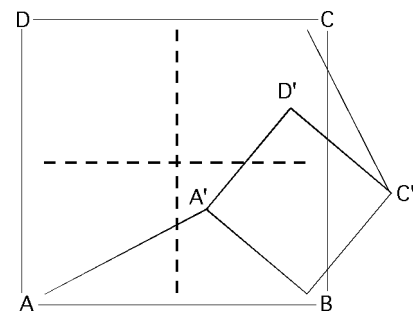


WHAT HAPPENS WHEN WE ROTATE A PIECE OF SQUARE PAPER?

PROBLEM

A small square $A'B'C'D'$ that has a side of 2 cm is placed on a larger square $ABCD$ that has a side of 4 cm as shown in the illustration. Find the area that the segment AA' sweeps when the square $A'B'C'D'$ is given a clockwise full turn about the point B .



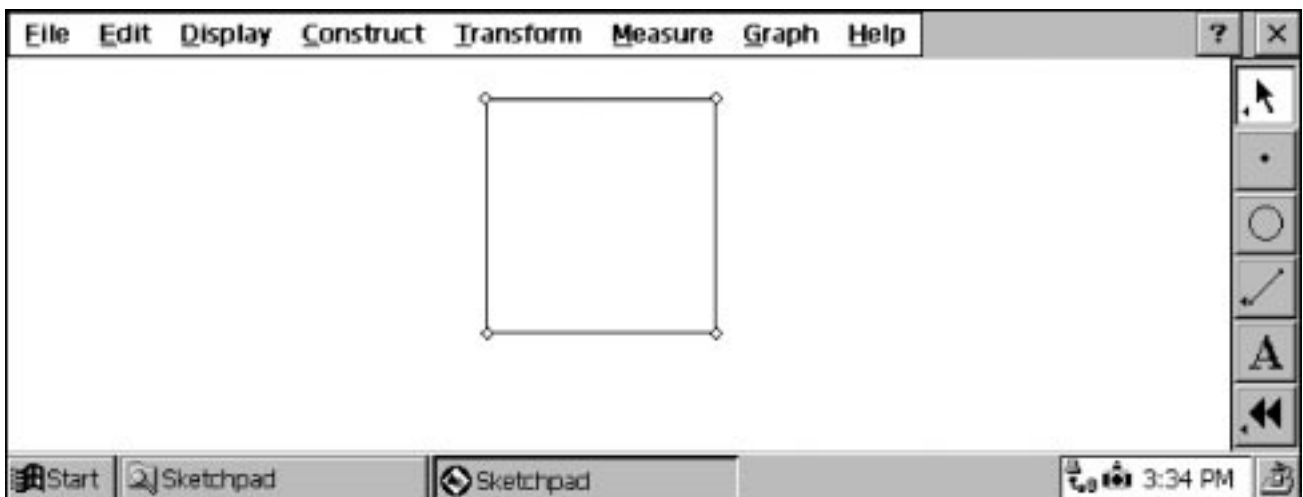
[Let's construct the figure of the problem using the Geometer's Sketchpad]

First of all, pay attention to the fact that the locus of the point A' is a circle of radius 2 when the square is given a full turn about the point B .

- 1) Construct the square $ABCD$.

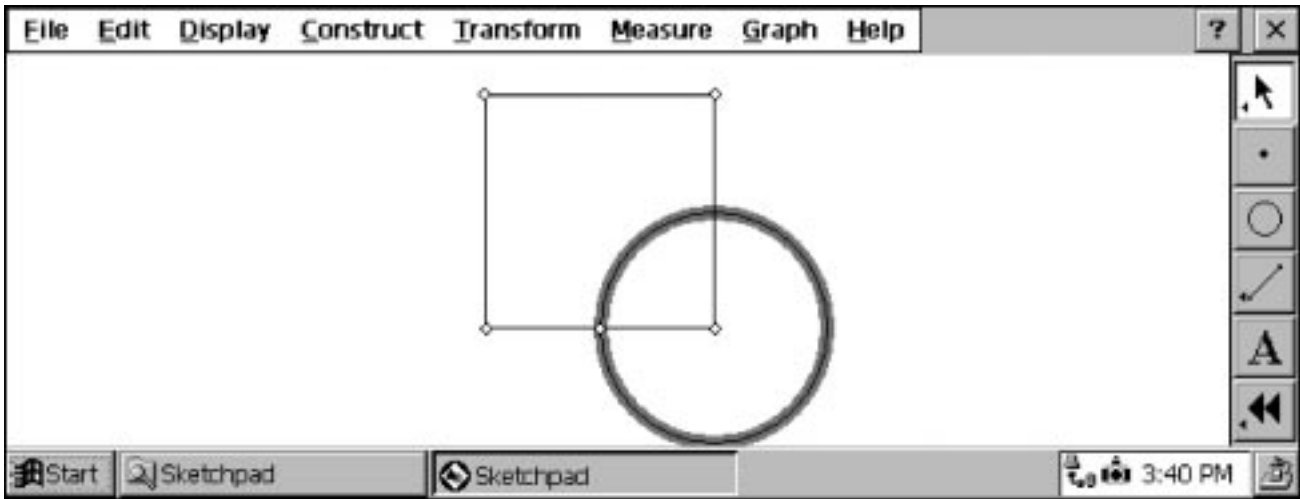
Plot the point A with the Plot palette.

