

The Slope of a Line

Name(s): _____

If you are a skier, you might describe the slope of a ski hill. If you are a carpenter, you might describe the slope of a roof you built. An economist might describe the slope of a graph. In this activity, you'll discover how to recognize the slopes of various lines. You will also play a game, with a partner, that challenges you to recognize various slopes.

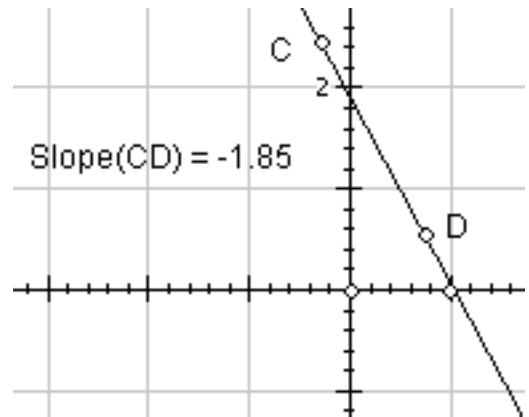
Sketch and Investigate

Preferences is in the
Edit menu. >

Hold the stylus down
on the **Segment** tool
to show the **Straight**
Object tools. Drag
right to choose the
Line tool. >

Select the line; then, in
the **Measure** menu,
choose **Slope**. >

1. In **Preferences**, on the **Units** tab, set the distance unit to cm.
2. In the **Graph** menu, choose **Show Grid**.
3. Draw any line.
4. Measure its slope.
5. Drag one of the line's control points and observe the effect on the line's slope.



6. Drag the line itself and observe the effect on its slope.

Q1 Continue to change the slope of your line. Investigate the following questions to prepare yourself for the Slope Game:

- a. Which lines have a positive slope and which have a negative slope?



- b. What is the slope of a horizontal line?



- c. How can you tell a steeper slope from a shallower slope?



- d. What is the slope of a vertical line?



The Slope of a Line (continued)

Playing the Slope Game

Play this game with a partner.

Choose the **Line** tool, then choose **Edit: Select: Select All Lines**. Now you can measure all the slopes at once.

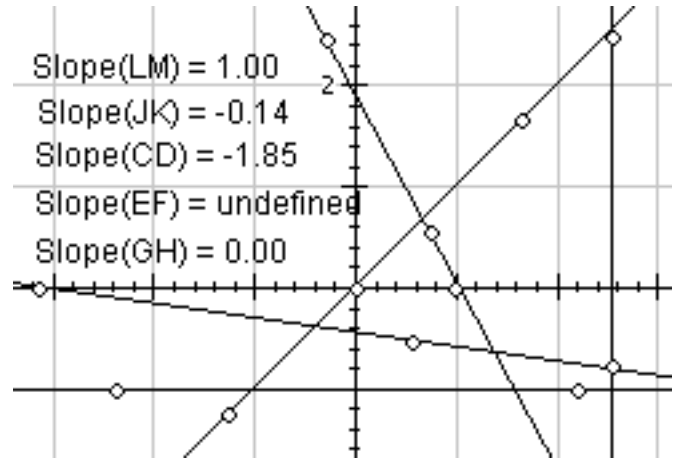
7. Draw five different random lines in your sketch.

8. Measure the slopes of the five lines.

9. Hide all the point labels.

10. Challenge your partner to match each measured slope with a line. Using the

Arrow, your partner is allowed to drag only measurements, to move them next to the lines they match. The lines and the points are “off limits” until all the measurements have been matched up with lines.



Tap in a blank area to make sure nothing is selected, then choose the **Point** tool. Choose **Edit: Select: Select All Points**. Then, in the **Display** menu, choose **Show Labels**.

11. To check how you scored, show all the point labels. Award one point for each correctly matched slope.

12. Switch roles, scramble the lines, and play the game again. Add a few more lines to make the game more challenging. Measure the slopes of the new lines.

Q2 Record your scores on a separate sheet.

Explore More

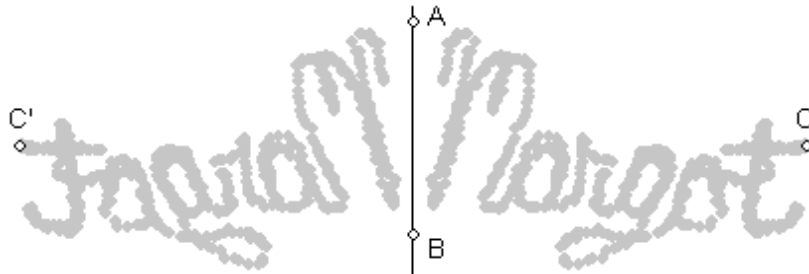
1. The *rise* of a sloped roof is the vertical distance between its lowest point and its highest point. The *run* is the horizontal distance between the lowest and highest points. Carpenters describe the slope of a roof by comparing the rise and the run. Sketch a scale drawing of a two-sided slanted roof that has a rise of 2 and a run of 3. Calculate the slopes of the two sides of the roof.

Properties of Reflection

Name(s): _____

When you look at yourself in a mirror, how far away does your image in the mirror appear to be? Why is it that your reflection looks just like you, but backwards? Reflections in geometry have some of the same properties of reflections you observe in a mirror. In this activity, you'll investigate the properties of reflections that make a reflection the "mirror image" of the original.

Sketch and Investigate: Mirror Writing



Holding Shift while you draw the line makes it easy to make the line vertical.

Double-tap on the line with the Arrow tool.

Select the two points; then, in the **Display** menu, choose **Trace Points**. A check mark indicates that tracing is turned on.

