

CASIO®

Input Your Future.

CASIO®



The Enjoyable Path to Math

A supplementary reader
for CASIO fx-991MS/fx-570MS/fx-115MS/fx-100MS
fx-95MS/fx-82MS/fx-350MS/fx-85MS



About this book...

This book is a collection of easy-to-understand middle and high school practical problems that can be solved using a CASIO Scientific Calculator. Problems have been carefully selected to provide practice in performing calculations that are commonly encountered in our modern digital information age.

As you will probably discover when you work through the problems in this book, the scientific calculator is a tool that makes it possible to perform very complex calculations that would be impossible to perform using manual calculation alone. We hope this lightening of the calculation load will provide you with more time to spend developing knowledge of theory and logic, and contribute towards making the study of mathematics more fun.

It is important to remember that this book is not intended to replace the User's Manual that comes with your CASIO Scientific Calculator. Make sure you also carefully read the User's Manual and familiarize yourself with the features and functions of your calculator before and while using this book.

When using this book, also remember that it was prepared for a global audience, so some of the units and contents contained herein may not apply in your particular country or region.

We at CASIO sincerely hope you enjoy working through the problems in this book, and that by doing so you are better prepared to meet the challenges of the future digital information age.

CASIO COMPUTER CO., LTD.

The History of the CASIO Calculator

As the company that developed the world's first relay-type calculator, CASIO boasts a proud history of continual innovation and development of new calculators that expand new horizons and meet the needs of students the world over. Ever since the 1972 introduction of the fx-1 with simple scientific function calculator capabilities, CASIO FX Series calculators have been meeting the needs of teachers, students and other calculator users for some 30 years.

Today, the lineup of CASIO FX Series Scientific Function calculators covers a wide variety of needs, and are featured in mathematics texts the world over.

● Contents ●

	Page	fx-82MS fx-85MS fx-350MS	fx-95MS	fx-100MS fx-115MS	fx-570MS fx-991MS
Negative Value Calculations	1	●	●	●	●
Expression Values and Calculations	2	●	●	●	●
Proportion and Inverse Proportion	3	●	●	●	●
Circles and Sectors	5	●	●	●	●
Square Roots	7	●	●	●	●
Pythagorean Theorem	9	●	●	●	●
Permutation, Combination	10	●	●	●	●
Trigonometric Function Addition Theorem	12	●	●	●	●
Exponential Functions	13	●	●	●	●
Logarithmic Functions	14	●	●	●	●
Solving Quadratic Equations and Cubic Equations ...	15		●	●	●
Solving Simultaneous Equations	18		●	●	●
Problems Using CALC and SOLVE	22			●	●
Problems Involving Complex Numbers, with an Emphasis on Polar Form	25			●	●
Statistics Problems	28	●	●	●	●
Solving Differentials, with an Emphasis on the Derivative	31			●	●
Solving Integrations, with an Emphasis on Definite Integrals	32			●	●
Matrix Problems	34				●
Vector Problems	38				●

*Please note that some calculator models cannot be used for certain activities.

*Operational procedures may differ depending on the calculator model you use.

Negative Value Calculations

Example

$$(-3) \times (-7) = ?$$

Operation

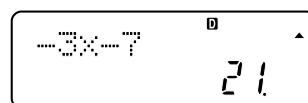
1. Enter the COMP Mode.

MODE **1**

2. Perform the calculation $(-3) \times (-7)$.

(-) **3** **×** **(-)** **7** **=**

Result: 21



Expression Values and Calculations

Example

Determine the value of $x^2 - \frac{2}{5}y$ when $x = 5$ and $y = -3$.

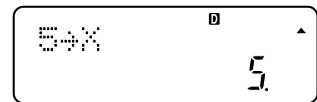
Operation

1. Enter the COMP Mode.

MODE **1**

2. Assign 5 to variable X.

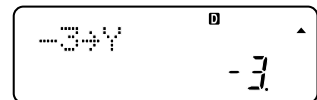
5 **SHIFT** **STO** **X**



5→X 5

3. Assign -3 to variable Y.

(-) **3** **SHIFT** **STO** **Y**

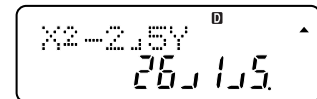


-3→Y -3

4. Determine the value of $X^2 - \frac{2}{5}Y$.

ALPHA **X** **x²** **=** **2** **a%** **5** **ALPHA** **Y** **=**

Result: $26\frac{1}{5}$



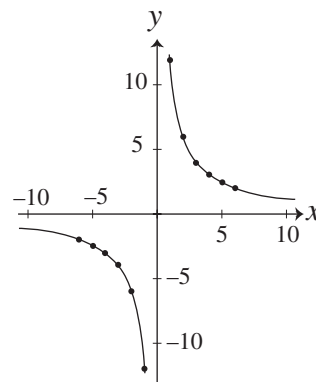
X²-2.5Y 26.2

Proportion and Inverse Proportion

Example

For $y = 12/x$, calculate the values of y for $x = 1, 2, 3, 4, 5, 6$, to determine how the value of y changes in proportion to a gradual increase in the value of x .

Next, calculate the values of y for $x = -1, -2, -3, -4, -5, -6$, to determine how the value of y changes in proportion to a gradual decrease in the value of x .



Operation

1. Enter the COMP Mode.

MODE 1

2. Determine the value of y when $x = 1$.

1 x^{-1} \times 12 =

Result: 12

1- \times 12⁰ 12

3. Determine the value of y when $x = 2$.

2 x^{-1} \times 12 =

Result: 6

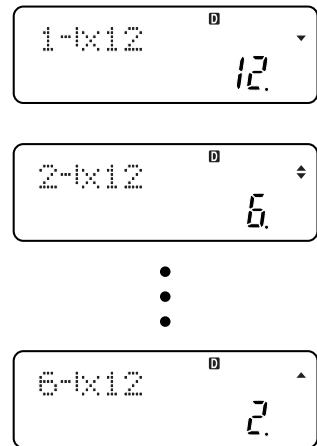
2- \times 12⁰ 6

4. Likewise, determine the values of y for $x = 3, 4, 5, 6$.

5. Press the \blacktriangle key five times to display the value of $y = 12$ when $x = 1$.

1- \times 12⁰ 12

6. Use the \square key to scroll through the values of y for $x = 2, 3, 4, 5, 6$.



The above reveals that the value of y decreases as the value of x increases.

7. Repeat steps 2 through 6 to calculate the values of y for $x = -1$ through -6 .

Doing so will reveal that the value of y increases as the value of x decreases.

