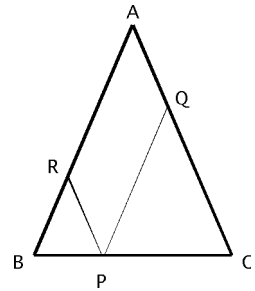


SECRET OF SEGMENTS IN AN ISOSCELES TRIANGLE

PROBLEM

The triangle ABC is an isosceles triangle. ($AB = AC$) Draw a point P on the base BC and draw the two segments PQ and PR so that the segment PQ is parallel to the segment AB and the segment PR parallel to the segment AC. Prove that the equation $RP + QP = AB$ is true.



There are two points to this problem.

- 1) The quadrilateral ARPQ is a parallelogram because $AR \parallel QP$ and $AQ \parallel RP$.
Then regarding the parallelogram ARPQ,
- 2) the triangle RBP is an isosceles triangle ($RP = RB$) because $QP = AR$ and $\angle RPB = \angle ACB = \angle RBP$.

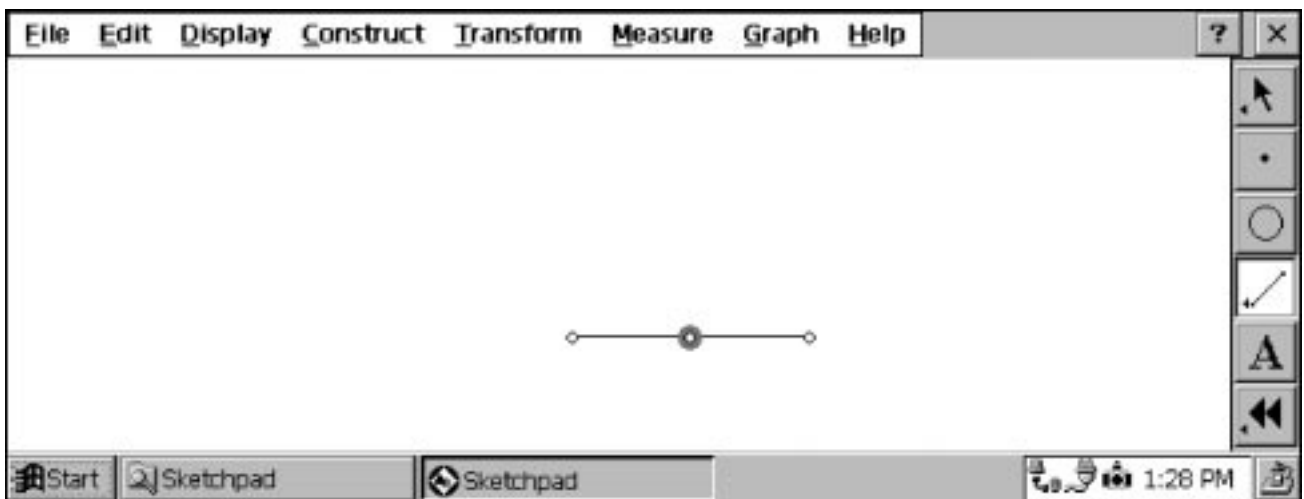
From 1) and 2), $RP + QP = AR + RB = AB$.

Unless the students find that the quadrilateral ARPQ is a parallelogram, it is impossible for them to solve this problem.

[Let's construct the figure of the problem using the Geometer's Sketchpad]

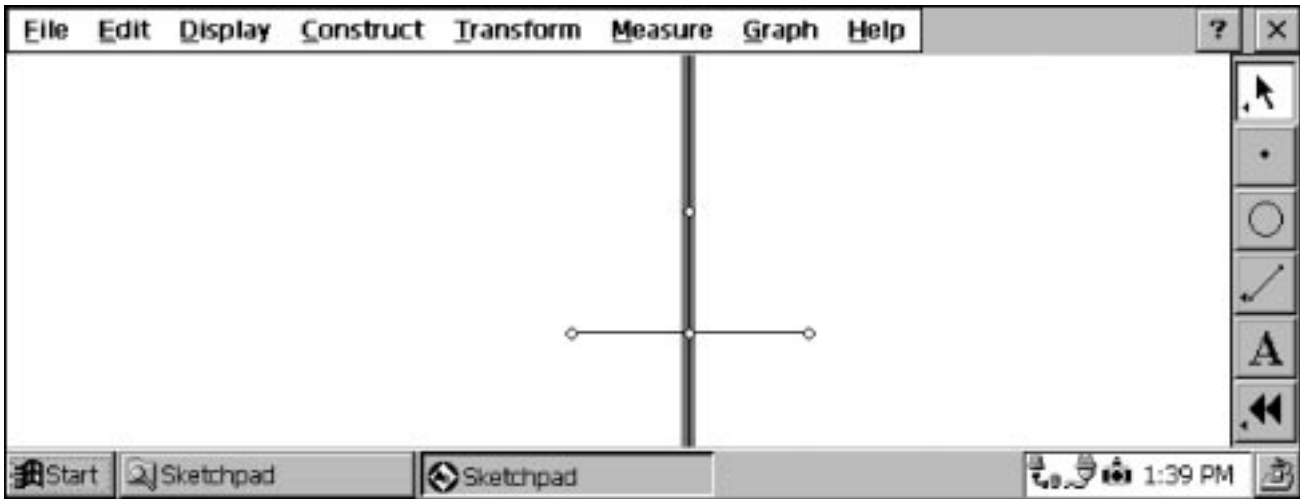
- 1) Construct the segment BC and the midpoint M of it.

