CLEMSON ALGEBRA PROJECT
UNIT 7: INTRODUCTION TO STATISTICS

PROBLEM 1: THE STROOP TEST

This investigation is an experiment from the field of cognitive psychology. Cognitive psychologists try to understand and explain how the brain functions. This particular experiment was developed by J. R. Stroop. In this experiment,

1. Each student will look at a list of color words - red, green, blue, or black. Each word in the list will be written in a variety of colors of ink. Two different types of lists will be displayed. In **matching** lists, the words are matched to the color of ink used (e.g., *red* is written in red ink; *blue* is written in blue ink). In **non-matching** lists, the words are not matched to the color of ink in which they are written (e.g., *blue* may be written in red ink).
2. The student will say the color of ink for each word in the list.
3. If possible, three different students will record the time a fourth student takes to name the color of ink of each word on the list. Additionally, the length of the list will be recorded. (Blank tables are provided on page 3.)

A. What questions could we try to answer with this experiment? What do you think we will find? Why is it important to use several timers?

B. Perform the experiment and record the results in two tables. In the first, record the results for those who identified colors in a matching list; in the second, record the results for those who identified colors in a non-matching list.

C. Calculate the descriptive statistics for the **average time per word** for the two tables.

D. Use median box plots to compare and contrast the spread of the two data sets.

E. Construct scatterplots, one for each table, using the **average time** as the dependent variable and **list length** as the independent variable. Describe any apparent trends. Compare and contrast the results.

F. Calculate and graph best-fit lines for each scatterplot.

G. Identify and interpret the *y*-intercept for your best-fit lines.

H. Identify and interpret the slopes for your best-fit lines.

I. Using your models, estimate how long it would take to name the colors in a list of 25 words. Estimate the time for a list of 10 words.

J. Using your equations, estimate the number of colors someone could name in two minutes.

K. Solve your equations for *x*. Why might we wish to use this form?

L. Using you equations, estimate the number of colors someone could name in one hour, in one day, and in one year. Explain how you found your answers and what your assumptions were. Do you trust your answers? Why or why not?

M. Are the results of the matching and non-matching experiments what you had expected? Compare and contrast these two experiments.

N. Suppose you were a cognitive psychologist who designed the experiment. Write up your conclusions.

MATERIALS

Matching and Non-Matching Word Lists

Three stop watches

Casio CFX-9850Ga Plus or ALGEBRA FX2.0 Graphing Calculator

EXTENSIONS

1. Examine gender differences in times it takes to read the matching and non-matching word lists.
2. Examine differences in times it takes to read the word lists for those wearing glasses and those not wearing classes.
3. Examine the differences in times with a blurred overhead image and a clear overhead image.

REFERENCE: William S. Hadley, Experiments from Psychology and Neurology, *Mathematics Teacher*, October, 1996.

MATCHING EXPERIMENT

List Length	Timer 1	Timer 2	Timer 3	Average Time	Average Time per Word

NON-MATCHING EXPERIMENT

List Length	Timer 1	Timer 2	Timer 3	Average Time	Average Time per Word

ONE SOLUTION TO PROBLEM 1: THE STROOP TEST

A. What questions could we try to answer with this experiment? What do you think we will find? Why is it important to use several timers?

We might investigate whether it takes longer to read the non-matching word lists. We might also look at whether there is there a gender difference in the time it takes to read the word lists. We might expect that it takes longer to identify colors in the non-matching lists because it is more difficult, but we may not expect there to be any gender differences.

It is important to use several timers and average their results to account for different reaction times. There is likely to be error in the times; by having several timers and computing the average, we should be able to arrive at more accurate results.

B. Perform the experiment and record the results in two tables. In the first, record the results for those who identified colors in a matching list; in the second, record the results for those who identified colors in a non-matching list.

Results from one class are shown in the following two tables.