1. Construct vertical line AB .
2. Construct point C to the right of the line.
3. Mark \overleftrightarrow{AB} as a mirror.
4. Reflect point C to construct point C' .
5. Turn on **Trace Points** for points C and C' .
6. Drag point C so that it traces out your name.

Q 1 What does point C' trace?

When you want to erase the traces, choose **Clear All Traces** in the **Display** menu.



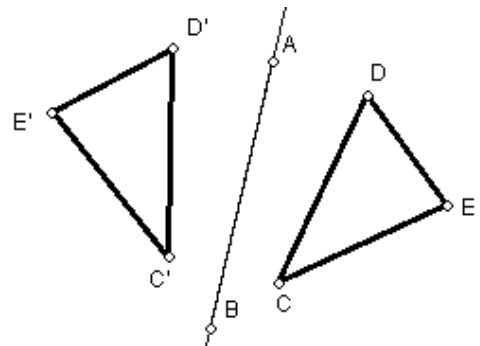
7. For a real challenge, try dragging point C' so that point C traces out your name.

Sketch and Investigate: Reflecting Geometric Figures

Select points C and C' . In the **Display** menu, you'll see **Trace Points** checked. Choose it to uncheck it.

Select the entire figure; then, in the **Transform** menu, choose **Reflect**.

8. Turn off **Trace Points** for points C and C' .
9. Construct CDE .
10. Reflect CDE (sides and vertices) over \overleftrightarrow{AB} .
11. Drag different parts of either triangle and observe how the triangles are related. Also drag the mirror line.



Properties of Reflection (continued)

Select three points that name the angle, with the vertex your middle selection. Then, in the **Measure** menu, choose **Angle**.

12. Measure the lengths of the sides of triangles CDE and $C'D'E'$.
13. Measure one angle in CDE and measure the corresponding angle in $C'D'E'$.

Q2 What effect does reflection have on lengths and angle measures?



Q3 Are a figure and its mirror image always congruent? State your answer as a conjecture.



Your answer to question Q4 demonstrates that a reflection reverses the **orientation** of a figure.

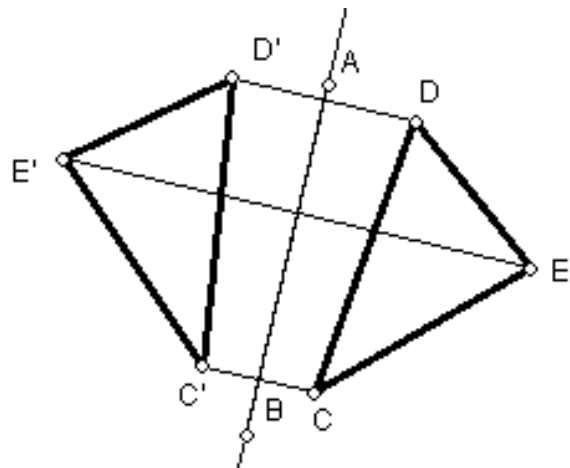
Q4 Going alphabetically from C to D to E in CDE , are the vertices oriented in a clockwise or counterclockwise direction? In what direction (clockwise or counterclockwise) are vertices C' , D' , and E' oriented in the reflected triangle?



14. Construct segments connecting each point and its image: C to C' , D to D' , and E to E' .

You may wish to construct points of intersection and measure distances to look for relationships between the mirror line and the connecting segments.

15. Drag different parts of the sketch around and observe relationships between the connecting segments and the mirror line.



Q5 How is the mirror line related to a segment connecting a point and its reflected image?



Explore More

1. Suppose Sketchpad didn't have a **Transform** menu. How could you construct a given point's mirror image over a given line? Try it. Start with a point and a line. Come up with a construction for the reflection of the point over the line using just the tools and the **Construct** menu. Describe your method.
2. Use a reflection to construct an isosceles triangle. Explain what you did.

Triangle Sum

Name(s): _____

This is a two-part investigation. First you'll investigate and make a conjecture about the sum of the measures of the angles in a triangle, then you'll continue sketching to demonstrate why your conjecture is true.

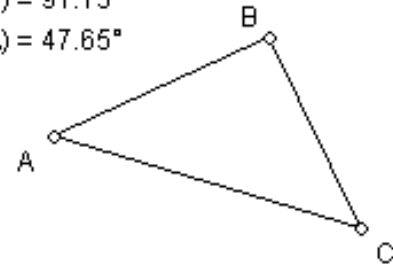
Sketch and Investigate

To measure an angle, select three points, with the vertex your middle selection. Then, in the **Measure** menu, choose **Angle**.

Choose **Calculate** from the **Measure** menu. Choose a measurement from the **Values** pop-up menu to enter it into a calculation.

1. Construct ABC .
2. Measure its three angles.
3. Calculate the sum of the angle measures.
4. Drag a vertex of the triangle and observe the angle sum.

Angle(C-A-B) = 41.20°
 Angle(A-B-C) = 91.15°
 Angle(B-C-A) = 47.65°



Q1 What is the sum of the angles in any triangle? _____

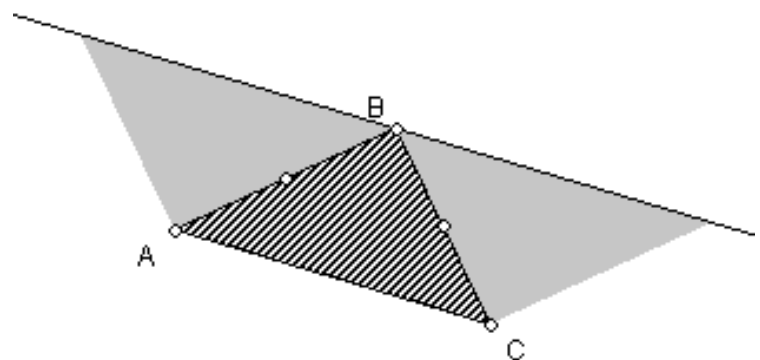
Select point B and \overline{AC} ; then, in the **Construct** menu, choose **Parallel Line**.