Circles and Sectors

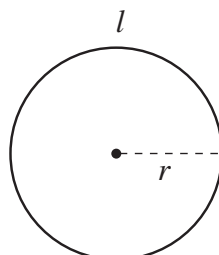
Example 1

Determine, up to one decimal place, the circumference and the area of a circle with a radius of 8 cm.

Explanation (Radius = r)

Circumference of a circle: $l = 2\pi r$

Area of a circle: $S = \pi r^2$



Operation

1. Enter the COMP Mode.

MODE **1**

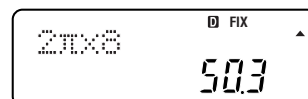
2. Press the **MODE** key a number of times until 1 (Fix1) is specified as the fixed number of decimal places.

MODE ... **1** (Fix) **1**

3. Calculate the circumference of the circle.

2 **SHIFT** **π** **\times** **8** **=**

Result: 50.3

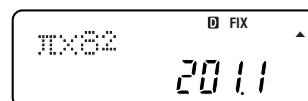


The circumference of the circle is approximately 50.3 cm.

4. Calculate the area.

SHIFT **π** **\times** **8** **x^2** **=**

Result: 201.1



The area is approximately 201.1 cm².

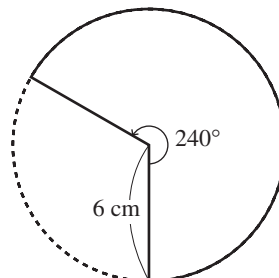
Example 2

Determine, up to one decimal place, the length of the arc and area of a sector that has a radius of 6 cm and a central angle of 240° .

Explanation (Radius = r , central angle = a°)

$$\text{Length of the arc of a sector: } l = 2\pi r \times \frac{a}{360}$$

$$\text{Area of a sector: } S = \pi r^2 \times \frac{a}{360}$$

**Operation**

1. Enter the COMP Mode.

MODE **1**

2. Press the **MODE** key a number of times until degrees (Deg) are specified as the angle unit.

MODE ... **1** (Deg)

3. Press the **MODE** key a number of times until 1 (Fix1) is specified as the fixed number of decimal places.

MODE ... **1** (Fix) **1**

4. Determine the length of the arc of the sector.

2 **SHIFT** **π** **\times** **6** **\times** **(** **240** **\div** **360** **)** **=**

2 π \times 6 \times (240 \div 360)
25.1

The length of the arc of the sector is approximately 25.1 cm.

5. Calculate the area.

SHIFT **π** **\times** **6** **x^2** **\times** **(** **240** **\div** **360** **)** **=**

π \times 6 2 \times (240 \div 360)
75.4

The area is approximately 75.4 cm².

Square Roots

Example 1

Arrange the following from smallest to largest.

$$\sqrt{7}, 2.7, \sqrt{22/3}, 18/7$$

Explanation

Calculate the approximate square root values and then compare the results.

Operation

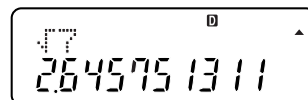
1. Enter the COMP Mode.

MODE 1

2. Determine the approximate value of $\sqrt{7}$.

$\sqrt{\square}$ 7 =

Result: $\sqrt{7} \approx 2.645751311$

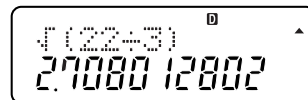


The calculator display shows the square root of 7. The top line shows the square root symbol over the number 7. The bottom line shows the decimal result 2.645751311. There is a small 'D' icon in the top right corner and an arrow pointing up.

3. Determine the approximate value of $\sqrt{22/3}$.

$\sqrt{\square}$ () 22 \div 3 () =

Result: $\sqrt{22/3} \approx 2.708012802$

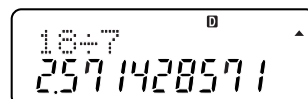


The calculator display shows the square root of 22 divided by 3. The top line shows the square root symbol over the expression (22÷3). The bottom line shows the decimal result 2.708012802. There is a small 'D' icon in the top right corner and an arrow pointing up.

4. Convert $18/7$ to its decimal equivalent.

18 \div 7 =

Result: $18/7 = 2.571428571$



The calculator display shows the division of 18 by 7. The top line shows the expression 18÷7. The bottom line shows the decimal result 2.571428571. There is a small 'D' icon in the top right corner and an arrow pointing up.

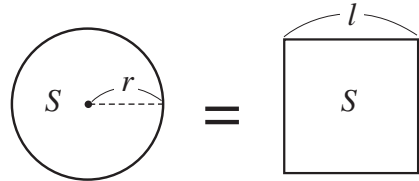
5. Using the \blacktriangleleft key to scroll through and compare the results produced by steps 2 through 4 reveals that the proper ascending order arrangement of the values is: $18/7 < \sqrt{7} < 2.7 < \sqrt{22/3}$.

Example 2

Determine the length of one side of a square whose area is equal to that of a circle with a radius of 10 cm.

Explanation

- First, determine the area of the circle.
- If the length of one side of a square is l , the area of the square is l^2 . This means that the length of one side of a square with the same area as the above circle can be determined by calculating the square root of the area.



Operation

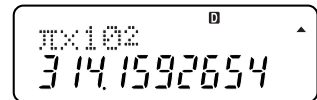
1. Enter the COMP Mode.



2. First, determine the area of the circle with a 10cm radius.



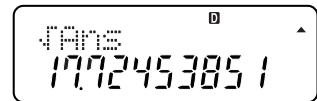
Result: $S = 314.1592654$



3. Calculate the square root of the area.



Result: $l = \sqrt{S} = 17.72453851$

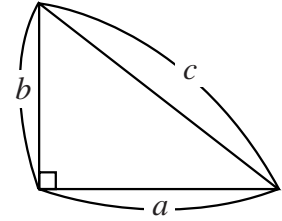


The above indicates that the length of one side of a square whose area is equal to that of a circle with a radius of 10 cm is 17.7 cm.

Pythagorean Theorem

Example

Determine the length of the hypotenuse of a right triangle whose sides are 9 cm and 12 cm long.



Explanation (Two sides = a , b ; Hypotenuse = c)

$$a^2 + b^2 = c^2$$

Operation

1. Enter the COMP Mode.

MODE 1

2. Determine the sum of the squares of the two sides of the right triangle.

9 x^2 + 12 x^2 =

9^2+12^2
225

3. Calculate the square root.

$\sqrt{\quad}$ =

Result: $\sqrt{225} = 15$

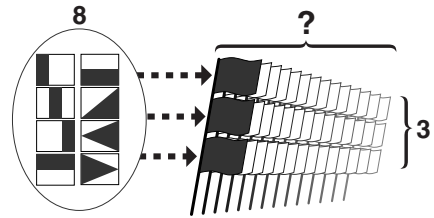
\sqrt{Ans}
15

The above shows that the length of the hypotenuse is 15 cm.

