

# Reflector

## — Reflector of a bicycle —

Yokohama National University

Toshikazu Ikeda

### 1. Problem

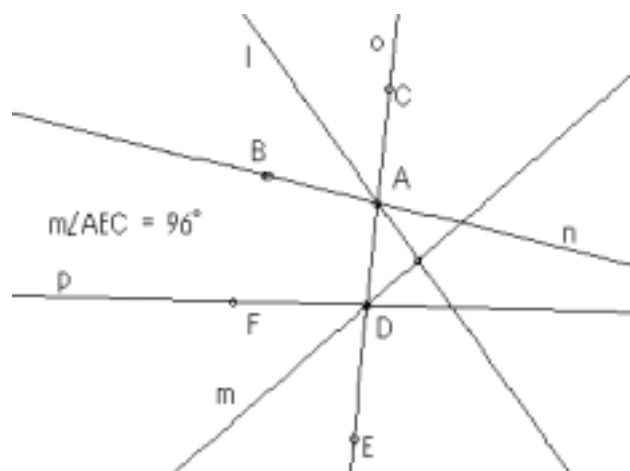
The reflector of a bicycle reflects light glaringly. What kind of structure does it have? If we conduct an experiment in a dark room using a penlight or the like based on the daily experiences, it seems that the reflector of a bicycle reflects light in the direction exactly opposite to that it comes. Taking the fact into account that the reflector adopts the principle of the mirror, and is composed of plural mirrors overlapped each other, let's investigate the angles with which each mirror is constructed.

### 2. Role of the Sketchpad

When we consider the problem in a two-dimensional plane, not in a three-dimensional space because of its difficulty, the Sketchpad can be used as a tool that shows how light is reflected according to the angle between mirrors. Though we can construct the paths of light using a compass and rulers, the Sketchpad can provide dynamic views of how light is reflected as the angles between mirrors are altered. By moving the figures, we can easily find the angle with which incident light and reflected light are parallel. In addition, we can observe the phenomena that incident light and reflected light become more parallel as the angle between mirrors decreased from 180 degrees to 90 degrees, and they cross each other and become less parallel as the angle decreases below 90 degrees. Furthermore, we can observe how the difference in size of mirrors affects the difference in distance between the reflected light and the luminous source.

### 3. Consideration in a two-dimensional plane

Let's consider the problem in a two-dimensional plane because it is difficult to do so in a three-dimensional space at first. Here we assume a special case in which light is reflected only by two mirrors. What is the measure of the angle if incident light and reflected light is parallel? Because it is troublesome to construct many figures, we use the Sketchpad. The procedure of constructing figures is as below. Screen 1 shows Steps 1 to 3.



(Screen 1)

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### Step 1

Construct two lines  $l$  and  $m$  (representing two mirrors), intersecting each other at a point  $G$ , and measure the angle between  $l$  and  $m$  (angle between two mirrors).

### Step 2

Construct a line  $n$  that intersects line  $l$  at a point  $A$ . Construct a point  $B$  on line  $n$  and a point  $C$  that is an image of point  $B$  with respect to line  $l$ .

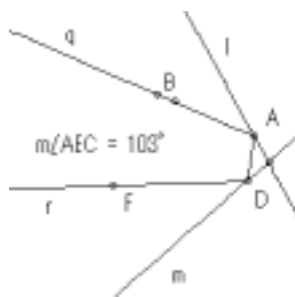
### Step 3

Construct a line  $o$  that passes through points  $C$  and  $A$ , and intersects line  $m$  at a point  $D$ . Construct a point  $E$  on line  $o$  and a point  $F$  that is an image of  $E$  with respect line  $m$ . Construct a line  $p$  that passes the points  $D$  and  $F$ .

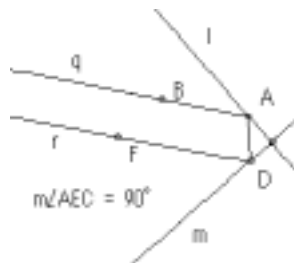
### Step 4

Construct a ray  $q$  that starts at point  $A$  and passes through point  $B$ , a segment  $AD$ , and a ray  $r$  that starts at point  $D$  and passes through point  $F$ . Then hide lines  $n$ ,  $o$ ,  $p$ , and points  $C$  and  $E$ .