Select the three vertices; then, in the **Construct** menu, choose **Triangle Interior**.

Double-tap on the point to mark it as a center. Select the interior; then, in the **Transform** menu, choose **Rotate**.

Follow these steps to investigate why your conjecture is true.

5. Construct a line through point B parallel to \overline{AC} .
6. Construct the midpoints of \overline{AB} and \overline{CB} .
7. Construct the interior of ABC .
8. Mark one of the midpoints as a center for rotation and rotate the interior by 180° about this point.
9. Mark the other midpoint as a center and rotate the interior by 180° about this point.
10. Drag point B and observe how the three triangles are related to each other and to the parallel line.



Q2 Explain how each of the three angles at point B is related to one of the three angles in the triangle. Explain how this demonstrates your conjecture from question 1.



The Euler Segment

Name(s): _____

In this investigation, you'll look for a relationship among four points of concurrency: the incenter, the circumcenter, the orthocenter, and the centroid.

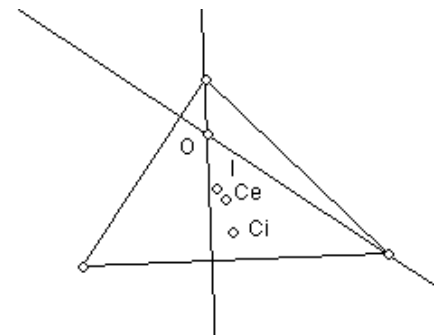
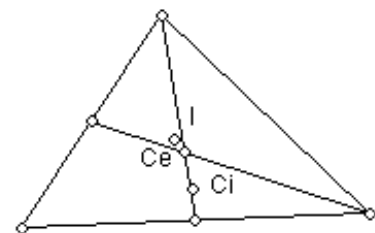
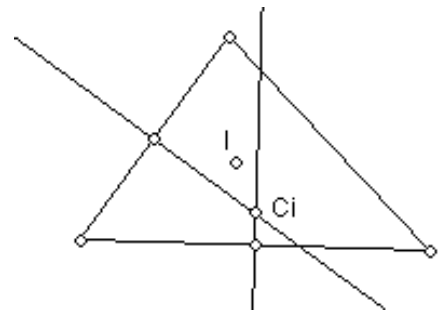
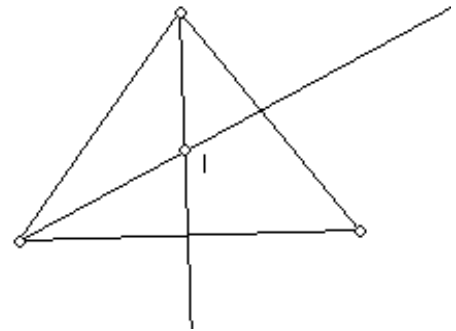
Select three points, with your middle selection the vertex of the angle you wish to bisect, then choose **Construct: Angle Bisector.**

Double-click on the point with the **Text** tool to edit its label.

Select the rays; then, in the **Display** menu, choose **Hide Rays.**

Sketch and Investigate

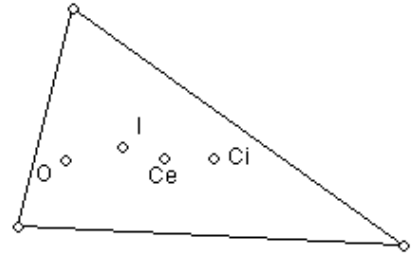
1. Construct a triangle.
2. Construct two angle bisectors in the triangle.
3. Construct the point of intersection of these angle bisectors and label it I for incenter.
4. Hide the angle bisectors.
5. Construct the midpoints of two sides of the triangle.
6. Construct a line through one midpoint perpendicular to the side. Repeat for the other midpoint and side.
7. Construct the point of intersection of these perpendicular bisectors and label it C_i for circumcenter.
8. Hide the perpendicular bisectors (but not the midpoints yet).
9. Construct two medians: segments connecting vertices to the midpoints of opposite sides.
10. Construct the point of intersection of these medians and label it C_e for centroid.
11. Hide the medians and the midpoints.
12. Construct two altitudes in the triangle.
13. Construct the point of intersection of these altitudes and label it O for orthocenter.



The Euler Segment (continued)

14. Hide the altitudes.
15. Drag your triangle around and observe how the points behave.

Q1 Three of the four points are always collinear. Which three?



16. Construct a segment that contains the three collinear points. This is called the **Euler segment**.
17. Drag the triangle again and look for interesting relationships on the Euler segment. Be sure to check special triangles, such as isosceles and right triangles.

Q2 Describe any special triangles in which the triangle centers are related in interesting ways or located in interesting places.



Q3 Which of the three points are always endpoints of the Euler segment and which point is always between them?



To measure the distance between two points, select the two points. Then, in the **Measure** menu, choose **Distance**. (Measuring the distance between points is an easy way to measure the length of part of a segment.)

18. Measure the distances along the two parts of the Euler segment.
 19. Drag the triangle and look for a relationship between these lengths.
- Q4 How are the lengths of the two parts of the Euler segment related? Test your conjecture using the calculator.



Explore More

1. Construct a circle centered at the midpoint of the Euler segment and passing through the midpoint of one of the sides of the triangle. This circle is called the **nine-point circle**. The midpoint it passes through is one of the nine points. What are the other eight? Hint: Six of them have to do with the altitudes and the orthocenter.
2. Once you've constructed the nine-point circle, as described above, drag your triangle around and investigate special triangles. Describe any triangles in which some of the nine points coincide.