Permutation, Combination

Example 1

What would be the maximum number of different flag signals possible using eight flags of eight different colors, when each signal consists of three flags?



Explanation

Determine ${}_8P_3$.

Operation

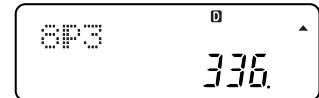
1. Enter the COMP Mode.

MODE **1**

2. Determine ${}_8P_3$.

8 **SHIFT** **nPr** **3** **=**

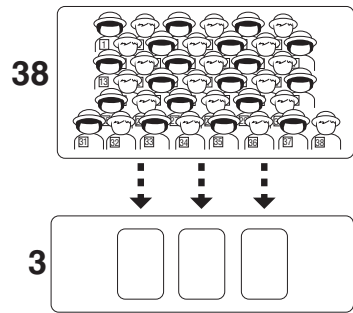
Result: ${}_8P_3 = 336$



The above indicates there are 336 different signals possible.

Example 2

How many different combinations are possible when selecting three individuals from a class of 38?



Explanation

Determine ${}_{38}C_3$.

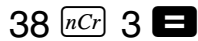
Operation

* The following shows operation using the fx-82MS/85MS/350MS/95MS.

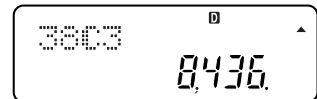
1. Enter the COMP Mode.



2. Determine ${}_{38}C_3$.



Result: ${}_{38}C_3 = 8436$



The above shows there are 8436 different combinations possible.

Trigonometric Function Addition Theorem

Example

Determine the validity of: $\sin 75^\circ$
 $= \sin (30^\circ + 45^\circ) = \sin 30^\circ \cos 45^\circ + \cos 30^\circ \sin 45^\circ$.

Explanation

sin Function Addition Theorem:

$$\sin(\alpha + \beta) = \sin\alpha \cos\beta + \cos\alpha \sin\beta$$

Operation

1. Enter the COMP Mode.

MODE **1**

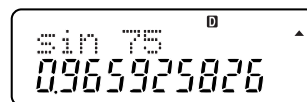
2. Press the **MODE** key a number of times until degrees (Deg) are specified as the angle unit.

MODE ... **1** (Deg)

3. Determine value of $\sin 75^\circ$.

sin 75 **=**

Result: $\sin 75^\circ = 0.965925826$



sin 75 °
0.965925826

4. Determine value of $\sin 30^\circ \cos 45^\circ + \cos 30^\circ \sin 45^\circ$.

sin 30 **×** **cos** 45 **+** **cos** 30 **×** **sin** 45 **=**

Result: $\sin 30^\circ \cos 45^\circ + \cos 30^\circ \sin 45^\circ = 0.965925826$



sin 30 x cos 45 +
cos 30 x sin 45
0.965925826

5. Use the **▲** key to display the result obtained in step 3 to see if it matches the values obtained in step 4.

Exponential Function

Example

Determine the value of $64^{(-2/3)}$.

Explanation

- When $a \neq 0$: $a^{(-p)} = 1/(a^p)$
- When $a > 0$: $a^{(q/p)} = \sqrt[p]{a^q}$

Operation

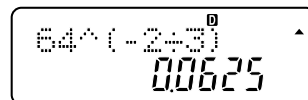
1. Enter the COMP Mode.

MODE **1**

2. Determine the value of $64^{(-2/3)}$.

64 **^** **(** **(-)** 2 **÷** 3 **)** **=**

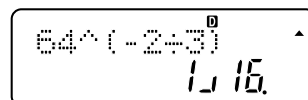
Result: $64^{(-2/3)} = 0.0625$



64^(-2/3)
0.0625

3. Convert to a fraction.

a%



64^(-2/3)
1/16

The above indicates that $64^{(-2/3)} = 0.0625 = \frac{1}{16}$.

Logarithmic Function

Example

Determine the value of $\log_2 \frac{1}{128}$.

Explanation

- This calculator has two logarithmic functions: \log whose base is e (natural logarithm), and \ln whose base is 10 (common logarithm).
- When the base is something other than that described above, the following formula can be used to convert the base: When $a > 0$, $b > 0$, $M > 0$ and $a \neq b \neq 1$, use $\log_a M = (\log_b M) / (\log_b a)$ for the calculation.

Operation

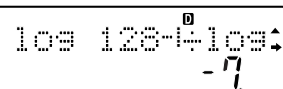
1. Enter the COMP Mode.

MODE **1**

2. Determine $\log_2 \frac{1}{128} = (\log \frac{1}{128}) / (\log 2)$.

log **128** **x⁻¹** **÷** **log** **2** **=**

Result: $\log_2 \frac{1}{128} = \log \frac{1}{128} / \log 2 = -7$



log 128⁻¹ ÷ log 2
-7

Solving Quadratic Equations and Cubic Equations

(fx-95MS/fx-100MS/fx-115MS/fx-570MS/fx-991MS only)

Example 1

Solve the equation $2x^2 - 5x + 2 = 0$.

Operation

*The following shows operation using the fx-100MS/115MS/570MS/991MS.

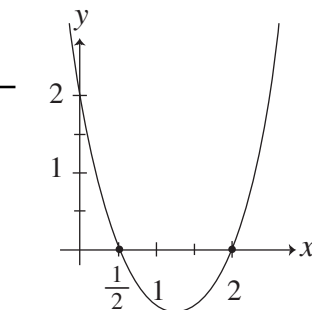
1. Select the EQN Mode and then specify the degree of the equation.

In this example we want to solve a quadratic equation, so you would specify 2.

MODE MODE MODE 1



(Degree?) 2



EQN 0
a? 0

2. Input values for a , b , and c in the quadratic equation $ax^2 + bx + c = 0$.

(a?) 2 =

EQN 0
b? 0

(b?) (-) 5 =

EQN 0
c? 0

(c?) 2 =

3. One of the solutions obtained appears on the display.

EQN 0
X1= 2

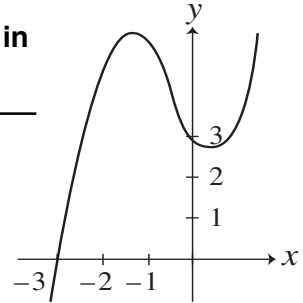
4. Display the next solution. ▾

EQN 0
X2= 0.5

The above produces the solutions $x = 2$ and $x = 0.5$.