MATCHING EXPERIMENT

List Length	Timer 1	Timer 2	Timer 3	Average Time	Average Time per Word
15	7.25	7.00	6.85	7.03	0.4687
12	5.87	6.69	6.56	6.37	0.5308
10	4.44	4.60	4.53	4.52	0.4520
20	7.03	7.22	7.66	7.30	0.3650
18		10.57	10.37	10.47	0.5817
8	3.22	3.41	3.41	3.35	0.4188

NON-MATCHING EXPERIMENT

List Length	Timer 1	Timer 2	Timer 3	Average Time	Average Time per Word
9	7.25	7.72	7.85	7.61	0.8456
12	7.75	7.63	8.03	7.8	0.65
16	11.50	11.82	11.65	11.65	0.7281
14	11.13	11.5	11.90	11.51	0.8221
9	4.84	5.15	5.03	5.00	0.5556
20	11.59	12.37	12.25	12.07	0.6035
8	7.87	7.83	7.91	7.87	0.9838

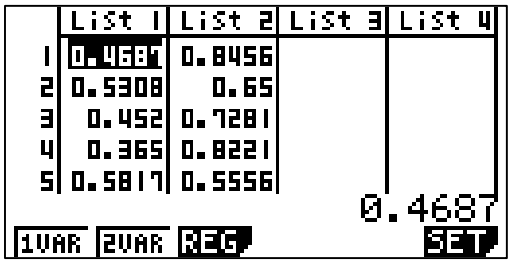
C. Calculate the descriptive statistics for the *average time per word* for the two tables.

We will first enter the data into the calculator. From the MAIN MENU, call up the "Statistics" menu.

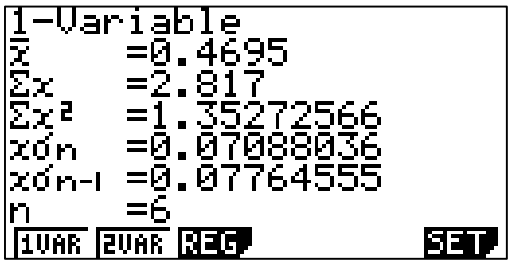
- To clear any data in List 1, press [F6] , [F4] for "Delete All," and [F1] to confirm the deletion. (If you do not wish to delete the data already in your calculator, from the MAIN MENU call up the "List" menu, access the set-up by pressing [SHIFT] [MENU] , and choose a different List File.)
- Move the cursor to List 2 and repeat the steps to delete any data.
- Type the numbers from the Matching Average Time per Word into List 1 and the Non-Matching Average Time per Word into List 2. Press [EXE] after each entry. The first part of your lists should look similar to the screen below left.

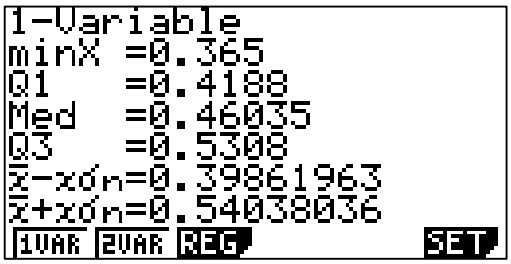
To calculate the statistics for the matching lists,

- Press [F2] for "Calculate," followed by [F6] for "Set."
- Make sure List1 is used as the 1Var Xlist and the frequency is 1. (The other values don't matter at this time.) See below right.

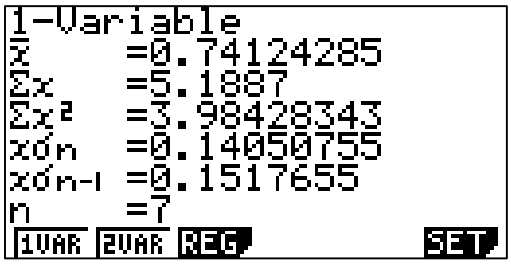


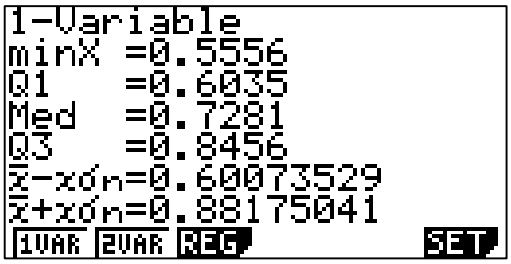
- After you have "Set" the calculations, press [EXIT] to return to the previous screen. Then press [F1] for the calculations. See the following screens.





x For the non-matching statistics, press F6 for "Set," but this time use List2 for the one variable statistics. Again press EXIT to return to the previous screen and F1 for the calculations. See below.





