WHY IS THE EPICENTER OF AN EARTHQUAKE LOCATED AS A SINGLE POINT?

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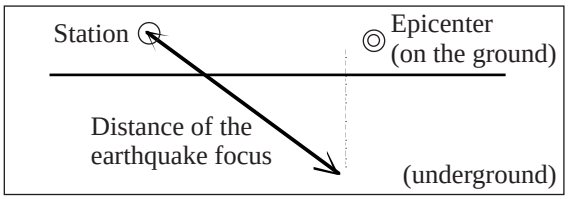
1. PROBLEM

It is known that the epicenter of an earthquake can be located from the distances between the earthquake focus and various seismograph stations. Why? Let's think it over with the Geometer's Sketchpad (referred to as GSP hereafter.)



2. THE AIM OF GUIDANCE AND THE ROLE OF GSP

(1) Aim of Guidance

1) We can find that the epicenter of an earthquake is located as a single point from distances between the earthquake focus and three stations by using GSP.
2) Some students may wonder why the epicenter is located as a single point and they try to prove it.
3) They can eventually prove the fact that the epicenter is located as a single point.

(2) Role of GSP

It prompts students to find the proposition to be proved, and the finding will give them an incentive to prove.

3. SUMMARY OF SOLVING STRATEGY

(1) Introduction

Teacher:

It is known that the epicenter of an earthquake can be located from the distances between the earthquake focus and various seismograph stations. Why? Let's think it over with the Geometer's Sketchpad (referred to as GSP hereafter.)

Albert: When only a distance between the earthquake focus and a station is known, we have only a circle that may include the epicenter and we cannot locate it.

Bob: When two distances between the earthquake focus and two stations are known, we have two circles and two crossings, and one of those may be the epicenter. (Figure 1)

Catherine: I do not think so. The epicenter is on the segment connecting the two crossings. The earthquake focus is under the ground and the epicenter is on the ground. (Figure 2) (Because the intersection of two spheres is a circle.)

Donald: If your remark is correct, we can locate the epicenter as a single point from three distances from the earthquake focus! (Figure 3)

(2) Checking that the epicenter is located as a single point using the Geometry Sketch Pad

Teacher: Is the epicenter located as a single point if three distances from the earthquake focus are given, according to Donald's remark?

Erick: The epicenter is not determined as a single point depending on conditions! (Figure 4)

Frank: It's true! The epicenter cannot be determined as a single point in this case.

George: But it does not happen actually.

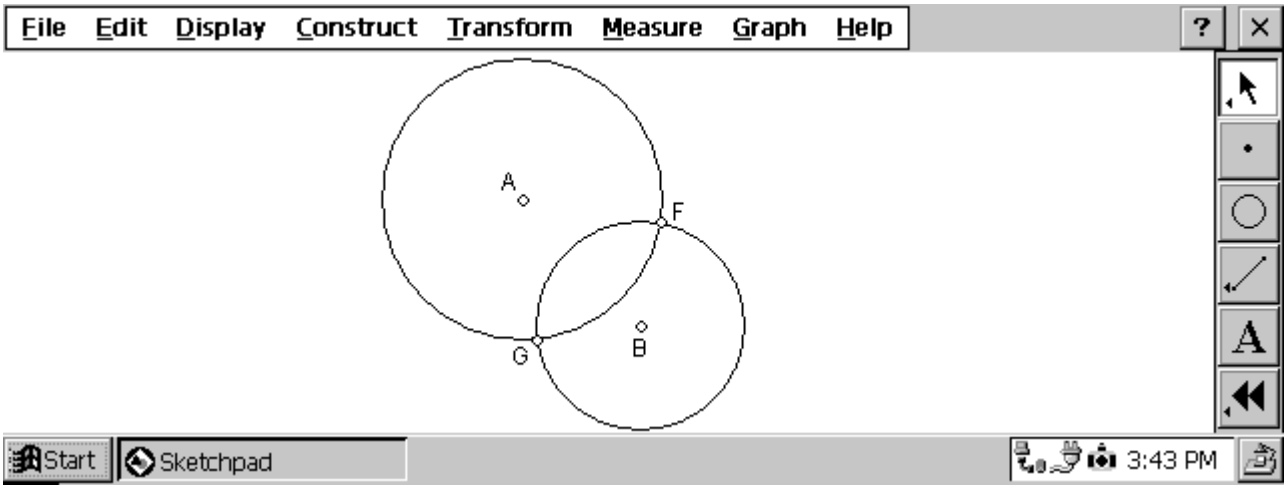


Figure 1: Epicenter is point F or G.