Return to the Arrow palette, specify the point and copy the point B using the Translate-Translate command. Similarly, specify the points A and B and construct the points C and D using the Translate-Translate command. Then specify the two points and construct the segment using the Construct-Straight and Object-Segment commands to construct the square.

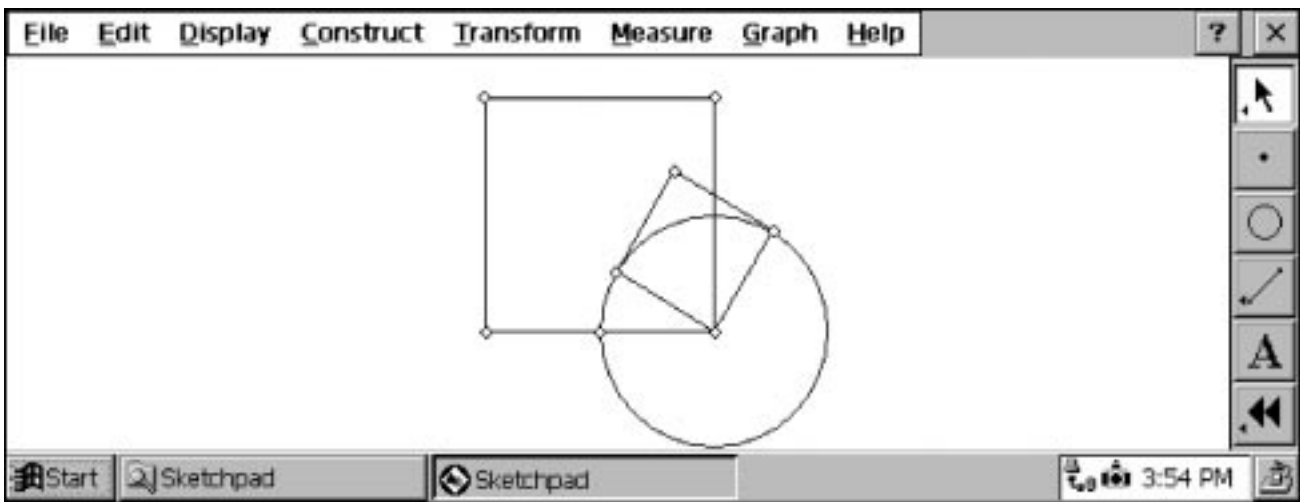


WHAT HAPPENS WHEN WE ROTATE A PIECE OF SQUARE PAPER?

- 2) To specify the point A' , first of all specify the midpoint of the segment AB using the Construct-Midpoint command because the point A' moves along a circle of the center B .
To construct a circle of radius being the distance between the point B and the point A'' , specify the point B and enter the Transform-Mark Center command.
Specify A'' also and perform the Construct-Circle by Center + Point command.



- 3) To construct A' on the circumference, specify the Construct-Point on Object with the circle being specified.
As C' is the mapped point of A' in a -90° rotation about B , apply the Transform-Mark Center command to B , specify A' and rotate it by -90° using the Transform-Rotate command.
As the point D' is the mapped point of B in a 90° rotation about A' , follow the same procedure and connect the four points to construct the square.