You may wish to record your results, rounding perhaps to the ten-thousandth.

	MATCHING	NON-MATCHING
MEAN	0.4695	0.7412
MODE	None	None
MEDIAN	0.4604	0.7281
LOWER QUARTILE	0.4188	0.6035
UPPER QUARTILE	0.5308	0.8456
STANDARD DEVIATION	0.0776	0.1518

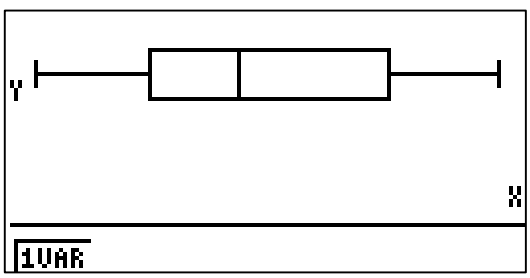
D. Use median box plots to compare and contrast the spread of the two data sets.

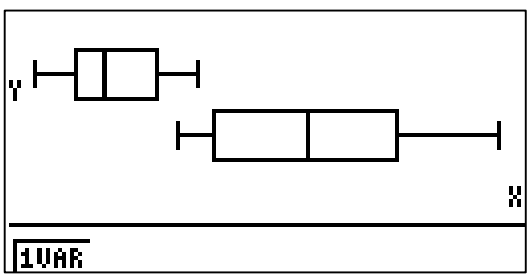
Return to the previous screen by pressing [EXIT] . Press [EXIT] again to return to the screen with the calculate/graph option. We'll now work on a box plot.

- Press [F1] for "Graph" and [F6] for "Set."
- Use the down arrow to "Graph Type. Press [F6] [F2] to make StatGraph1 a box plot.
- Use the down arrow to make sure List1 is used as the Xlist and the frequency is 1.
- Press [EXIT] to return to the previous screen and [F1] to draw the first box and whisker plot. If your graph does not look like the one shown below left, press [SHIFT] [MENU] for the "Set Up." Make sure the Stat Wind is set to "Auto" and the Background is set to "None." Press [EXIT] to return.

We'll now also set up a box plot using List2.

- Press [F6] for "Set" and [F2] for StatGraph2,
- Down arrow to Graph Type and press [F6] for more options, followed by [F2] for box plot. Set the Xlist as List2 with a frequency of 1. If you are using a 9850Ga Plus, you may also wish to use a different color. To return, press [EXIT] .
- Press [F4] for "Select," and make sure StatGraph1 and StatGraph2 are both on. If needed, down arrow to StatGraph2 and press [F1] to turn on the second statistics plot. Press [F6] to see both graphs. See below right.





From the two box plots, you should note that the average times per word from the matched list are much smaller than for the non-matched. You should also note that the spread is much smaller; people reading the matched lists were more consistent than those reading the non-matched lists.

E. Construct scatterplots, one for each table, using the *average time* as the dependent variable and *list length* as the independent variable. Describe any apparent trends. Compare and contrast the results.

We are now changing from uni-variate (single variable) statistics to bi-variate statistics. We are looking at the possible relationships between the length of the word lists and the amount of time it takes to identify the colors of the words on the list. It should be natural to expect some type of positive association; that is, for both matched and non-matched lists, it should take longer to read longer lists.

First, we must enter the data. Note that we are using different variables from those we just used. We will again separate the matched data from the non-matched, but we will need two lists for each. In List 1 we will put the List Length for the matched data and in List 2 the corresponding Average Time. Similarly, we will put the List Length for the non-matched data in List 3 and the corresponding Average Time in List 4.