Example 2

Solve the equation $x^3 + 2x^2 - 2x + 3 = 0$ in the range of complex numbers.



Operation

*The following shows operation using the fx-100MS/115MS/570MS/991MS.

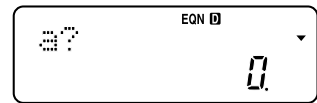
1. Select the EQN Mode and then specify the degree of the equation.

In this example we want to solve a cubic equation, so you would specify 3.

MODE MODE MODE 1

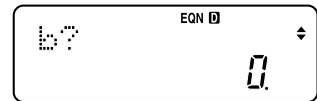


(Degree?) 3

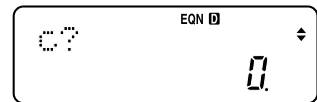


2. Input values for a , b , c , and d in the cubic equation $ax^3 + bx^2 + cx + d = 0$.

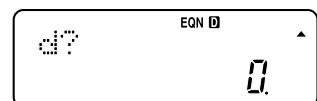
(a?) 1



(b?) 2

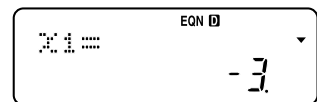


(c?) (-) 2



(d?) 3

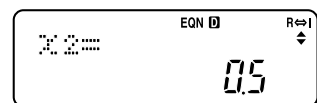
3. One of the solutions obtained appears on the display.



4. Display the next solution.

This causes the symbol $R \leftrightarrow I$ to appear in the upper right corner of the display.

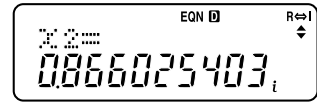
This indicates that the displayed solution is a complex number, and the real part is 0.5.



Solving Quadratic Equations and Cubic Equations

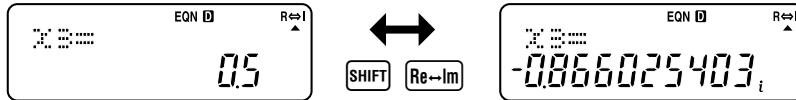
5. Display the imaginary part of the solution.

SHIFT **Re↔Im**



Each press of **SHIFT** **Re↔Im** toggles the display between the real part and imaginary part.

6. Display the other solution. **▼**



This solution is also a complex number, which is the conjugate of the previous solution.

The above example shows us that the solutions for the equation are $x = -3$, $x = 0.5 - 0.866025403i$, and $x = 0.5 + 0.866025403i$.

Solving Simultaneous Equations

(fx-95MS/fx-100MS/fx-115MS/fx-570MS/fx-991MS only)

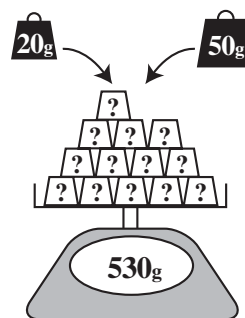
Example 1

A group of 13 weights, some weighing 50 grams and some weighing 20 grams, weigh a combined total of 530 grams. How many of each type of weight are there?

Explanation

If the number of 50g weights is x and the number of 20g weights is y , the relation $\begin{cases} x + y = 13 \\ 50x + 20y = 530 \end{cases}$ can be

established and solved as simultaneous equations.



Operation

*The following shows operation using the fx-100MS/115MS/570MS/991MS.

1. Select the EQN Mode and then specify the number of unknowns.

In this example we have two unknowns, x and y , so we would specify 2.

MODE MODE MODE 1

(Unknowns?) 2

2. Input the values for the coefficients of the linear equations with two unknowns $\begin{cases} a_1x + b_1y = c_1 \\ a_2x + b_2y = c_2 \end{cases}$.

(a1?) 1 =

(b1?) 1 =

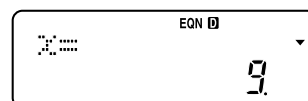
(c1?) 13 =

(a2?) 50 =

(b2?) 20 =

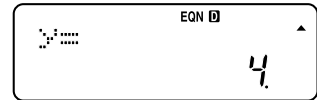
(c2?) 530 =

3. One of the solutions obtained appears on the display.



Solving Simultaneous Equations

4. Display the next solution.



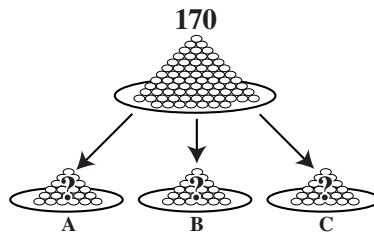
The above tells us there are nine 50g weights and four 20g weights.

Example 2

170 beans are divided among three individuals named A, B, and C. The ratio of beans given to A and B is 3:5, while the number of beans given to C is 22 less than the combined total given to A and B. How many beans each were given to A, B, and C?

Explanation

Let us say A received x number of beans, B received y number of beans, and C received z number of beans. This means the total number of beans is $x + y + z = 170$. Also, since we know the ratio of beans received between A (x) and B (y) is 3:5, we can say $5x = 3y$, which can be transformed to $5x - 3y = 0$. Finally, since C (z) received 22 beans less than A and B combined, we can say $z = x + y - 22$.



Combining all of this leaves us with the set of simultaneous equations $\begin{cases} x + y + z = 170 \\ 5x - 3y = 0 \\ x + y - z = 22 \end{cases}$, which we should be able to solve and come up with the answer we need.

Operation

*The following shows operation using the fx-100MS/115MS/570MS/991MS.

1. Select the EQN Mode and then specify the number of unknowns.

In this example we have three unknowns, x , y , and z so we would specify 3.

MODE MODE MODE 1

(Unknowns?) 3

2. Input the values for the coefficients of the linear equations with three unknowns

$$\begin{cases} a_1x + b_1y + c_1z = d_1 \\ a_2x + b_2y + c_2z = d_2 \\ a_3x + b_3y + c_3z = d_3 \end{cases}$$

(a1?) 1 =

(b1?) 1 =

(c1?) 1 =

(d1?) 170 =

(a2?) 5 =

(b2?) (-) 3 =

(c2?) 0 =

(d2?) 0 =

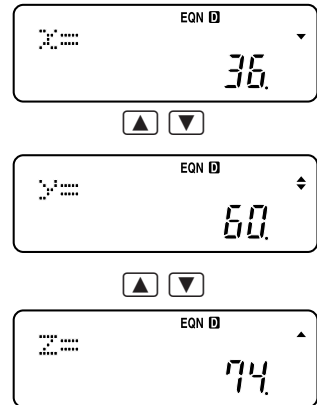
(a3?) 1 =

(b3?) 1 =

(c3?) (-) 1 =

(d3?) 22 =

3. Display the solutions.



The above shows us that A received 36 beans, B 60 beans, and C 74 beans.