Napoleon's Theorem

Name(s): _____

French emperor Napoleon Bonaparte fancied himself as something of an amateur geometer and liked to hang out with mathematicians. The theorem you'll investigate in this activity is attributed to him.

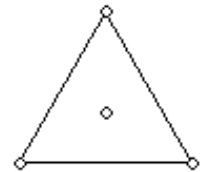
Sketch and Investigate

1. Construct an equilateral triangle. You can use a premade tool or construct the triangle from scratch. Make sure that the triangle is controlled by two vertices (not the center). If you use a premade tool, delete the interior if necessary.

One way to construct the center is to construct two medians and their point of intersection.

2. Construct the center of the triangle.

3. Hide anything extra you may have constructed to construct the triangle and its center so that you're left with a figure like the one at right.



Select the entire figure. Tap the Custom tool, then choose Create Tool.

4. Create a tool of this construction.

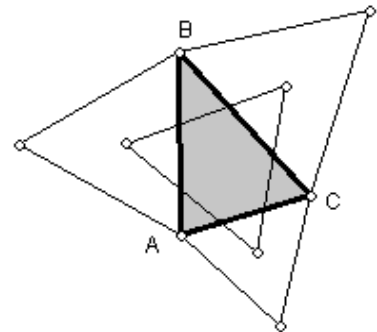
Next, you'll use your tool to construct equilateral triangles on the sides of an arbitrary triangle.

5. Open a new sketch.

Select points A, B, and C. Then, in the Construct menu, choose Triangle Interior.

6. Construct $\triangle ABC$.

7. Construct the interior of $\triangle ABC$.



Be sure to attach each equilateral triangle to a pair of triangle ABC's vertices. If your equilateral triangle goes the wrong way (overlaps the interior of ABC) or is not attached properly, undo and try attaching it again.

8. Use the new tool to construct equilateral triangles on each side of $\triangle ABC$.

9. Drag to make sure each equilateral triangle is stuck to a side.

10. Construct segments connecting the centers of the equilateral triangles.

11. Drag the vertices of the original triangle and observe the triangle formed by the centers of the equilateral triangles. This triangle is called the outer Napoleon triangle of $\triangle ABC$.

Q1 State what you think Napoleon's theorem might be.



Explore More

1. Construct segments connecting each vertex of your original triangle with the most remote vertex of the equilateral triangle on the opposite side. What can you say about these three segments?

Midpoint Quadrilaterals

Name(s): _____

In this investigation, you'll discover something surprising about the quadrilateral formed by connecting the midpoints of another quadrilateral.

Sketch and Investigate

If you select all four sides, you can construct all four midpoints at once.

1. Construct quadrilateral $ABCD$.

2. Construct the midpoints of the sides.

3. Connect the midpoints to construct another quadrilateral, $EFGH$.

4. Drag vertices of your original quadrilateral and observe the midpoint quadrilateral.

5. Measure the four side lengths of this midpoint quadrilateral.

6. Measure the slopes of the four sides of the midpoint quadrilateral.

Q1 What kind of quadrilateral does the midpoint quadrilateral appear to be? How do the measurements support that conjecture?

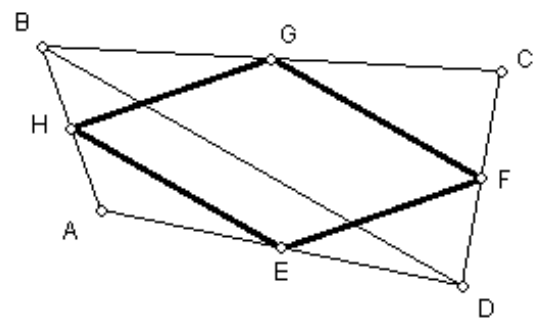
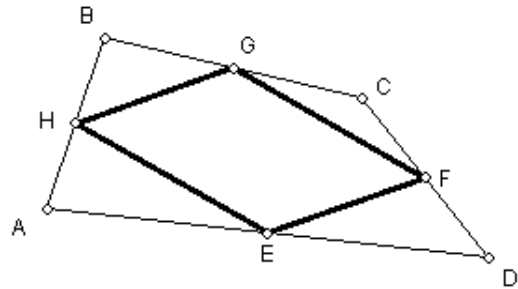


7. Construct a diagonal.

8. Measure the length and slope of the diagonal.

9. Drag vertices of the original quadrilateral and observe how the length and slope of the diagonal are related to the lengths and slopes of the sides of the midpoint quadrilateral.

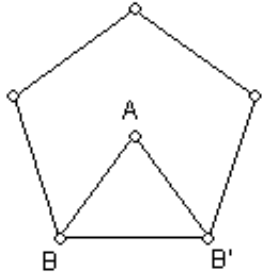
Q2 The diagonal divides the original quadrilateral into two triangles. Each triangle has as a midsegment one of the sides of the midpoint quadrilateral. Use this fact and what you know about the slope and length of the diagonal to write a paragraph explaining why the conjecture you made in question 1 is true. Use a separate sheet of paper if necessary.



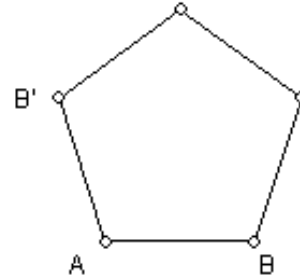
Constructing Regular Polygons

Name(s): _____

A **regular polygon** is a polygon whose sides all have equal length and whose angles all have equal measure. The easiest way to construct regular polygons with Sketchpad is to use rotations. The figures below show pentagons constructed by two different methods.