- To return to the previous screen, press [EXIT] . Press [F6] for more options, [F4] to delete all items in a list, and [F1] to confirm the deletion. In this manner, clear out Lists 1-4 as necessary. (Alternately, call up the "List" function from the MAIN MENU and, using the SET UP options, put all data in a different List File.)
- Enter the data into Lists 1 – 4 as discussed, pressing [EXE] after each entry.

The calculator allows us to sort the lists, while keeping the relationship between Lists 1 and 2 and between Lists 3 and 4 intact.

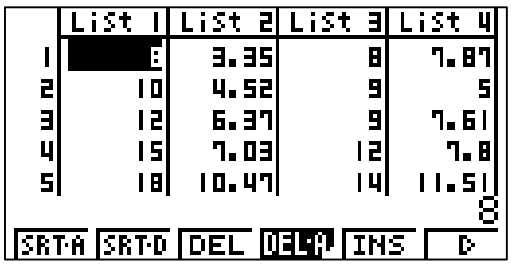
- After entering the data press [F1] to sort the list from lowest to highest. Type in 2 and press [EXE] to tell the calculator that two lists are related.

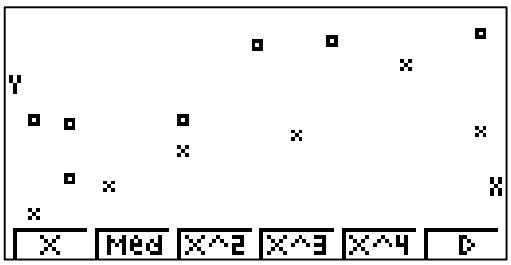
The calculator then asks for the base list; type in 1 and press [EXE] . It then asks for the second list; type 2 and press [EXE] . The data in Lists 1 and 2 are then sorted together.

- Repeat the process, but with List 3 as the base list and List 4 as the second list. See below left for the beginnings of the four lists.

We now wish to set up StatGraphs 1 and 2 as scatterplots.

- Press [F6] for more options and [F1] for graph. Then press [F6] again to "Set" the graphs.
- For StatGraph1, use the down arrow to graph type and press [F1] for "Scatterplot." Make sure the Xlist is set as List1 and the Ylist as List2, with Frequency of 1. Other choices are optional.
- Up arrow to StatGraph and press [F2] to set up StatGraph2. Make sure it is also a "Scatterplot," but with the Xlist set as List3 and the Ylist as List4. Use a different symbol (and color if using the 9850Ga Plus) from that used with StatGraph1.
- Press [EXIT] when finished.
- Press [F4] for "Select," and make sure StatGraphs 1 and 2 are on. Then press [F6] to draw the graphs. See below right.

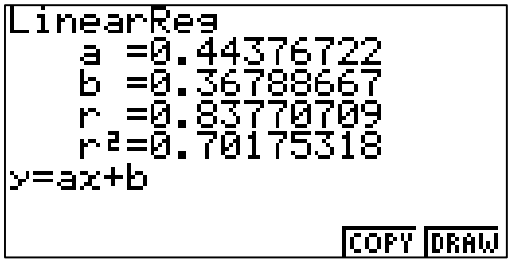


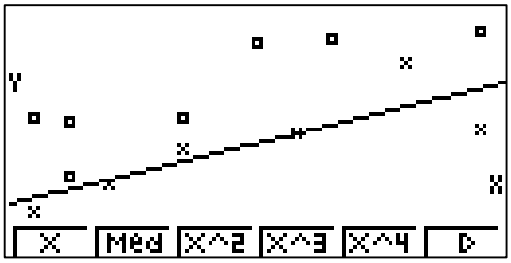


Even with the different symbols used in the scatterplot, it is somewhat difficult to analyze these. Clearly, however, there is a slight tendency upward in both and one scatterplot is below the other.