Construct the segment BC with the Segment palette and perform the Midpoint command in the Construct menu.



2) Construct a line L that passes through M and is perpendicular to BC.

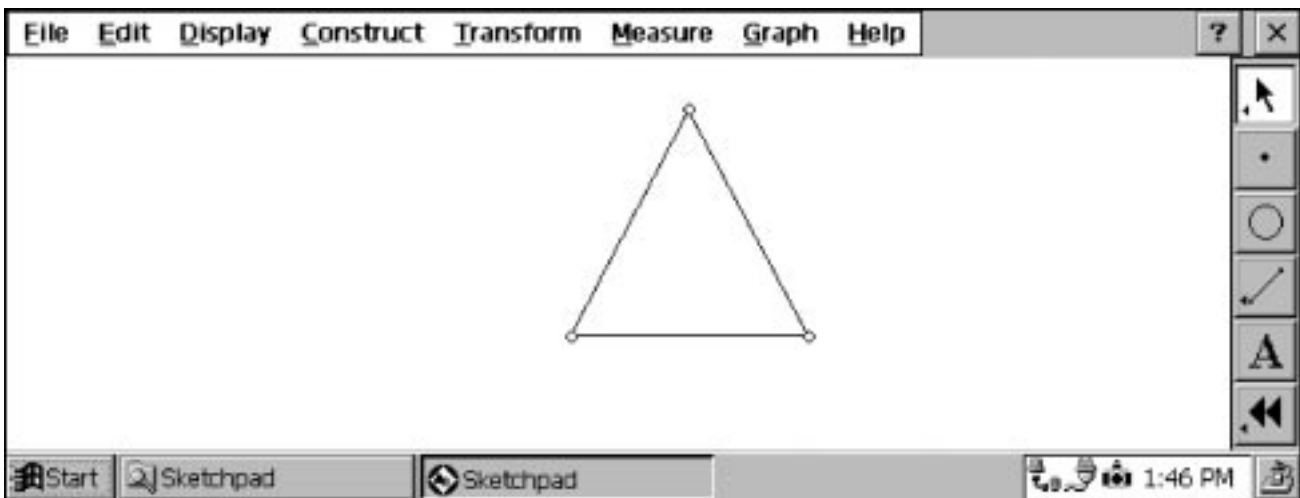
With the Arrow palette, specify the segment BC and then the midpoint M, and perform the Perpendicular Line command in the Construct menu to construct the line L.



3) Draw a point A arbitrarily on the line L using the Point on Line command in the Construct menu.