The pentagon above was constructed by rotating vertex B around center point A . BAB' is a **central angle** of the polygon.



The pentagon above was constructed by rotating vertex B around vertex A . $B'AB$ is an **interior angle** of the polygon.

Before you rotate anything, you must mark a center of rotation. Double-tap on a point to mark it as a center. Select what you want to rotate; then, in the Transform menu, choose **Rotate**.

Experiment with using rotations to construct different regular polygons. Figure out central angle measures to rotate by and interior angle measures to rotate by. Each time you make a polygon that seems correct, drag points to make sure it holds together. Create tools of your successful constructions to use later. Fill in the chart below with the central and interior angle measures for the named regular polygons, whether you have time to construct them all or not. Indicate which constructions you created tools for.

To create a tool, select the entire figure. Then tap the **Custom** tool and choose **Create tool**.

Polygon	Central Angle Measure	Interior Angle Measure	Saved tool? (Y or N)
Equilateral			
Square			
Regular Pentagon (5)			
Regular Hexagon (6)			
Regular Octagon (8)			
Regular Nonagon (9)			
Regular Decagon (10)			
Regular n -gon			

Explore More

1. The regular heptagon (seven sides) doesn't appear on the chart because the angle measures aren't "nice." What are they? To construct the regular heptagon, use the calculator to calculate an expression for the desired angle, then mark that measurement as an angle for rotation.

The Cycloid

Name(s): _____

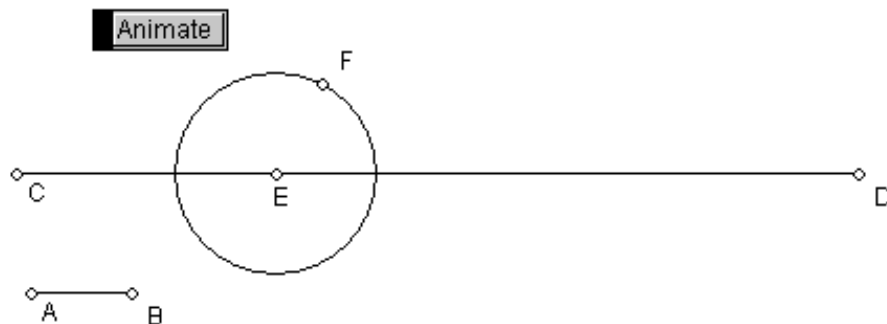
Imagine that a bug is clinging tightly to a bicycle wheel as the bicycle travels down the road. What path does the bug travel? Is the path the same whether the bug is at the center of the wheel or at its edge? In this activity, you'll investigate those questions.

Sketch and Investigate

1. Construct a small segment AB in the bottom left corner of your sketch.
2. Construct a long, horizontal segment CD .
3. Construct a point E on \overline{CD} .
4. Construct a circle with center point E and radius AB .
5. Construct a point F on this circle.
6. While this point is selected, choose **Trace Point** in the **Display** menu.
7. Make an action button that animates point E forward (left to right) along the segment slowly and point F backward (clockwise) around the circle slowly.

Select point E and \overline{AB} ; then, in the **Construct** menu, choose **Circle By Center+Radius**.

Select points E and F . Choose **Edit: Action Button: Animation**. Use the pop-up menus in the **Animate Properties** dialog box to choose direction and speed.



8. Tap on the **Animate** button to watch the wheel roll.

The path of point F as the wheel rolls is called a **cycloid**. Unless you got lucky, your cycloid traces probably start to make a mess if you leave the animation running for a while. You can make the traced point start in the same place every time by making the segment length a multiple of the circle's circumference. Follow these steps.

9. Stop the animation and clear the traces.
10. Measure the segment's length and the circle's circumference.
11. Drag point B to make the circumference close to a convenient number (such as 5 cm or 2 in.), then drag point C or D to make CD as close as you can to twice the circumference.

Choose **Display: Motion: Stop Animation**. Then choose **Display: Clear All Traces**.

The Cycloid (continued)

12. Tap the action button. Point F should trace two **cycles** of the cycloid curve, then return to the start of the segment and trace over these cycles again. (Don't worry if it's off just a bit.)

Follow the steps below to extend your investigation to points inside or outside the circle.

13. Stop the animation and clear the traces.
14. Construct \vec{EF} .
15. Construct point G on \vec{EF} .
16. Turn on **Trace Point** for point G , and turn it off for point F .
17. Try the animation with point G outside the circle and again with point G inside the circle.

Q1 In the space below, sketch what the cycloid looks like when point G is inside the circle, on the circle, and outside the circle.

Inside the circle	On the circle	Outside the circle

18. Adjust your sketch so that point G traces three cycles of the curve.

Q2 How are the circumference of the circle and the length of the segment related when point G traces three cycles?



Q3 Because the cycloid curve repeats itself, it is called **periodic**. The distance from a point on one cycle to the corresponding point on the next cycle (for example, the distance from a peak to a peak) is called the **period** of the curve. What would be the period of the curve if the circle had a radius of 1 cm?



Q4 Is the period different if point G is outside the circle instead of inside or on the circle?



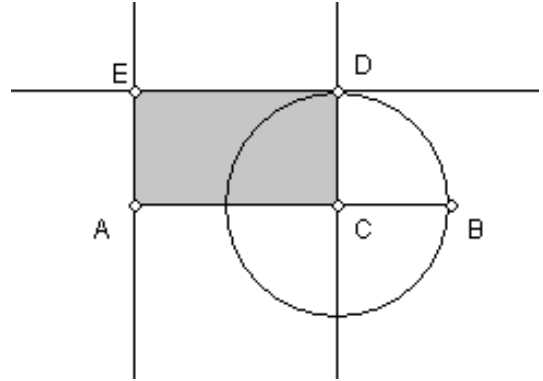
A Rectangle with Maximum Area

Name(s): _____

Suppose you had a certain amount of fence and you wanted to use it to enclose the biggest possible rectangular field. What rectangle shape would you choose? In other words, what type of rectangle has the most area for a given perimeter? You'll discover the answer in this investigation. Or, if you have a hunch already, this investigation will help confirm your hunch and give you more insight into it.