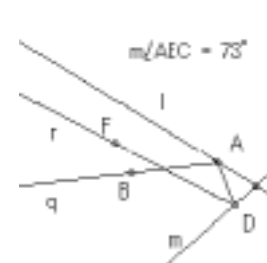
By moving lines  $l$  and  $m$  (corresponding to mirrors), we can observe how the incident light is reflected visually. When the angle between two mirrors is 90 degrees, the incident light and the reflected light are almost parallel. (Refer to Screens 2-1, 2-2, and 2-3.)



(Screen 2-1)

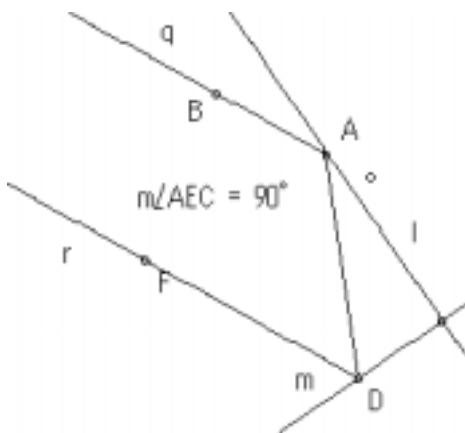


(Screen 2-2)

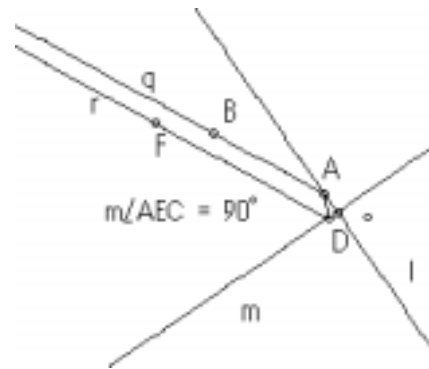


(Screen 2-3)

It will attract students' interest to disassemble the reflector actually to investigate the inside. A unit of mirrors is comprised of three mirrors, arranged protrusively in a form of three faces around a common vertex of a cubic, and many units overlap each other. These three faces are very small. In the case of two mirrors, the incident light and the reflected light are parallel when the angle between two mirrors is 90 degrees, as explained above. This fact can be observed in an experiment using two large mirrors. Why are these three mirrors so small? Now, let's move the incident light  $q$  closer and closer to the crossing of the two mirrors in the figure constructed by the Sketchpad before, with the angle between two mirrors fixed to 90 degrees. As a result, the distance between the incident light and the reflected light becomes shorter and shorter. (Refer to Screen 3-1.) On the contrary, the distance becomes longer and longer when we move the incident light further and further from the crossing. (Refer to Screen 3-3.) The larger the mirror, the further the reflected light from the crossing. In fact, it is more preferable if the reflected light comes back nearer to the luminous source, therefore, smaller mirrors are more useful in the reflector. This is the reason why small mirrors are used and overlapped each other.



(Screen 3-1)



(Screen 3-2)

#### 4. Consideration in a three-dimensional space

##### (1) Consideration in a plane

As in the two-dimensional consideration, let's consider in a plane including the path of light. When is the reflected light, which is reflected by three mirrors, parallel to the incident light? Using the Sketchpad, let's construct a figure as shown in Screen 3 as before.



(Screen 3)

We can select any incident angle and the angle between two adjacent mirrors changes accordingly. It is, however, impossible to adjust the angles so that the reflected light is parallel to the incident light. Does this mean that a reflector comprised of three-mirror shaped units is useless? A reflector comprised of two-mirror units is sufficient. In fact if we conduct the experiment again, we can observe the incident light and the reflected light are parallel. What is wrong?

Let's examine the consumption again. It is assumed so far that the paths of the incident light and reflected light are included in a plane. Is this true?