F. Calculate and graph best-fit lines for each scatterplot.

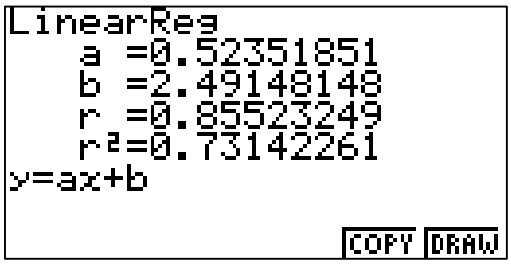
- Before proceeding, press [EXIT] to return to the primary "Statistics" screen. Press [SHIFT] [MENU] for SET UP. Use the down arrow to move to the "Residual List" and make sure it is set to "None." Press [EXIT] to return.
- Press [F1] for graph, [F4] for "Select," and, once StatGraphs1 and 2 are on, [F6] to redraw the scatterplots.
- While looking at the scatterplot, press [F1] to select the option for linear regression. Because there are two StatGraphs, the calculator asks you to select the scatterplot. Since StatGraph1 is first suggested, simply press [EXE] to do the regression on the matched data. Results are shown below left.
- Press [F5] and [EXE] to store this equation into Y1 and then [F6] to draw the graph. See below right.

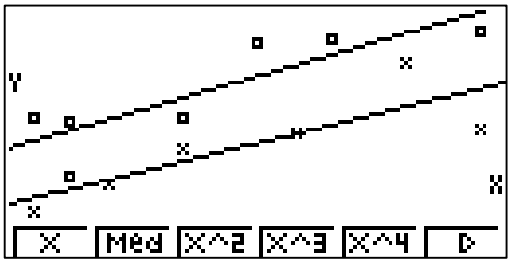




We will now repeat the process for the non-matched data.

- With the screen above right in view, press [F1] for linear regression. Use down arrow so that StatGraph2 is shown on the screen. Press [EXE] . Results are shown below left.
- Press [F5] , the down arrow, and [EXE] to store this equation into Y2 and then [F6] to draw the graph. See below right.





G. Identify and interpret the *y*-intercept for your best-fit lines.

Our two regression equations were as follows:

Matching Experiment: $y = 0.4438x + 0.3679$

Non-Matching Experiment: $y = 0.5235x + 2.4915$

The *y*-intercepts are 0.3679 and 2.4915, respectively. The *x*-values represent the number of words in the list and *y*-values represent the average time, measured in seconds. A literal interpretation of the *y*-intercept for the matching experiment suggests that it would take .3679 seconds to read a list with 0 words in it. For the non-matching, it would take 2.4915 seconds. This, of course, is not reasonable; nevertheless, the *y*-intercept is not without meaning. We could think of the *y*-intercepts as representing the average time it takes to start reading the word list.

H. Identify and interpret the slopes for your best-fit lines.

The slopes of our two regression lines are 0.4438 and 0.5235, respectively. Since slope represents the change in *y* over the change in *x*, here our slope represents the change in average time over the change in list length. For the matched data, we could say that for every additional word, it takes an additional 0.4438 seconds to read the list. For the non-matched, each additional word adds 0.5235 seconds to our time.

I. Using your models, estimate how long it would take to name the colors in a list of 25 words. Estimate the time for a list of 10 words.

This is a simple matter of substituting first 25 and then 10 for x into our regression equations. Again, these are:

Matching Experiment: $y = 0.4438x + 0.3679$

Non-Matching Experiment: $y = 0.5235x + 2.4915$

We can solve our problem several ways. One way is simply to do the arithmetic. From the MAIN MENU, select the "Run" function.

- Type in .4438 times 25 + .3679 and press EXE . The model predicts that the average time to read a list of 25 words in a matched list will be 11.4629 seconds.
- Similarly we find that we expect it to take 15.579 seconds to read a list of 25 words from the non-matched words. See below left.

