

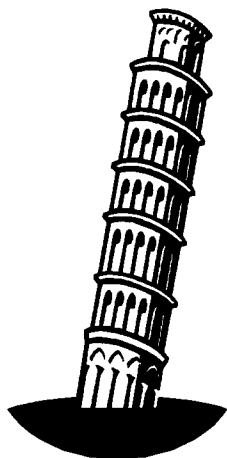
Tower of Hanoi

Exponents
Patterns
Problem Solving
Bases
Generalizations

Standards: Problem Solving, Communication, Reasoning, Connections, Number Theory, Computation and Estimation, Patterns, and Algebra

Materials: fx-7400G, Tower of Hanoi puzzle, and stop watch.

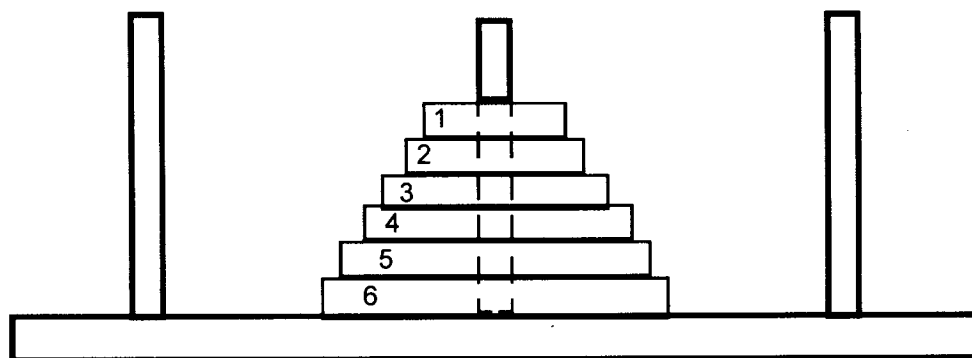
Calculator Use: RUN Menu, x^2 and \wedge keys.



? represents a high level question

The objective of the Tower of Hanoi is to move the stack of disks from one peg to another in the fewest possible number of moves using two rules:

1. Move only one disk at a time.
2. No disk may be placed on a smaller disk.

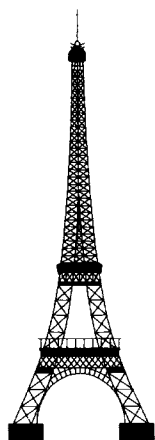


If there was only one disk present on the Tower of Hanoi, the disk could be transferred to another peg in 1 move.

If there were only two disks present on the Tower of Hanoi, the disks could be transferred to another peg in 3 moves. Perform the moves to verify this statement.

If there were only three disks present on the Tower of Hanoi, the disks could be transferred to another peg in how many moves. **A.** _____

If there were only four disks present on the Tower of Hanoi, the disks could be transferred to another peg in how many moves. **B.** _____



A legend states that a group of monks has 64 disks on a Tower of Hanoi. They move the disks continuously, around the clock, following the rules stated earlier. The monks make one move per second, and never have an incorrect move. They supposedly believe the world will end when they complete their task.

Disk	Moves
1	1
2	3
3	7
4	15
5	
6	
64	
n	

Table 1

Tower of Hanoi

Assuming they started moving disks on the first second of January 1, 1900, when will they complete their task?

The answer in seconds. **C.** _____ The answer in years. **D.** _____

This question can most easily be answered if there is a generalization (formula) which can be developed by completing Table 1 with a few more disks. Time yourself while doing the Tower of Hanoi Problem with 5 and 6 disks. Did you average one move per second? **E.** _____

Explain your reasoning. **F.**

Table 2 is an expanded form of Table 1. Table 2 will provide additional information for the generalization needed. Complete Table 2.

Disks	Moves	Moves - 1	Exponents
1	1	2 - 1	2 ¹ - 1
2	3	4 - 1	2 ² - 1
3	7	8 - 1	2 ³ - 1
4	15	16 - 1	2 ⁴ - 1
5			
6			
64			
n			

The "tower" puzzle can be related to Base 2 by writing base 10 counting numbers in base 2 and noting a pattern.

Base 10	Base 2	Zeros right of first 1 counting from the right
1	1	0
2	10	1
3	11	0
4	100	2
5	101	0
6	110	1
7	111	0
8	1000	3

Table 3

Do the "tower" puzzle using 3 disks and notice that you will move the disk (with the top disk as number 1) in the order listed in the third column of the table above. You still have to move to the proper peg to avoid placing a large disk on a small one. Check this with 4 disks.

Can you devise a pattern using the ones in Table 3? **G.**

How to Find a Square

To find the square of 15:

Press 15
x² key
EXE

15 ²	225
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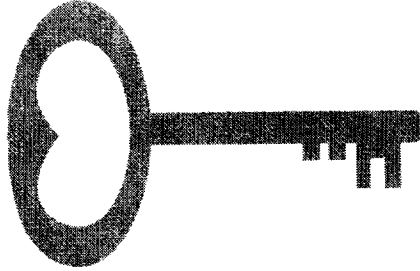
How to Find a Number Raised to the Power of Another Number

The carat key ^ is used to take a number to a power. To find 3⁵:

Press 3
^ key
5
EXE

3 ⁵	243
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Solution Key



Tower of Hanoi

- A.** 7 **B.** 15 **C.** 1.8446744×10^{19} seconds **D.** $5.84542046 \times 10^{11}$ years
- E.** Answers will vary, but probably not. **F.** Only if you know exactly what move to make which would not allow any thinking or reasoning.
- G.** The pattern with the ones is the same as the pattern with the zeros.