Problems Using CALC and SOLVE

(fx-100MS/fx-115MS/fx-570MS/fx-991MS only)

Example 1

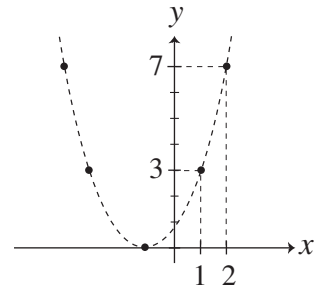
Perform the following steps to sketch the general graph of the function

$$y = x^2 + x + 1.$$

- Substitute various values for x , and solve for y .
- Plot the points (x, y) on a plane.
- Connect adjacent points.

Explanation

The CALC function comes in handy when substituting various different values for multiple instances of the same variable within a single function.



Operation

1. Select the COMP Mode.

MODE 1

2. Input the function and store it.

ALPHA Y ALPHA = ALPHA X x^2 + ALPHA X + 1
CALC

X?
0

3. Input values for x to display the values obtained for y .

(Input 1.)

1 =

Y=X²+X+1
3

(Input 2.)

CALC 2 =

Y=X²+X+1
7

< Note >

In this way, use CALC to solve for other values and then draw the graph to see what it looks like.

Example 2

An object is dropped from a height of A (m) (free fall). If the speed of the object when it reaches the ground is B (m/s) and gravitational acceleration is $C = 9.8$ (m/s²), we can set up the following relation: $(1/2) B^2 = CA$.

Determine the speed of the object when it reaches the ground if it is dropped from a height of 30 meters.

Convert the speed to kilometers per hour.

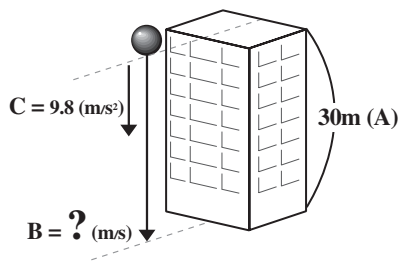
Explanation

The SOLVE function lets you solve for the value of a given relation without changing its form.

A unit conversion function (fx-570MS/fx-991MS only) lets you convert from meters per second to kilometers per hour.

$$gh = \frac{1}{2} V^2$$

$$g = C = 9.8, h = A = 30, V = B = ?$$

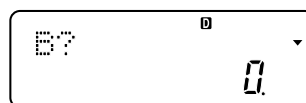
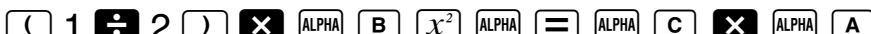


Operation

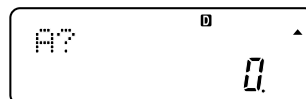
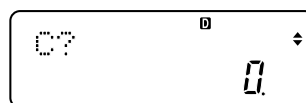
1. Select the COMP Mode.



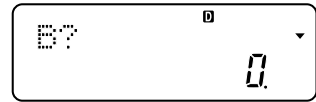
2. Input the function and store it.



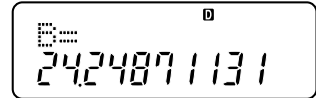
3. Input values for the variables. Do not input anything for the variable for which you want to solve. In this example we want to solve for B, so we input values for A and C.



4. Display the variable for which you want to solve, and then run SOLVE.



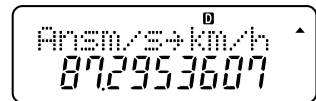
SHIFT **SOLVE** (It will take some time before the solution appears.)



5. Now convert the speed to kilometers per hour.

The unit conversion number for converting meters per second to kilometers per hour is 20.

SHIFT **CONV** 20 **=**



The above shows us that the approximate speed of the object would be 24.25 meters per second, which converts to 87.3 kilometers per hour.

Problems Involving Complex Numbers, with an Emphasis on Polar Form

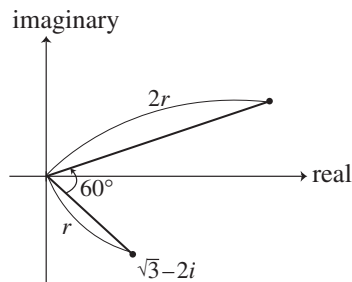
(fx-100MS/fx-115MS/fx-570MS/fx-991MS only)

Example 1

Rotate $\sqrt{3} - 2i$ 60 degrees around the origin of the complex plane, and then determine the point with a ratio of 2 with the origin as the center.

Explanation

Defining r as the distance of point (a, b) from the origin on the complex plane and θ as the angle formed with the positive part of the x -axis makes it possible to express complex number $z = a + bi$ as $z = r(\cos\theta + i \sin\theta)$. This is called polar representation of complex number z . Using polar representation for z_2 in the complex number multiplication $z_1 \times z_2$ gives us $z_2 = r_2(\cos\theta_2 + i \sin\theta_2)$. Now we can rotate z_1 θ_2° around the origin of the complex plane, giving us a value with a ratio of r_2 with the origin as the center.



Operation

1. Select the CMPLX Mode.

MODE 2

2. Specify the angle unit .

MODE MODE MODE MODE 1 (Deg)

3. Input the polar form of the complex number, with $r = 2$, and $\theta = 60$. The values you input are automatically converted to rectangular form on the display, but you can also display them in polar form.

2 SHIFT ∟ 60 =

CMPLX D R↔I
2∠60
i

SHIFT Re↔Im

CMPLX D R↔I
2∠60
1.732050808i

4. Multiply by $\sqrt{3} - 2i$.

\times ($\sqrt{}$ 3 $-$ 2 i) $=$

SHIFT Re \leftrightarrow Im

CMPLX \uparrow R \leftrightarrow I
ANSX($\sqrt{3}-2i$)
5.196152423

CMPLX \uparrow R \leftrightarrow I
ANSX($\sqrt{3}-2i$)
 i

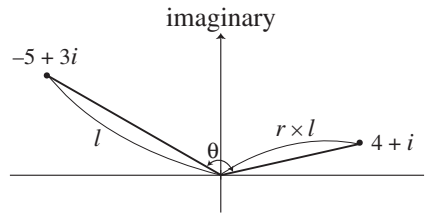
The above obtains the complex number $5.196152423 + i$.

Example 2

Examine the relationship between $-5 + 3i$ and $4 + i$, which are two points on the complex plane.

Explanation

You can examine the relationship between two points on the complex plane (z_1 and z_2) by determining the absolute value (r) and the argument (θ) of z_1/z_2 .