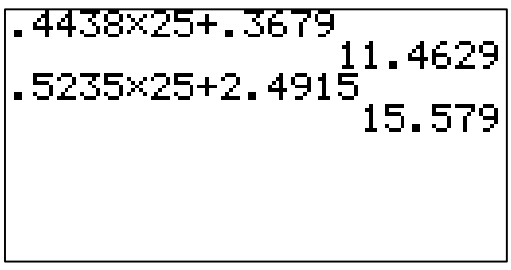
To substitute 10 in for *x*, we could repeat the process. Alternatively, we can use the calculator's "deep" memory.

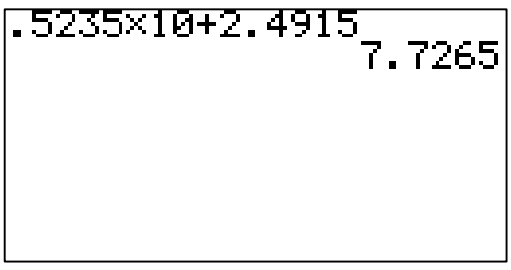
- Press AC/ON to clear the screen.
- Use the up arrow until you see the operation you typed in for the matched experiment. Use the right arrow key and change the 25 to 10. Press EXE . Our model predicts it will take us 4.8059 seconds to read a matched list of 10 seconds.

Repeat the process for the non-matched experiment.

- Press AC/ON to clear the screen.
- Use the up arrow until you see the operation you typed in for the non-matched experiment. Use the right arrow key and change the 25 to 10. Press EXE .

Our model predicts it will take us 7.7265 seconds to read a non-matched list of 10 seconds. See below right.





J. Using your equations, estimate the number of colors someone could name in two minutes.

Because we measured *y* in seconds, we first need to change the 2 minutes to 120 seconds.

For the matching experiment, it takes approximately 11 seconds to read a list of 18 words. Therefore it would take approximately 110 seconds to read a list of 180 words could be read. Thus, a reasonable estimate would be that it would take about 2 minutes (120 seconds) to read **200** words.

For the non-matching, it takes approximately 12 seconds to read 16 words. Therefore a reasonable estimate would be that in two minutes (120 seconds) we could read **160** words.

K. Solve your equations for *x*. Why might we wish to use this form?

If we are not interested in simplifying, our results will look as follows:

Matching: $x = \dfrac{y - 0.3679}{0.4438}$

Non-Matching: $x = \dfrac{y - 2.4915}{0.5235}$

With the equations in this form, we can more readily calculate the word length that could be read in a given time.

L. Using you equations, estimate the number of colors someone could name in one hour, in one day, and in one year. Explain how you found your answers and what your assumptions were. Do you trust your answers? Why or why not?

The number of seconds in one hour is 60 times 60, or 3,600 seconds. The number of seconds in one day is 3,600 times 24, or 86,400 seconds. The number of seconds in one year is 86,400 times 365, or 31,536,000 seconds. If we are to use our models, we must substitute these values in for *y*.

Matching: Our equation was $x = \frac{y - 0.3679}{0.4438}$. Substituting the indicated values for *y* tells us that we could read about 8,000; 195,000; and 71,000,000 words from a matched list in one hour, one day, and one year, respectively.

Non-Matching: Our equation was $x = \frac{y - 2.4915}{0.5235}$. Again making the appropriate substitutions, we find that we could read about 7,000; 165,000; and 60,000,000 words from a non-matched list in one hour, one day, and one year, respectively.

For these to be reasonable, we must assume that one would be reading constantly, with no breaks, and continue to read at the same rate throughout. Consequently, we do not have much faith in these results. The word lengths in the original experiments varied from 5 words to 20 words. Any extrapolation beyond the domain of the original experiment is questionable.

M. Are the results of the matching and non-matching experiments what you had expected? Compare and contrast these two experiments.

Student answers will vary.

N. Suppose you were a cognitive psychologist who designed the experiment. Write up your conclusions.

Student answers again will vary. Conclusions should include that on average it takes longer to read a non-matching list than to read a matching list.

REFERENCE: William S. Hadley, Experiments from Psychology and Neurology, *Mathematics Teacher*, October, 1996.