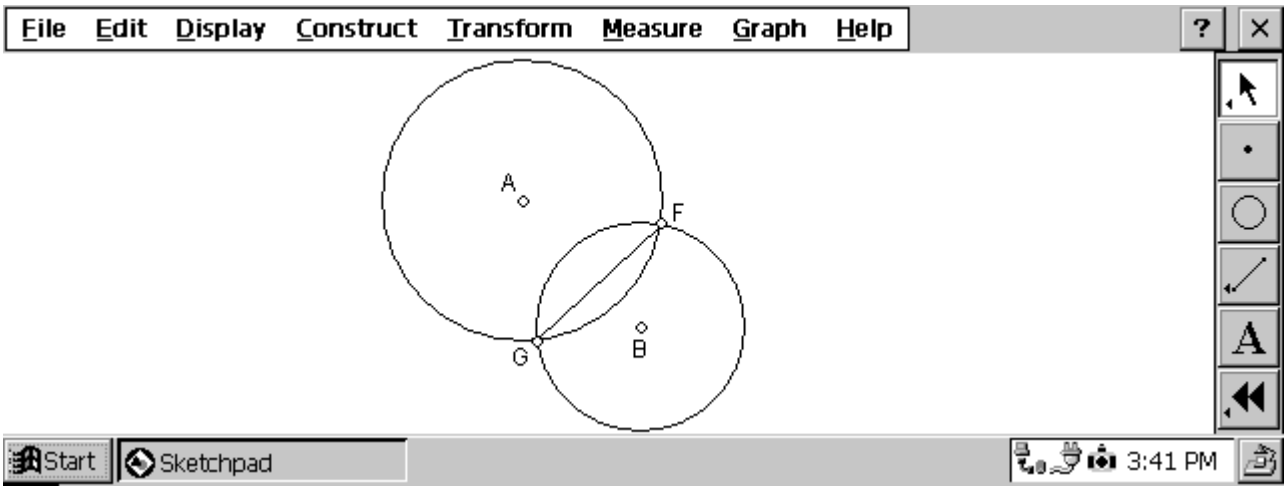


Figure 2: Epicenter is on segment FG.

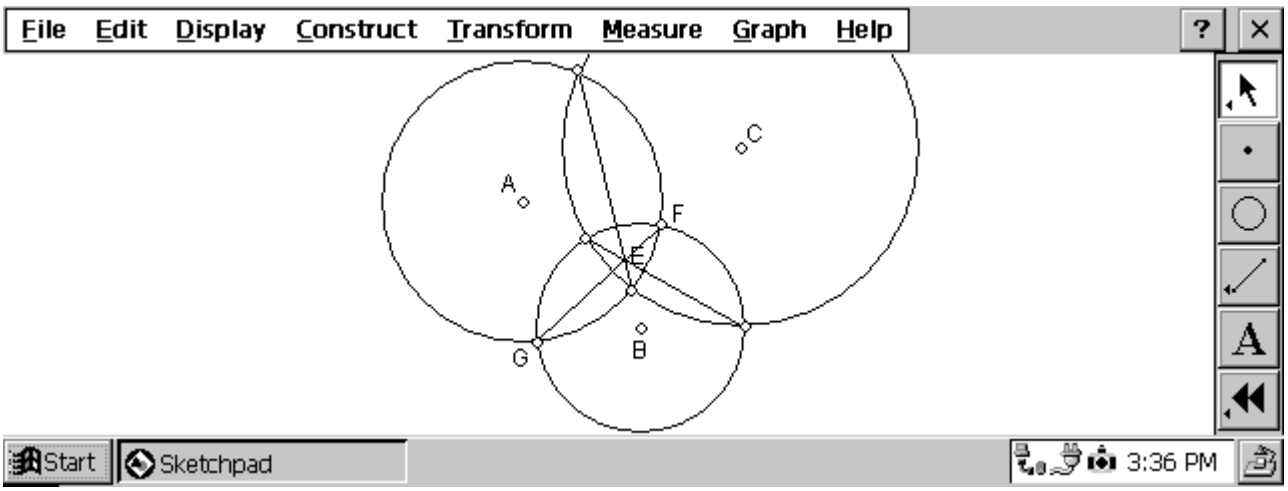


Figure 3: Epicenter is point E.

