CLEMSON ALGEBRA PROJECT
UNIT 9: POLYNOMIALS

PROBLEM 1: CONES FOR SHAVED ICE

The Parent Teacher Student Association would like to sell cones of flavored shaved ice at each football game to raise money for a new computer lab. The Fund Raising Committee has searched for paper cones to use, but the cones they have found are either too large or too small. The PTSA asks the Math Team to design a conical cup from a circle that has a diameter of 17 centimeters and has the greatest volume.

A. Recall that a cone is made from a circle by removing a sector of arc length x. Cut out a circle and remove a sector from the circle and build a cone.
B. What is the slant height of the cone?
C. Without measuring, determine the radius of the base of the cone you have made in terms of the arc length that you removed from the circle.
D. Without measuring, determine the height of the cone in terms of the arc length you removed from the circle.
E. Write a formula for the volume of the cone in terms of the arc length that you removed.
F. Determine the arc length that must be removed that will result in the maximum volume of the cone.
G. Determine the dimensions of the cone. Make a cone with these dimensions.
H. Could you easily manufacture a cone with these dimensions? Explain your reasoning.

MATERIALS

Casio CFX-9850Ga Plus or ALGEBRA FX2.0 Graphing Calculator

Scissors

Paper

Tape

ONE SOLUTION TO PROBLEM 1: CONES FOR SHAVED ICE

A. Recall that a cone is made from a circle by removing a sector of arc length *x*. Cut out a circle and remove a sector from the circle and build a cone.

A template may be best for drawing and then cutting out the circle. As the problem suggests, make sure the circle has a radius of 8.5 centimeters.

B. What is the slant height of the cone?

The slant height of the cone is the radius of the original circle; therefore the slant height is 8.5 cm.

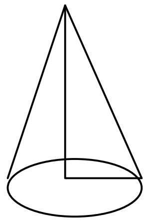
C. Without measuring, determine the radius of the base of the cone you have made in terms of the arc length that you removed from the circle.

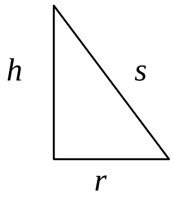
The circumference of the base of the cone is the circumference of the original circle minus the arc length that was removed. If we let *R* represent the radius of the original circle from which the cone was made and *x* represent the length of the arc that was removed, the circumference of the base is $2\pi R - x$. Since we know that *R* is 8.5 centimeters, the circumference is $2\pi * 8.5 - x = 17\pi - x$ centimeters.

If we now let *r* represent the radius of the base, then $2\pi r = 17\pi - x$. Solving for *r*, we find that

$$r = \frac{17\pi - x}{2\pi}.$$

D. Without measuring, determine the height of the cone in terms of the arc length you removed from the circle.





Let x = the length of the arc removed from the original circle

Let h = the height of the cone

Let r = the radius of the base of the cone, which is $r = \frac{17\pi - x}{2\pi}$

Let s = the slant height of the cone, which is 8.5.

Using the Pythagorean Theorem,

$$s^2 = r^2 + h^2$$

$$8.5^2 = \left(\frac{17\pi - x}{2\pi}\right)^2 + h^2$$

$$8.5^2 - \left(\frac{17\pi - x}{2\pi}\right)^2 = h^2$$

$$h = \sqrt{8.5^2 - \left(\frac{17\pi - x}{2\pi}\right)^2}$$

E. Write a formula for the volume of the cone in terms of the arc length that you removed.

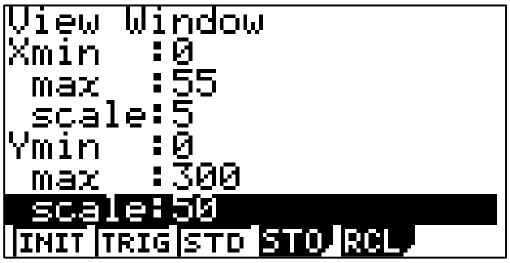
The general form for the volume of a cone is $V = \frac{\pi}{3} r^2 h$. Substituting our values, we obtain the following:

$$V = \frac{\pi}{3}\left(\frac{17\pi - x}{2\pi}\right)^2 \sqrt{8.5^2 - \left(\frac{17\pi - x}{2\pi}\right)^2}$$

F. Determine the arc length that must be removed that will result in the maximum volume of the cone.

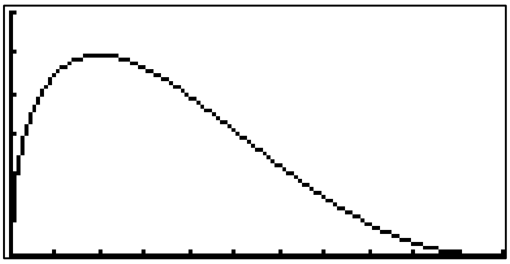
We will solve this with a graph. From the MAIN MENU, choose “Graph.”

