Name: _____ Date: _____

The Best Pi

Approximating π

Mathematicians throughout history have tried to come up with ways to calculate approximations of π. Two of the methods that have been developed are described below.

John Wallis was a mathematician who lived in England. In the mid 1800s he proved that an approximation of π can be calculated by evaluating the expression $4 \times \frac{2}{3} \times \frac{4}{3} \times \frac{4}{5} \times \frac{6}{5} \times \frac{6}{7} \times \frac{8}{7} \times \frac{8}{9} \times \ldots$. Use your calculator to help you fill in the chart below. Write each answer both as a mixed number and a decimal. What do you notice about the products?

	Mixed Number	Decimal
Product of the first three factors		
Product of the first four factors		
Product of the first five factors		
Product of the first six factors		
Product of the first seven factors		
Product of the first eight factors		

Gottfried William Leibniz was a mathematician who lived in Germany. In 1674 he proved that an approximation of π can be calculated by evaluating the expression $4 \times (1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \ldots)$. Use your calculator to help you fill in the chart below. Write each answer both as a mixed number and a decimal. What do you notice about the products?

	Mixed Number	Decimal
Find $4 \times (1 - \frac{1}{3})$.		
Find $4 \times (1 - \frac{1}{3} + \frac{1}{5})$.		
Find $4 \times (1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7})$.		
Find $4 \times (1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9})$.		

Thinking Cap

Press the [π] key on your calculator. Compare your answers from above with the number on the calculator. Which approximation of π do you think is best? Why?

The Best Pi

Topic: Approximating π

Objective: To use the calculator to explore approximations of π.

NCTM Standards: Connections, Number and Number Relationships, Computation and Estimation

Approximating π

Using the Activity

Students use the calculator in this activity to calculate approximations for π.

- The [b/c] key can be used to enter fractions.
- The [(] and [)] keys can be used to group expressions.
- The [F-D] key can be used to change mixed numbers to decimals.

Example To find the product of the first three factors in the first table, enter 4 [X] 2 [b/c] 3 [X] 4 [b/c] 3 [=]. The result is $3\frac{5}{9}$. To change this to a decimal, press the [F-D] key. The decimal is 3.5555556. Then change the decimal back to a mixed number by pressing the [F-D] key. This mixed number can be used to calculate the next product.

Assessment Students should be sure their decimal answers are all close to 3.14. Remind them that they are using different methods to find approximations of π and that their answers will vary.

Answers

Table 1: first three factors: see example above.; first four factors: [X] *4* [b/c] *5* [=] $2\frac{38}{45}$ [F-D] *2.8444444* [F-D] $2\frac{38}{45}$ *, first five factors:* [X] *6* [b/c] *5* [=] [=] $3\frac{31}{75}$ [F-D] *3.4133333* [F-D] $3\frac{31}{75}$*; first six factors:* [X] *6* [b/c] *7* [=] [=] $2\frac{162}{175}$ [F-D] *2.9257143* [F-D] $2\frac{162}{175}$*; first seven factors:* [X] *8* [b/c] *7* [=] $3\frac{421}{1225}$ [F-D] *3.3436735* [F-D] $3\frac{421}{1225}$*; first eight factors:* [X] *8* [b/c] *9* [=] *2.9721542; All of the products are close to 3.14.*

Table 2: first row: 4 [X] [(] *1* [−] *1* [b/c] *3* [)] [=] $2\frac{2}{3}$ [F-D] *2.6666667; second row: 4* [X] [(] *1* [−] *1* [b/c] *3* [+] *1* [b/c] *5* [)] [=] $3\frac{7}{15}$ [F-D] *3.4666667; third row: 4* [X] [(] *1* [−] *1* [b/c] *3* [+] *1* [b/c] *5* [−] *1* [b/c] *7* [)] [=] $2\frac{94}{105}$ [F-D] *2.8952381; fourth row: 4* [X] [(] *1* [−] *1* [b/c] *3* [+] *1* [b/c] *5* [−] *1* [b/c] *7* [+] *1* [b/c] *9* [)] [=] $3\frac{107}{315}$ [F-D] *3.3396825; All of the answers are close to 3.14.*

Thinking Cap Answers

The [π] key gives the approximation 3.1415927 for π. The best approximation is the entry in the first seven factors column of the first table. This product is the closest to the value given by pressing the [π] key.