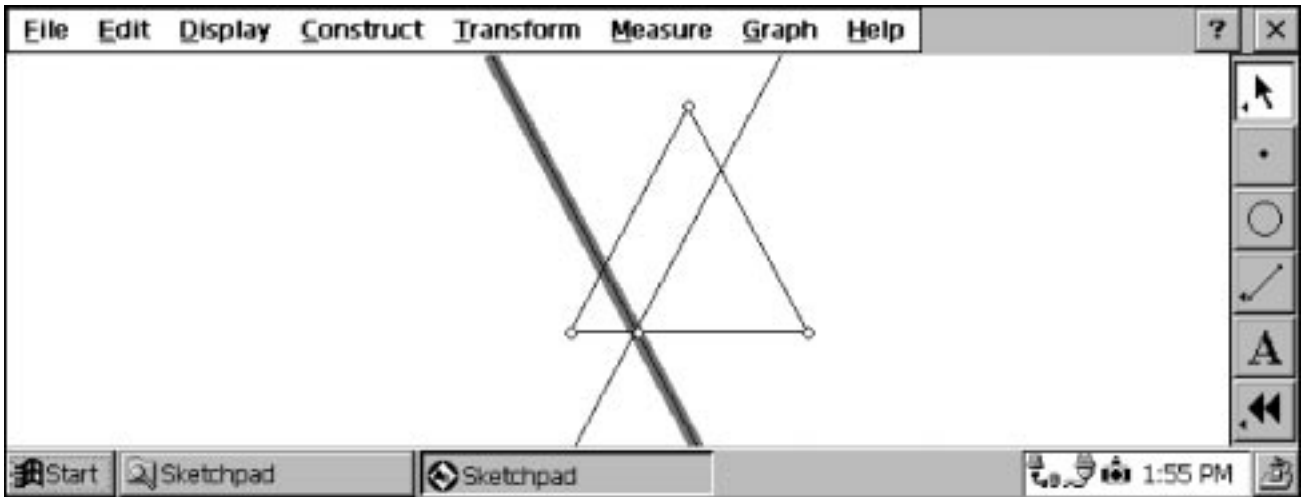
Draw the segment AB using the Straight Object-Segment command in the Construct menu, and then the segment AC in the same way.

Hide the line L, the midpoint M and points used to draw the line L using the Hide Object command in the Display menu.



- 4) To draw a point P on the segment BC, specify the segment BC and perform the Point on Segment command in the Construct menu.

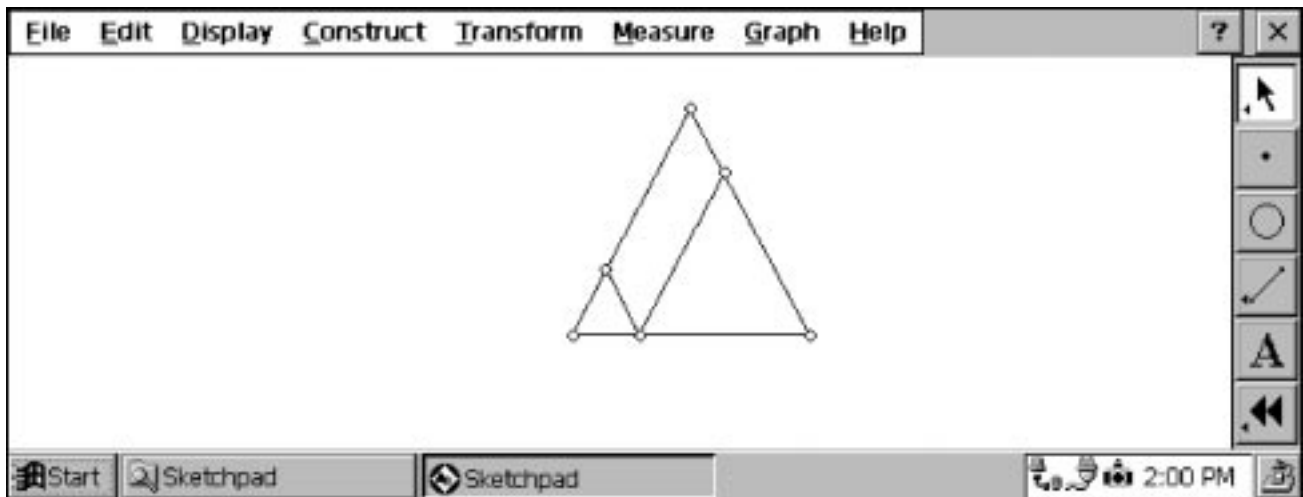
Draw the lines L1 and L2 that pass through the point P and are parallel to AB and AC respectively, as follows. To draw the line L1, specify the segment AB and the point P in this order and perform the Parallel Line command in the Construct menu. Draw the line L2 in the same way.



Draw the crossing R of L1 and AB, and the crossing Q of L2 and AC, and then draw the segments RP and QP, and hide the lines L1 and L2 using the Hide command.

[Let's move the figure]

Let's move the point P along the segment BC with the Arrow palette in the constructed figure.



By moving the point B along the segment BC, the students will be able to understand that the quadrilateral ARPQ is a parallelogram. At the same time they understand that $RP + QP = AB$.

[Development]

Pinch the constructed point A and move it a little so that the triangle ABC is not an isosceles anymore. Check that the equation $RP + QP = AB$ is false.

* To move the point A, specify the point A and perform the Split Point from Line in the Edit menu.