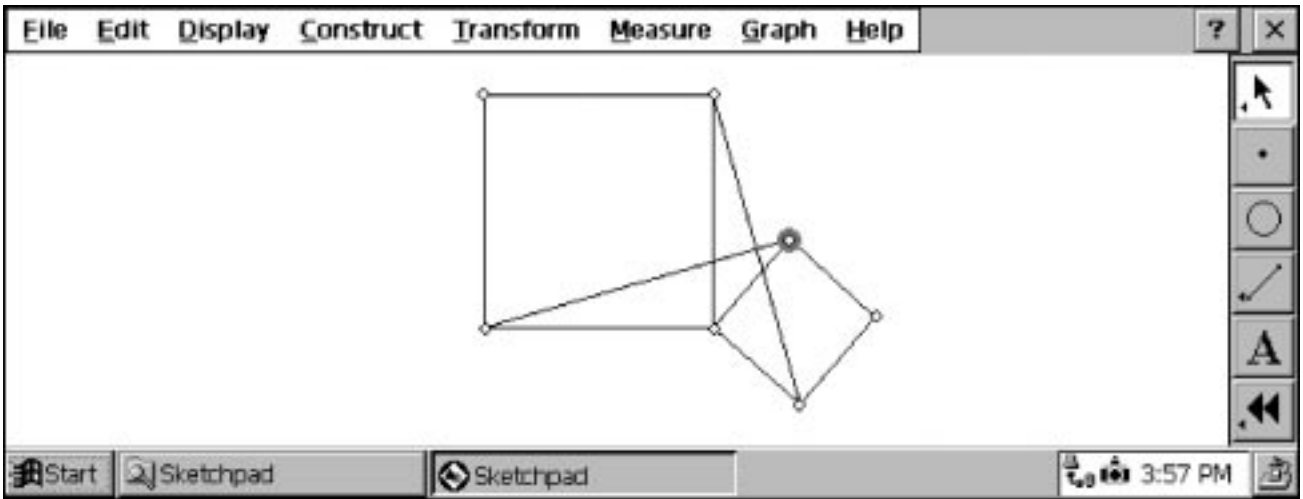
4) Delete unnecessary figures such as the auxiliary circle used to construct figures.

Hide the circle using the Display-Hide Circle command.

Hide A'' in the same way.

Draw the segments AA' and CC'.

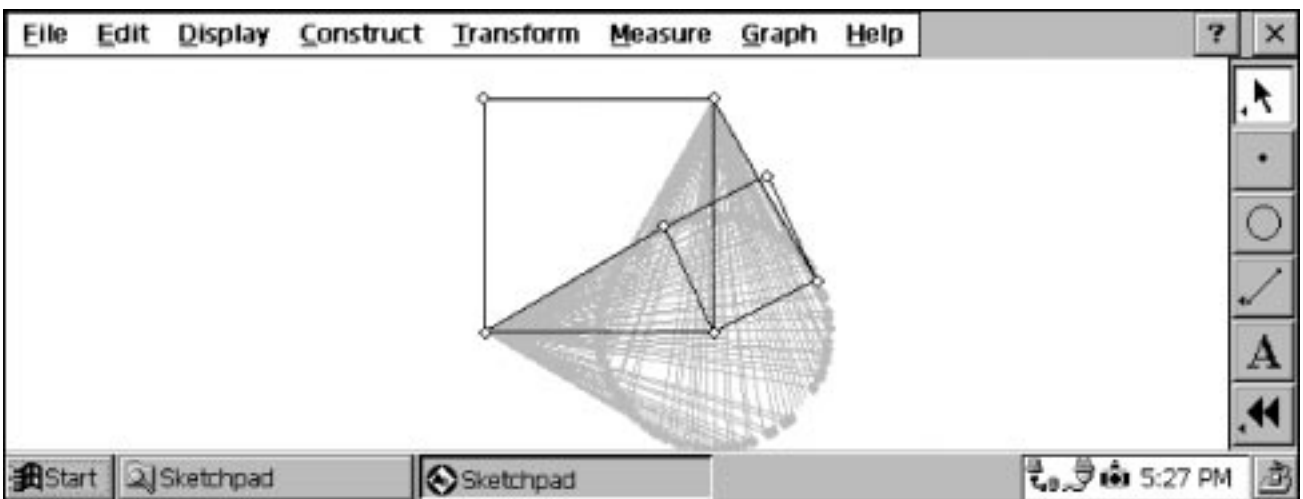
We can rotate the square by moving the point A'.



5) Draw the locus.

To draw the locus of the segment AA', specify the segment and specify the Trace Point command in the Display menu.

Move A' to draw the locus.



From above, we can find that the area swept by the segment AA' is equal to the area surrounded by the circumference of the circle B of radius 2 and its center B, plus the area surrounded by the two tangents to the circle B that pass through the point A and its circumference.

Solution:

When the segment AA' is a tangent to the circle B, $\angle AA'B = 90^\circ$.

AB = 4 and BA' = 2, therefore, the triangle AA'B is a right angle with $\angle ABA' = 60^\circ$.

The area is calculated as follows.

$$\begin{aligned} & 2 \times \left(\frac{1}{2} \times 2 \times 2\sqrt{3} \right) + \frac{240}{360} \times \pi \times 2^2 \\ & = 4\sqrt{3} + 8\pi/3 \text{ cm}^2. \end{aligned}$$

[We want the students to be as follows.]

As mentioned above, the students can understand visually which part changes and which part does not, by moving the figures constructed by them. At the same time, they can understand the reason why some parts do not change.

From these experiences, we can give them the mathematical definition and develop their aptitude for solving geometric problems.