Sketch and Investigate

1. Construct \overline{AB} .
2. Construct point C on \overline{AB} .
3. Construct lines perpendicular to \overline{AB} through points A and C .
4. Construct circle CB .
5. Construct point D where this circle intersects the perpendicular line.



6. Construct a line through point D , parallel to \overline{AB} .
7. Construct point E , the fourth vertex of rectangle $ACDE$.
8. Construct polygon interior $ACDE$.
9. Measure the area and perimeter of this polygon.
10. Drag point C back and forth and observe how this affects the area and perimeter of the rectangle.
11. Measure AC and AE .

Q1 Without measuring, state how AB is related to the perimeter of the rectangle. Explain why this rectangle has a fixed perimeter.



Q2 As you drag point C , observe what rectangular shape gives the greatest area. What shape do you think that is?



In Steps 12–14, you'll explore this relationship graphically.

12. Plot the measures AC and Area $ACDE$ as (x, y) . You should get axes and a point H as shown on the next page.

Select \overline{AB} , point A , and point C . Then, in the **Construct** menu, choose **Perpendicular Line**.

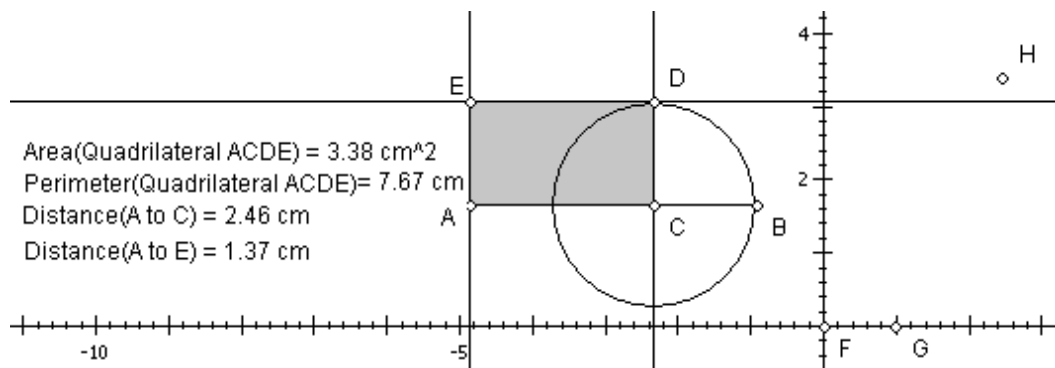
Select the vertices of the rectangle in consecutive order. Then, in the **Construct** menu, choose **Quadrilateral Interior**.

Select point A and point C . Then, in the **Measure** menu, choose **Distance**. Repeat to measure AE .

Select measurements AC and Area $CDEA$, in that order. Then, in the **Graph** menu, choose **Plot As (x, y)** . If you can't see point H , drag point G to scale the axes.

A Rectangle with Maximum Area (continued)

13. Drag point C to see point H move to correspond to different side lengths and areas.



Select point H and point C ; then, in the **Construct** menu, choose **Locus**.

14. To see a graph of all possible areas for this rectangle, construct the locus of point H as defined by point C . It should now be easy to position point C so that point H is at a maximum value for the area of the rectangle.

You may wish to select point H and measure its coordinates.

- Q3 Explain what the coordinates of the high point on the graph are and how they are related to the side lengths and area of the rectangle.



15. Drag point C so that point H moves back and forth between the two low points on the graph.

- Q4 Explain what the coordinates of the two low points on the graph are and how they are related to the side lengths and area of the rectangle.



Explore More

- Investigate area/perimeter relationships in other polygons. Make a conjecture about what kinds of polygons yield the greatest area for a given perimeter.
- What's the equation for the graph you made? Let AC be x and let AB be $(1/2)P$, where P stands for perimeter (a constant). Write an equation for area, A , in terms of x and P . What value for x (in terms of P) gives a maximum value for A ?

The Pythagorean Theorem

Name(s): _____

In this investigation, you'll create a tool for constructing a square, then you'll construct squares on the sides of a right triangle. The areas of these squares illustrate perhaps the most famous relationship in mathematics—the Pythagorean theorem.

Sketch and Investigate

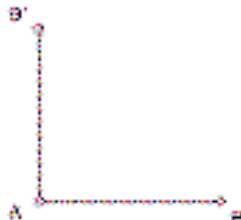
Double-tap on point A to mark it as a center. Select point B and \overline{AB} , then, in the **Transform** menu, choose **Rotate**.

Select the vertices in consecutive order; then, in the **Construct** menu, choose **Quadrilateral Interior**.

1. Construct \overline{AB} .
2. Mark point A as a center and rotate point B and \overline{AB} by 90° .
3. Mark point B' as a center and rotate point A and $\overline{B'A}$ by 90° .
4. Construct $\overline{A'B}$ to finish the square.
5. Construct the square's interior.



Step 1



Step 2



Step 3



Steps 4 and 5

Use the **Text** tool and tap on each point.

Select the entire figure; then tap the **Custom** tool and choose **Create Tool**.

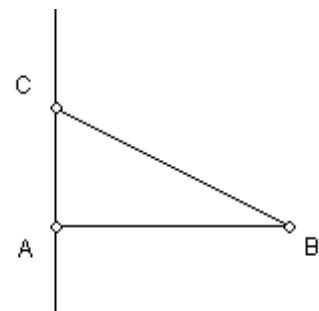
6. Drag each vertex of the square to make sure it holds together.
7. Hide the labels.
8. Create a tool of this construction.