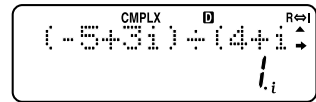


Operation

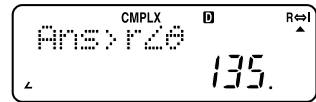
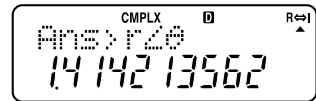
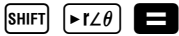
1. Select the CMPLX Mode.



2. Perform the calculation $(-5 + 3i) \div (4 + i)$.



3. Convert to polar form and display the result.



The above indicates that $-5 + 3i$ represents a 135° rotation of $4 + i$ around the origin, and that it is a point with a ratio of $1.414213562 (= \sqrt{2})$ with the origin as the center.

Statistics Problems

Example 1

The following are the measured 100-meter race times for five students.
12.5 11.6 10.8 12.8 11.4 (Unit: Seconds)

Calculate the mean and the sample standard deviation for these results. Replace the times of the 12-second runners (runner 1 and runner 4) with 11.1 and 11.7 seconds (which are times run by other students), and calculate the mean and sample standard deviation again.

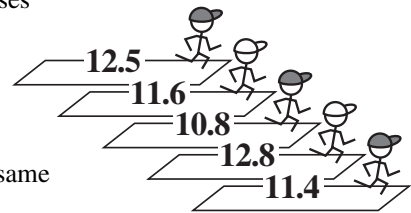
Explanation

Sample standard deviation is a value that expresses the level of variation among data.

When the number of data samples is n , the value of each data is x_i ($i = 1 \dots n$), and the mean is \bar{x} :

Sample standard deviation $\sigma_{n-1} = \sqrt{(\sum x_i^2 - n\bar{x}^2)/(n-1)}$.

Instead of using these complex formulas, you can obtain the same results using a simple key operation on the calculator.



Operation

*The following shows operation using the fx-82MS/85MS/350MS/95MS.

1. Enter the SD Mode, and clear the statistical memory area.

MODE **2**
SHIFT **CLR** **1** (**Sci**) **=**

2. Input the data.

In the SD Mode, the **M+** key operates as the **DT** key.

Input a value and then press the **DT** key to register it.

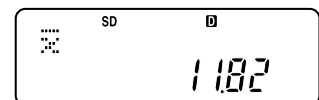
To input two sequential values that are identical, input the value and then press **DT** twice.

The display shows how many values have been input (number of samples = n).

12.5 **DT** 11.6 **DT** 10.8 **DT** 12.8 **DT** 11.4 **DT**

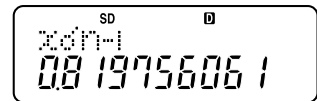
3. Calculate the mean of the data.

AC **SHIFT** **S-VAR** **1** **=**



4. Calculate the sample standard deviation of the data.

AC **SHIFT** **S-VAR** **3** **=**



The above shows that the mean is 11.82 seconds, and the sample standard deviation is 0.82.

5. You can view values you have already input by using the **▼** and **▲** keys to scroll them on the display.

You can also edit a displayed value and recalculate, if you want.

For the second part of this problem, first display the first value (12.5 seconds).

AC **▼**

6. Input the new value (11.1) and then press **=**.

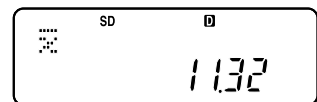
11.1 **=**

7. Press the **▼** key a number of times to display the other data item to be edited (12.8 seconds for runner 4 in this example), and then input the new data (11.7 seconds).

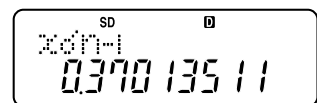
▼ **▼** **▼** **▼** **▼** **▼** 11.7 **=**

8. Calculate the mean and sample standard deviation of the data.

AC **SHIFT** **S-VAR** **1** **=**



AC **SHIFT** **S-VAR** **3** **=**



The above shows that the mean is 11.32 seconds, and the sample standard deviation is 0.37.

Example 2

The following table shows the mathematics and English test results for eight students.

Math	64	72	85	53	61	86	61	58
English	80	77	68	84	75	96	68	71

Use this data to determine the correlation coefficient of the test scores.

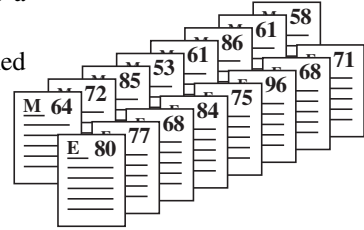
Explanation

The correlation coefficient expresses the extent to which a change in one type of data tends to correspond to a change in another type of data.

The correlation coefficient r for the two types of data x and y is obtained using the following expression.

$$r = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^n (x_i - \bar{x})^2 \cdot \sum_{i=1}^n (y_i - \bar{y})^2}} = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{n \cdot x\sigma_n \cdot y\sigma_n}$$

(\bar{x} , \bar{y} : x , y mean; $x\sigma_n$, $y\sigma_n$: x , y standard deviation)



Operation

*The following shows operation using the fx-100MS/115MS/570MS/991MS.

1. Enter the REG Mode and select liner regression (Lin).

First, clear statistical memory.

MODE MODE 2 1
 SHIFT CLR 1 (Sci) =

2. Input the data.

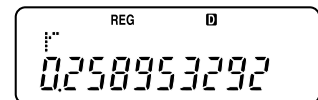
Data items are input in pairs.

Separate each item that makes up a pair with \square , and press \square to register the pair.

64 \square 80 \square 72 \square 77 \square 85 \square 68 \square 53 \square 84 \square
 61 \square 75 \square 86 \square 96 \square 61 \square 68 \square 58 \square 71 \square

3. Calculate the correlation coefficient.

SHIFT S-VAR \blacktriangleright \blacktriangleright 3 (r) =



The correlation coefficient of the above data is about 0.259.

Solving Differentials, with an Emphasis on the Derivative

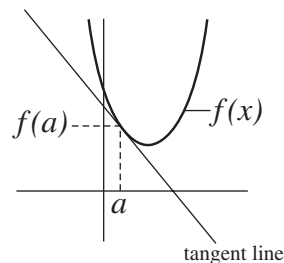
(fx-100MS/fx-115MS/fx-570MS/fx-991MS only)

Example 1

Determine the equation for a tangent line at (1, 3) on $y = x^2 - 3x + 5$.

Explanation

Derivative $f'(a)$ at $x = a$ on $y = f(x)$ is the slope of the tangent line at $x = a$. Also, the tangent line passes through $(a, f(a))$, which means that its equation is $y = f'(a)(x - a) + f(a)$.