##### (2) Consideration in a projective plane

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Let's change the standpoint slightly and try to utilize the consideration in a two-dimensional plane. When light is reflected by three mirrors, we need to think the problem in a  $x$ - $y$ - $z$  space, but here, we think in a projected  $x$ - $y$  plane. A mirror  $Z$  is placed horizontally and two mirrors  $X$  and  $Y$  are placed so that they are perpendicular to mirror  $Z$ , and we can observe from above. In this manner, even junior high school students can observe how light is reflected by mirrors  $X$ ,  $Y$ , and  $Z$ , sequentially. When viewed from above, the relation between the incident light and the reflected light with respect to mirrors  $X$  and  $Y$  is the same as in a two-dimensional plane. As for mirror  $Z$ , however, light proceeds along a line when viewed from above. When light is reflected at a point  $p$  on mirror  $X$ , a point  $q$  on mirror  $Y$ , and a point  $r$  on mirror  $Z$ , we can construct the path of light when viewed from above as shown in FIG.1

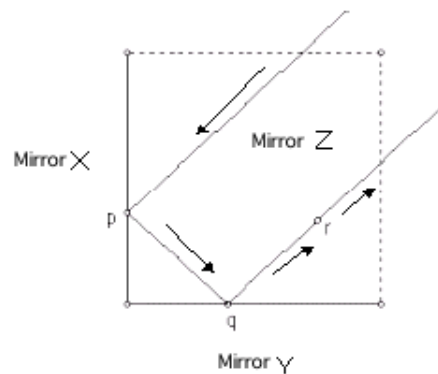


FIG.1 Path when viewed from above

As explained above, we can use the consideration on the path of light in a two-dimensional plane. Though the path of light in a vertical plane, that is, in the  $Z$  direction, is not cleared, we can observe that the reflected light and the incident light are parallel in the  $x$ - $y$  plane. To consider on the path of light in a vertical plane, we are required only to repeat the observation viewed from a different point. For example, if we place mirror  $X$  horizontally in the abovementioned experiment, we can observe the path of light in a plane parallel to the  $z$ -axis, and we can find the same phenomenon. As a result, we can conclude that the incident light and the reflected light are parallel in a three-dimensional space.

### (3) Consideration using vectors

High school students can solve the problem using vectors. First we explain using two-dimensional vectors, and then proceed to the cases using three-dimensional vectors.

Let an incident light vector in a two-dimensional plane  $A_0=(a, b)$ . As shown in FIG.2, when the incident light is reflected by  $Y$ -axis, only  $x$ -component is reversed in direction, the reflected light vector is  $A_1=(-a, b)$ . Similarly, when reflected by  $X$ -axis, only  $y$ -component is reversed in direction, therefore, the reflected light vector is  $A_2=(-a, -b)$ . As a result, we can conclude the incident light vector  $A_0=(a, b)$  and the light vector  $A_2=(-a, -b)$  reflected by  $Y$ - and  $X$ -axis are parallel. It is essential that  $x$ -component is reversed in direction when reflected by  $Y$ -axis, and  $x$ -component is reversed by  $X$ -axis.

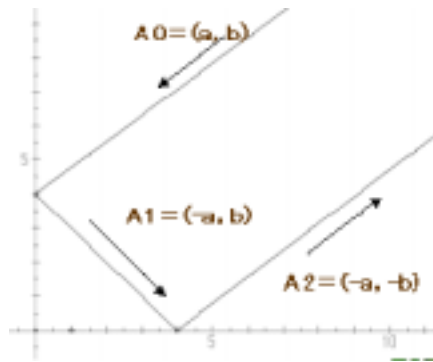


FIG.2

We can make use of the consideration on a two-dimensional plane when we consider the problem in a three-dimensional space. Let the incident light vector in a three-dimensional space be  $A_0 = (a, b, c)$ . When it is reflected by X-Z plane, y-component is reversed in direction, and the reflected vector is  $A_1 = (a, -b, c)$ . Similarly, when reflected by Y-Z plane, the vector is  $A_2 = (-a, -b, c)$ , and when reflected by X-Y plane, the vector is  $A_3 = (-a, -b, -c)$ . By comparing the incident light vector  $A_0 = (a, b, c)$  with the light vector  $A_3 = (-a, -b, -c)$ , we can find that the incident light and the reflected light are parallel but opposite in direction when the light is reflected by three mirrors in the reflector. In addition, we can see that mirrors as small as possible are used so that the reflected light will come back as nearer as possible to the luminous source.

Finally, I would like to thank Mr. Hirotake Ohno, a teacher of The Kamakura Junior High School attached to Yokohama National University, for his great assistance during the development of this educational material.