Henry: The case in Figure 5 does not exist actually. Only in Figure 3 we can determine the epicenter as a single point.

Ivan: Why is it always determined even when the center of the circle is moved or the radius is changed? It's mysterious.

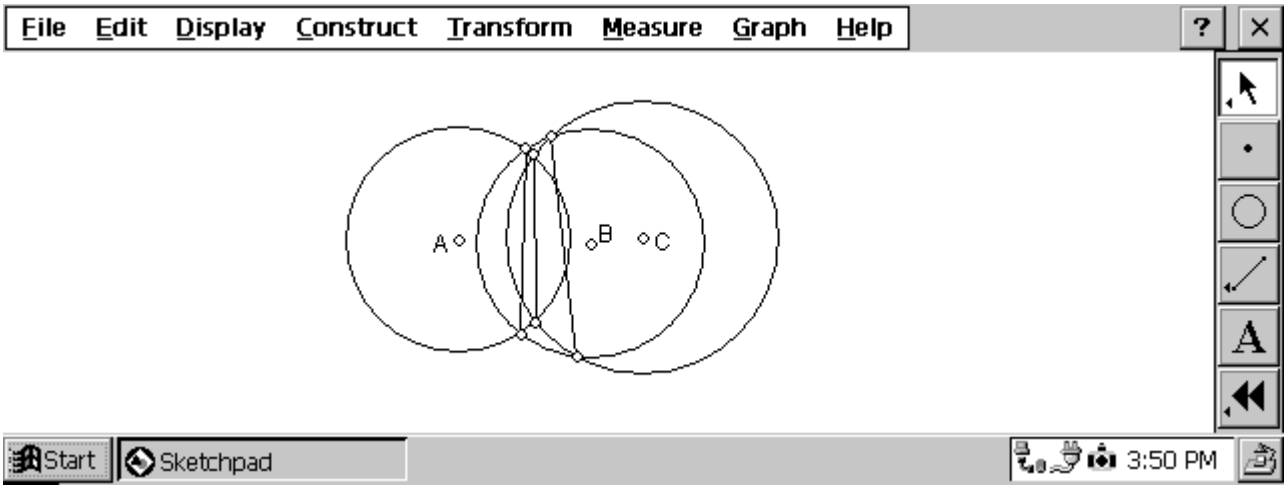


Figure 4: Example when the epicenter is not determined as a single point.