Operation

1. Select the COMP Mode.

MODE 1

2. Determine derivative $f'(1)$ at $x = 1$ on $y = x^2 - 3x + 5$.

SHIFT d/dx ALPHA X X² - 3 ALPHA X + 5 , 1) =

3. Determine $f'(1) = -1$.

d/dx (x² - 3x + 5) = -1

4. The form of an equation for a tangent line is $y = -x + c$, which can be transformed to $c = y + x$. Since this tangent line passes through (1, 3), substitute these values.

$$\begin{aligned} c &= y + x \\ &= 3 + 1 \end{aligned}$$

5. This produces $c = 4$.

Based on the above, the equation for the tangent line is $y = -x + 4$.

Solving Integrations, with an Emphasis on Definite Integrals

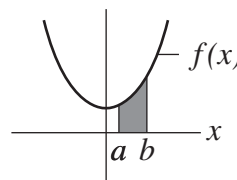
(fx-100MS/fx-115MS/fx-570MS/fx-991MS only)

Example 1

Determine the area enclosed by $y = x^2 + 2$, the x -axis, and $x = 1, x = 2$.

Explanation

The area enclosed by $y = f(x)$, the x -axis, and $x = a, x = b$ is generally determined using the definite integral $\int_a^b f(x)dx$.



Operation

1. Select the COMP Mode.

MODE 1

2. Calculate the value of the definite integral values for $y = x^2 + 2$ from $x = 1$ to $x = 2$.

$\int dx$ ALPHA X x^2 + 2 , 1 , 2) =

3. This produces a definite integral value of 4.333333333.

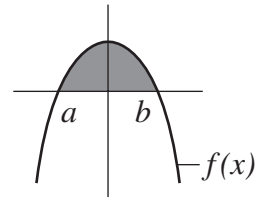
The above indicates that the area being calculated is 4.333333333 (= 13/3).

Example 2

Determine the area enclosed by $y = 4 - x^2$ and the x -axis.

Explanation

First, solve $y = 4 - x^2$ and find the x -intercepts. Use this to determine the integration partition, whose definite integral value is the area.



Operation

1. Select the EQN Mode, and then solve $y = 4 - x^2$.

MODE MODE MODE 1 ▶

(Degree?) 2

(a?) (-) 1 =

(b?) 0 =

(c?) 4 =

2. The solutions are 2 and -2.

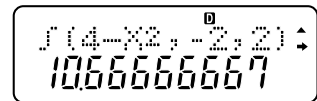


3. Select the COMP Mode and then calculate integration values for $y = 4 - x^2$ from $x = -2$ to $x = 2$.

MODE 1

∫dx 4 - ALPHA X x^2 , (-) 2 , 2) =

4. This produces a definite integral value of 10.66666667.



This above tells us that the area being calculated is 10.66666667 (= 32/3).

Matrix Problems

(fx-570MS/fx-991MS only)

Example 1

Determine $X^2 - Y^2$ for two 2×2 matrices named X and Y

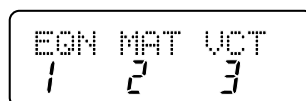
when $X + Y = \begin{bmatrix} 3 & 2 \\ 3 & -1 \end{bmatrix}$ and $X - Y = \begin{bmatrix} 1 & -4 \\ 7 & 1 \end{bmatrix}$.

Explanation

Matrix multiplication normally does not follow the commutative law $XY = YX$. Consequently, $(X + Y)(X - Y) = X^2 - XY + YX - Y^2$ must be used.

Operation

1. Select the MAT Mode.

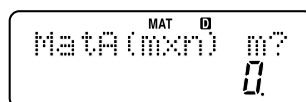
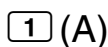
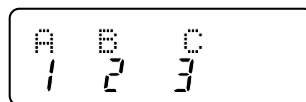
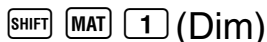


2. Specify the matrix name, and the number of rows and columns, and then input its elements.

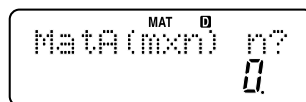
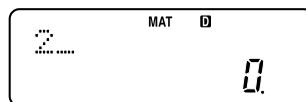
For this problem, we need to input $X + Y$ into Matrix A and $X - Y$ into Matrix B.

The following shows input of the elements of Matrix A.

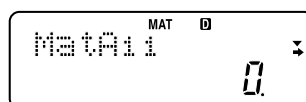
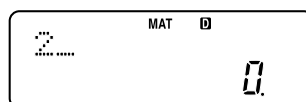
• Specify Matrix A.



• Specify 2 for the number of rows (m).



• Specify 2 for the number of columns (n).

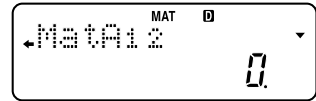
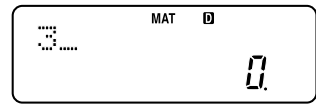


Matrix Problems

- 2×2 Matrix A is displayed as $\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$.

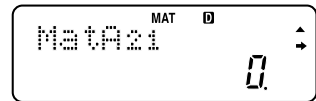
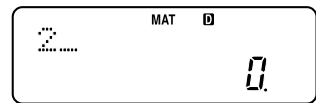
First, input 3 for A_{11} .

3 **=**

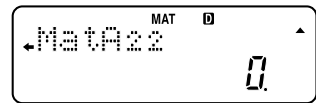
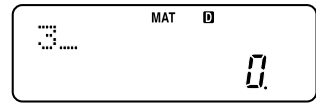


- Next, input 2 for A_{12} , 3 for A_{21} , and -1 for A_{22} .

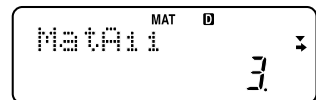
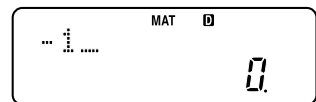
2 **=**



3 **=**



(-) 1 **=**



- The display returns to the initial element (A_{11}) of the matrix after input of values for all elements is complete.

AC

Use the same procedure to input Matrix B elements.

SHIFT **MAT** **1** (Dim) **2** (B) **2** **=** **2** **=**

1 **=** **(-)** **4** **=** **7** **=** **1** **=** **AC**

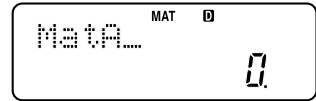
3. Determine X and Y from $X + Y$ and $X - Y$.

First, use $(X + Y) + (X - Y) = 2X$ to determine X.