Q1 What properties of a square did you use in this construction?



9. Experiment with using the tool to get a feel for the way it works. Note that the direction in which the square is constructed depends on which way you drag when you use the tool.

10. Open a new sketch.
11. Construct \overline{AB} .
12. Construct a line through point A perpendicular to \overline{AB} .
13. Construct \overline{BC} , where point C is a point on the perpendicular line.



Select point A and \overline{AB} and, in the **Construct** menu, choose **Perpendicular Line**.

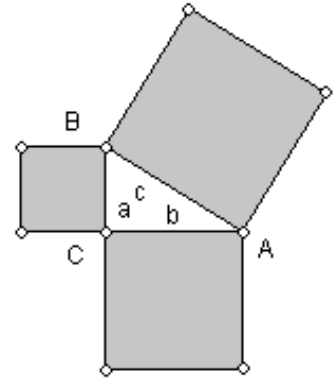
The Pythagorean Theorem (continued)

14. Hide the perpendicular line and construct \overline{AC} .
 15. Drag each vertex to confirm that your triangle stays a right triangle.
- Q2 What property of a right triangle did you use in your construction?



To change a label, double-tap on the label with the **Text** tool. (The reason for changing these labels is so that your figure will match the way the theorem is usually stated. This may make it easier to remember the theorem.)

16. Change labels so that the right-angle vertex is labeled C and the other two vertices are labeled A and B .
17. Show the labels of the sides. Change them to a , b , and c so that side a is opposite A , side b is opposite B , and side c is opposite C .



Be sure to attach each square to a pair of the triangle's vertices. If your square goes the wrong way (overlaps the interior of your triangle) or is not attached properly, undo and try attaching the square to the triangle's vertices in the opposite order.

18. Use your square tool to construct squares on the sides of your triangle.
19. Drag the vertices of the triangle to make sure the squares are properly attached.
20. Measure the areas of the three squares.
21. Measure the lengths of sides a , b , and c .
22. Drag each vertex of the triangle and observe the measures.

Choose **Calculate** from the **Measure** menu. Choose a measurement from the **Values** pop-up menu to enter it into a calculation.

- Q3 Describe any relationship you see among the three areas. Use the calculator to create an expression that confirms your observations.



- Q4 Based on your observations about the areas of the squares, write an equation that relates a , b , and c in any right triangle. (Hint: What's the area of the square with side length a ? What are the areas of the squares with side lengths b and c ? How are these areas related?)



Explore More

1. Do a similar investigation using other figures besides squares. Does your conjecture about the areas still hold?
2. Investigate the converse of the Pythagorean theorem: Construct a non-right triangle and squares on its sides. Measure the areas of the squares and sum two of them. Drag until the sum is equal to the third area. What kind of triangle do you have?

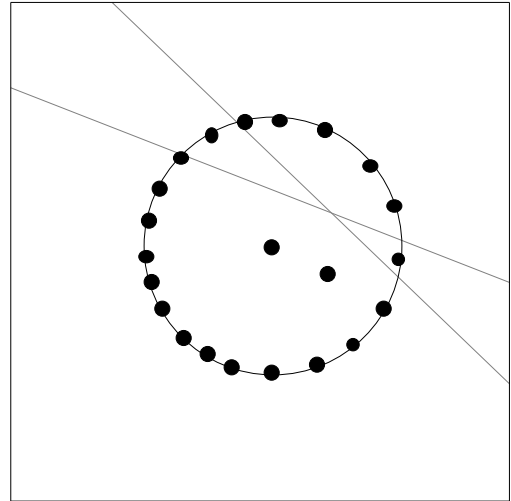
Constructing Conic Sections

In this activity you'll construct conic sections two ways, by paper-folding and by an equivalent construction method using Sketchpad. Your Sketchpad constructions will have the advantage that you can manipulate them dynamically.

Investigate

First you'll do a paper-folding construction.

1. On a piece of tracing paper (patty paper works well), draw a circle with about a 3-inch diameter. (Use a compass or draw around the rim of a cup or other circular object.)
2. Draw a point inside the circle, not at the center and not too close to the circumference.
3. Draw about 20 points on the circumference of the circle.
4. Fold the paper so that the point inside the circle lands on one of the points on the circle, and crease the paper. Unfold, then fold the inside point onto the next point on the circle. Repeat, folding and unfolding so that the inside point is folded onto each of the 20 points around the circle.



Q1 The creases on your paper should form a pattern called an “envelope” of lines. What shape does the border of this envelope appear to be?



5. Repeat steps 1–4 above on a new piece of paper, but this time, instead of drawing a point inside the circle in step 2, draw a point outside the circle.

Q2 What kind of shape is this envelope?

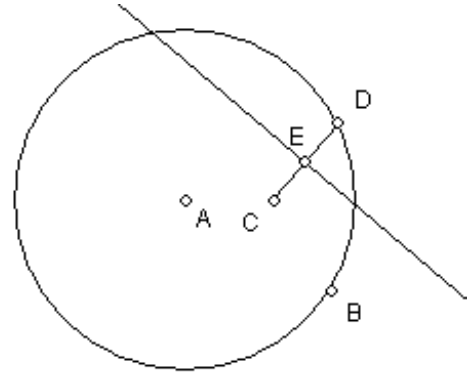


Constructing Conic Sections (continued)

Sketch and Investigate

Now you'll do an equivalent construction using Sketchpad.