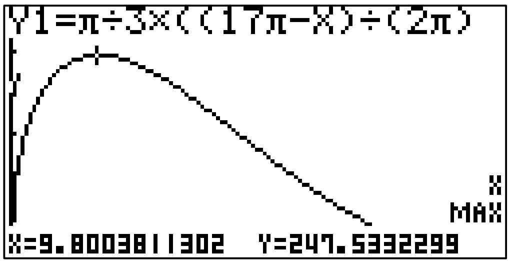
- Delete any functions that are there by highlighting them and pressing [F2] followed by [F1] . Alternatively, de-select them (turn them off) by highlighting them and pressing [F1] .
- Type the formula for the volume of the cone into Y1 and press [EXE] . This is fairly complicated. Be careful with the use of parentheses. The keys are: $\pi \div 3 \times ((1\ 7\ \pi - x) \div (2\ \pi))^2 \times \sqrt{}\, (8.5^2 - ((17\ \pi - x) \div (2\ \pi))^2)$. See below left for the beginning of this.
- Next, we will set the viewing window. Press [SHIFT] [F3] to obtain the viewing window. Values of *x* less than 0 have no meaning in this problem, so set the minimum *x*-value at 0. If *x* should be greater than 17π, the volume would be negative, so restrict *x* to values less than 55. Negative values for the volume are also meaningless; therefore set the minimum *y*-value at 0. Assume that the volume will be no greater than 300 cubic centimeters. Remember to press [EXE] after each entry. See below right for a possible window.



- When the window has been set, press [EXIT] and then [F6] to draw the graph. See below left.
- To determine the value of *x* that produces the greatest volume, press [F5] for the graph solver followed by [F2] for maximum. The calculator shows us

that if we cut an arc length of 9.8 centimeters, we will achieve the maximum volume, which is 247.5 cubic centimeters. See below right.

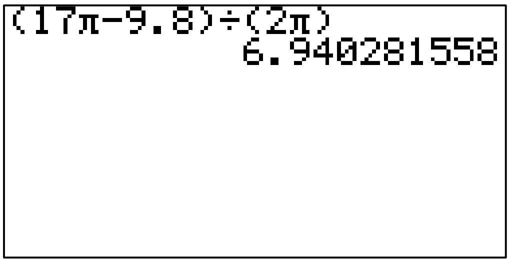


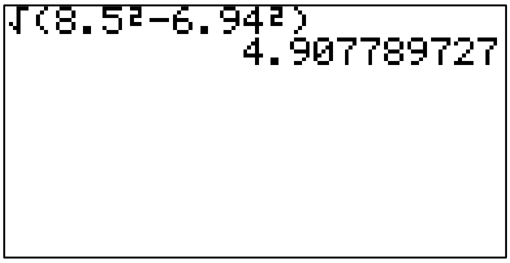


G. Determine the dimensions of the cone. Make a cone with these dimensions.

We know that $r = \dfrac{17\pi - x}{2\pi}$. To find the radius of the cone, we need to substitute 9.8 for x into the formula. From the MAIN MENU, choose "Run."

- Type in the formula as shown below left and press EXE . The radius is 6.940 centimeters.
- To find the height of the cone, we can substitute 8.5 for s and 6.94 for r into the formula $s^2 = r^2 + h^2$. Since $h = \sqrt{s^2 - r^2}$, we need to find the value of $\sqrt{8.5^2 - 6.94^2}$. See below right to see how we obtained a height of 4.9 cm.





H. Could you easily manufacture a cone with these dimensions? Explain your reasoning.

Since there is no such thing as a perfect measure, the quality of our machinery would determine the precision to which we can build the cones. Note that the radius is approximately twice the height. Consequently, these dimensions may not be optimal if a scoop of shaved ice is to be placed on top the ice in the cup.

