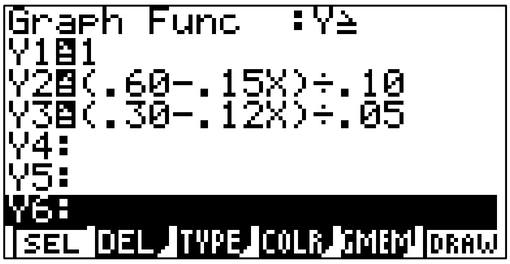
4) $y \geq (.30 - .12x) \div .05$

We will now set the window. Again, for the first inequality, we set the minimum x value at 1. Although we could set the minimum y value at 1 also because of the second inequality, we will want to see the bottom of the graph clearly. Consequently, we will set it at –1. Inequality 3) shows that the biggest x and y can be are 4 and 6, respectively. From the GRAPH menu, press [SHIFT] [F3] and enter the values, pressing [EXE] after each entry. Your screen should look like the one shown below left. To return to the primary "Graph" screen, press [EXIT] .

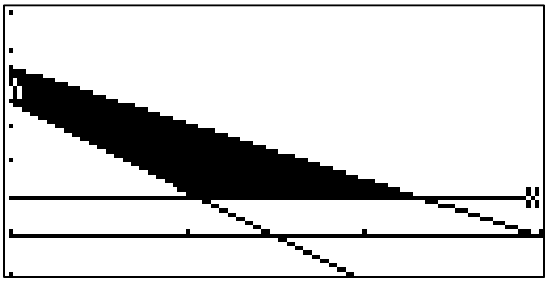
To enter inequalities 2), 3) and 4), from the primary “Graph” screen, delete or de-select and relations that have already been input. Then:

- Press F3 to change the type.
- Press F6 for more options.
- Press F3 for less than or equal to.

Type in inequality 2) and press EXE . Repeat this process to enter inequalities 3) and 4), but press F3 when you need greater than or equal to. After typing in the three inequalities (and using the arrow keys to view them simultaneously), your screen should look like the one shown below right.



To view the feasible region, press F6 . The result is shown below.



To address question B of this problem, we need to find the coordinates of the vertices of our polygonal region. Since we intend to minimize the cost, we need only look at the vertices that are left and below. However, for students who have not yet made this connection, you may want to find all four vertices. Pressing F1 will give us the trace feature of the calculator. When we first do this, we obtain the point (1, 1), which is not a vertex of the feasible region. Pressing the down arrow gives us the point (1, 4.5), the upper left vertex. Pressing the down arrow again gives us the other vertex on the left, the second of the two in which we are interested. This is the point (1, 3.6).

To find the other two vertices, we can once again take advantage of the calculator.

- Press [F5] to access the graph solver.
- Press [F5] for "Intersection."
- Press [EXE] to accept Y1 as one of the relations we're interested in.
- Press [EXE] to accept Y2 as the other.

The cursor will move to our intersection point, $(3.333,1)$. This is the farthest right vertex of our feasible region. To find the fourth point,

- Press [F5] to access the graph solver.
- Press [F5] for "Intersection."
- Press [EXE] to accept Y1 as one of the relations we're interested in.
- Press the down arrow to show Y3 as the other.
- Press [EXE] to accept Y3.

The cursor will move to our final vertex, which is at $(2.083,1)$. All that is left to do is to determine how much each will cost, and select the one that yields the minimum. Since each serving of peanuts costs 12.4 cents and each serving of potato chips costs 23.2 cents, our total cost is determined by computing $12.4x + 23.2y$. Plugging in our values we obtain the following:

$$(1, 4.5) \rightarrow 12.4 \times 1 + 23.2 \times 4.5 = 116.8$$

$$(1,3.6) \rightarrow 12.4 \times 1 + 23.2 \times 3.6 = 95.92$$

$$(3.333,1) \rightarrow 12.4 \times 3.333 + 23.2 \times 1 = 64.53$$

$$(2.083,1) \rightarrow 12.4 \times 2.083 + 23.2 \times 1 = 49.03$$

Note that the last point gives us the minimum cost. Interpreting this, we find that to meet our criteria, we should have just over two servings of peanuts and exactly one serving of potato chips each day. Our cost for doing this is just over 49 cents.