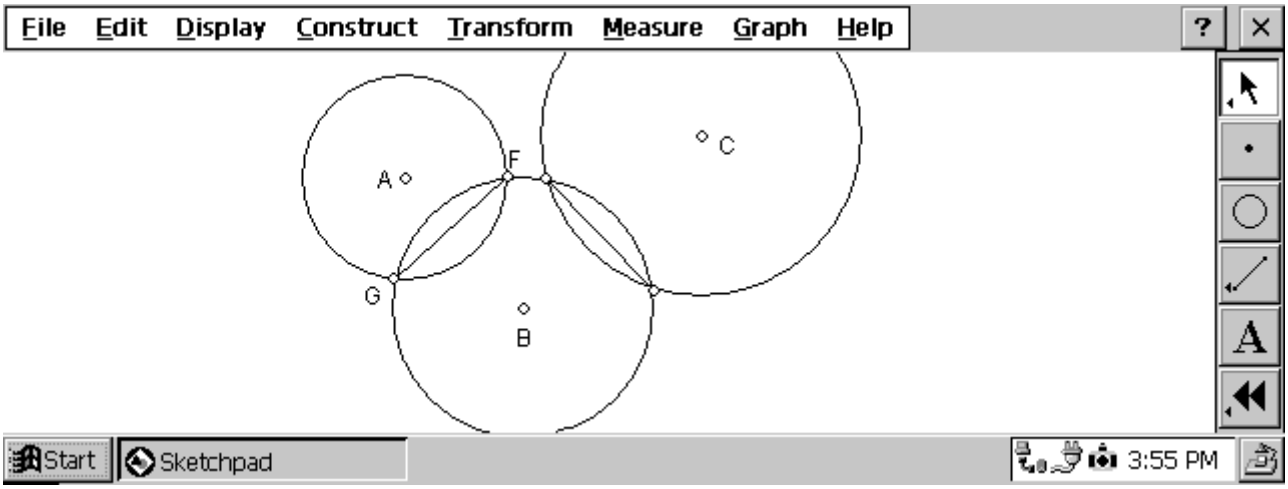


Figure 5: Example when the epicenter is not determined as a single point.

(3) Proving that the epicenter is determined as a single point

Teacher: Why located as a single point? Can you prove it?

John: I will do it by drawing three circles in the coordinate plane.

<Proof> We can express the three circles in the following expressions.

(Figure 6)

Circle O_A: $x^2 + y^2 = 1$

Circle O_B: $(x - b)^2 + y^2 = r_1^2$

Circle O_C: $(x - c)^2 + (y - d)^2 = r_2^2$

I will prove that the crossing of the two segments coincide with that of one of these two segment and the third segment.

Kelly: I will prove it using the reductio ad absurdum.

<Proof> We assume that the crossing E_1 of the common chord of Circles O_A and O_B and that of Circles O_A and O_C does not coincide with the crossing E_2 of the common chord of Circles O_A and O_B and that of Circles O_B and O_C. From this assumption, we will induce absurdity using the Pythagorean theorem.

Leon: I will prove differently.

(4) Presentation and discussion

We let the students present their solutions and discuss on them. We teach them that there are diverse ways of proving this proposition.

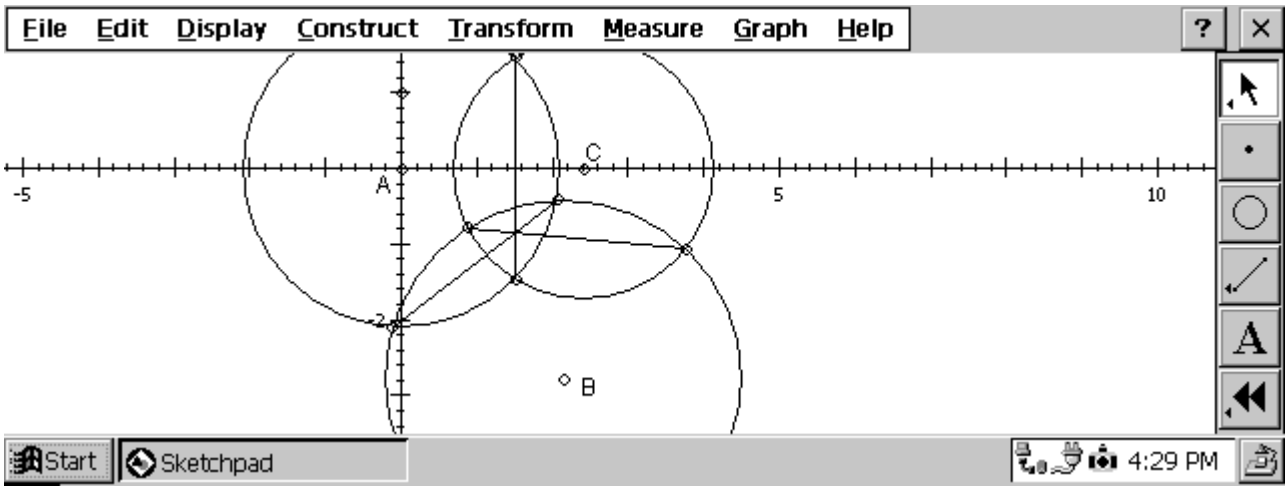


Figure 6

4. SUMMATION

As shown in this case, we can use GSP as an incentive to prove propositions. When we handle proofs in the class, we are apt to give them propositions that will turn out true. This is an inflexible and one-way teaching. I hope the students can find the proposition itself using GSP while they are wondering “Why is it so?” or “Can it be proved?” or “I want to prove it.” This will make the class of proofs more animated.