6. Construct a circle AB .
7. Construct a \overline{CD} , where point C is a point inside the circle and point D is a point on the circle.
8. Construct the midpoint E of \overline{CD} .
9. Construct a line through point E , perpendicular to \overline{CD} . This line is the perpendicular bisector of \overline{CD} . Can you see why it would be the fold line if you were to fold point C onto point D ?



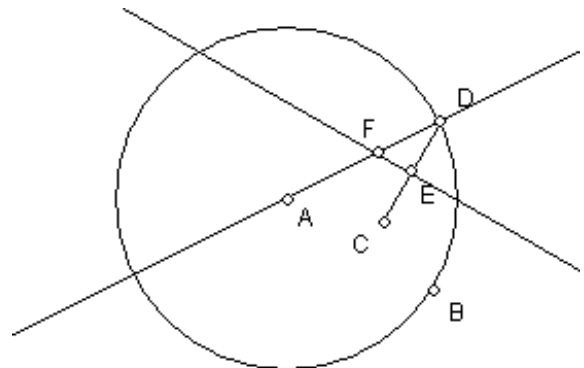
Select the line and point D ; then, in the **Construct** menu, choose **Locus**.

10. Construct the locus of this perpendicular line as defined by point D .
- Q3** The locus is an envelope of lines, just like the lines you folded. What shape is this envelope? Drag point C and observe how it changes the locus. What's the shape of the envelope when you drag point C outside the circle?



So far, you have an envelope of lines that surround a conic section, but you don't have the actual curve. To construct the curve, you need to construct a point locus instead of a line locus. If you want an extra challenge, see if you can figure out how to construct the point whose locus is the curve you want. If you get stuck, follow these steps.

11. Hide the line locus (the gray envelope of lines, but not the perpendicular bisector).
12. Construct line AD .
13. Construct the point of intersection, F , of \overleftrightarrow{AD} and the perpendicular bisector.
14. Construct the locus of point F as defined by point D .
15. Drag point C and observe how this locus changes as point C moves from inside the circle to outside it.



Constructing Conic Sections (continued)

Q4 Move point C back inside the circle and explain why the locus of point F is an ellipse. See the hints below.

- The definition of an ellipse as applied to the points in this sketch is “the set of points F such the the sum of the distances from point F to two fixed points A and C (the foci) is a constant.”
- In this construction, the constant sum is equal to the radius of the circle.



Q5 Move point D outside the circle and explain why the locus of point F is an hyperbola. Recall that the definition of a hyperbola is the set of points F such that the *difference* between the distances from point F to two fixed points is a constant.



Explore More

On a sheet of tracing paper, draw a line and a point not on the line. Mark several points on the line, and fold the point off the line onto each of these points. What curve does the envelope of creases surround? Construct this curve in Sketchpad, first as an envelope (a line locus), then as a curve (a point locus).

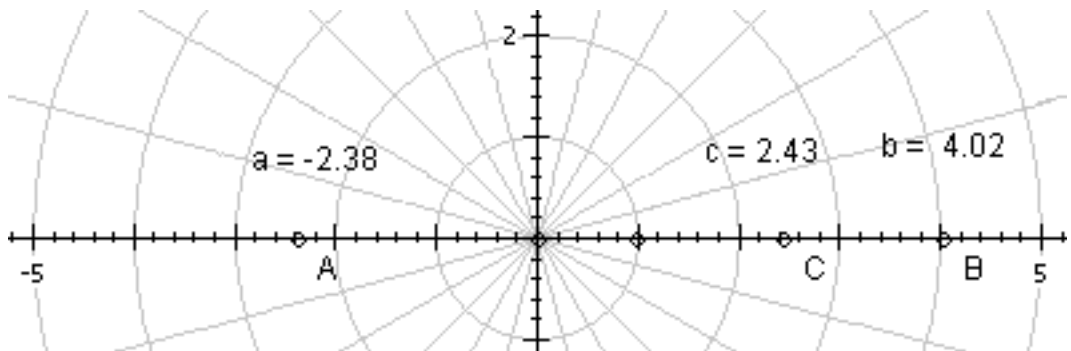
Graphing a Family of Polar Curves

In this activity, you'll plot a curve representing the family of polar curves that have equations of the form $r = a + b\sin(c\theta)$. These equations are functions in which θ is the independent variable and r is the dependent variable. The values a , b , and c are parameters that you'll be able to change dynamically to see how they affect the curve.

Sketch and Investigate

1. Choose **Graph: Grid Form: Polar Grid**.
2. Hide the origin point and the unit point.
3. Construct three points, A , B , and C , on the horizontal axis.
4. Measure their Abscissa (X) values. These are the values you'll use for the parameters a , b , and c .
5. Use the Text tool to change the prefixes of these measures from "X[A] =" to " a =", from "X[B] =" to " b =", and from "X[C] =" to " c =".

Select the points and
choose **Measure:**
Analytic: Abscissa
(X).



6. Choose **Graph: Plot Function**. To input a function r as a function of θ (instead of y as a function of x), tap the **Graph** pop-up menu and choose $r = f(\text{theta})$. Now use the values in the **Values** pop-up menu, the **sin** function in the **Trig** submenu of the **Functions** pop-up menu, and the operators on the keypad to create the equation:

$$r = a + b*\sin(c*\text{theta})$$

7. Drag points A , B , and C along the horizontal axis and observe how the values of a , b , and c affect the curve.
 8. Construct a point on the plotted curve and measure its coordinates.
 9. Animate the point on the curve to observe how, in a given equation, r varies as a function of θ .
- Q1 On a separate sheet, write a report in which you describe, in as much detail as you can, what kinds of curves you get with different values for a , b , and c . Sketch examples to illustrate your description. Be sure to try special values for these parameters, such as 0 and 1.