SHIFT MAT 3 (Mat)

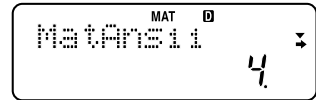


1 (A)

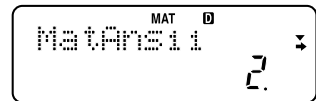


+

SHIFT MAT 3 (Mat) 2 (B) =



1 a/b 2 x SHIFT MAT 3 (Mat) 4 (Ans) =



This produces the result $X = \begin{bmatrix} 2 & -1 \\ 5 & 0 \end{bmatrix}$.

You can use \blacktriangle , \blacktriangledown , \blacktriangleleft , and \blacktriangleright to view the other elements of the matrix.

4. Input result obtained for X into Matrix C.

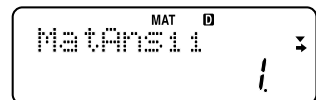
SHIFT MAT 1 (Dim) 3 (C) 2 = 2 =

2 = (←) 1 = 5 = 0 = AC

5. Now determine Y from $(X + Y) - X = Y$.

SHIFT MAT 3 (Mat) 1 (A) =

SHIFT MAT 3 (Mat) 3 (C) =

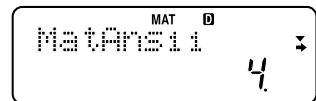


This produces the result $Y = \begin{bmatrix} 1 & 3 \\ -2 & -1 \end{bmatrix}$.

6. Now perform the calculation Matrix C × Matrix C – Matrix Ans × Matrix Ans (= $X^2 - Y^2$).

SHIFT MAT 3 (Mat) 3 (C) x² =

SHIFT MAT 3 (Mat) 4 (Ans) x² =



The above steps show us that $X^2 - Y^2 = \begin{bmatrix} 4 & -2 \\ 10 & 0 \end{bmatrix}$.

Example 2

Use a matrix to solve the simultaneous equations

$$\begin{cases} 2x - 3y = 18 \\ 3x + 2y = 1 \end{cases}$$

Explanation

Placing simultaneous equations $\begin{cases} ax + by = k \\ cx + dy = l \end{cases}$ inside matrices $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, and $B = \begin{bmatrix} k \\ l \end{bmatrix}$

makes it possible to express them as $AX = B$.

This means that if inverse matrix A^{-1} exists for Matrix A, then $X = A^{-1}B$.

Operation

1. Select the MAT Mode.

MODE MODE MODE 2

2. Input $\begin{bmatrix} 2 & -3 \\ 3 & 2 \end{bmatrix}$ for Matrix A and $\begin{bmatrix} 18 \\ 1 \end{bmatrix}$ for Matrix B.

SHIFT MAT 1 (Dim) 1 (A) 2 = 2 =
2 = (-) 3 = 3 = 2 = AC

SHIFT MAT 1 (Dim) 2 (B) 2 = 1 =
18 = 1 = AC

3. Determine in inverse for Matrix A.

SHIFT MAT 3 (Mat) 1 (A) x^{-1} =

4. Multiply the inverse matrix by Matrix B (from the left).

×

SHIFT MAT 3 (Mat) 2 (B) =

The above procedure tells us that $x = 3$ and $y = -4$.

Vector Problems

(fx-570MS/fx-991MS only)

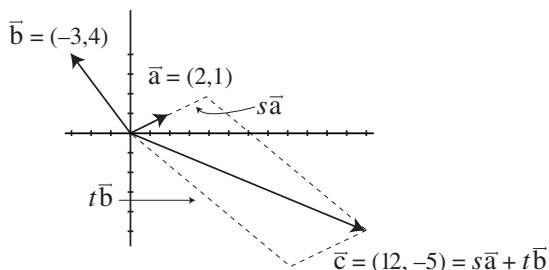
Example 1

Express $\vec{c} = (12, -5)$ in the form $\vec{c} = s\vec{a} + t\vec{b}$ when $\vec{a} = (2, 1)$, $\vec{b} = (-3, 4)$.

Explanation

Though this is a vector problem, it can be solved by solving the following simultaneous equations for each component.

$$\begin{cases} 2s - 3t = 12 \\ s + 4t = -5 \end{cases}$$



Operation

1. Select the EQN Mode and specify 2 as the number of unknowns.

MODE MODE MODE 1

(Unknowns?) 2

2. Input the function and store it.

(a1?) 2 =

(b1?) (-) 3 =

(c1?) 12 =

(a2?) 1 =

(b2?) 4 =

(c2?) (-) 5 =

3. This shows that the solutions of the simultaneous equations are 3 and 2.



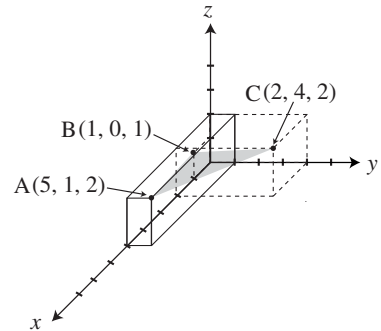
The above tells us that $\vec{c} = 3\vec{a} - 2\vec{b}$.

Example 2

What type of triangle would be produced by the vertices of the three points $A = (5, 1, 2)$, $B = (1, 0, 1)$, and $C = (2, 4, 2)$?

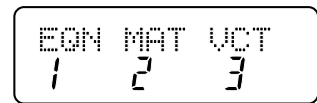
Explanation

When determining the properties of a triangle, the length of a side is determined using the absolute value of its vector, while the angle formed by two sides is determined using the inner product of their vectors. For this problem, first let's determine the lengths of the three sides of triangle ABC.



Operation

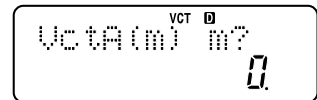
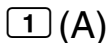
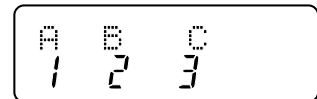
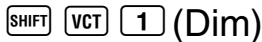
1. Select the VCT Mode.



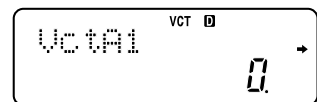
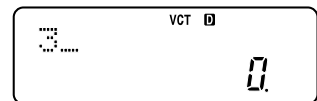
2. Specify the vector name and dimension, and then input its elements.

The following shows input of the elements of Vector A.

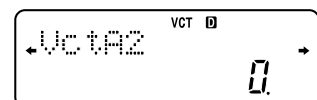
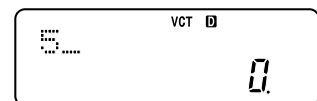
• Specify Vector A.



• Specify three dimensions.

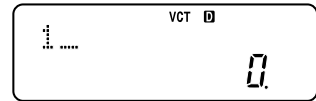


• 3-dimensional Vector A is displayed as (A1 A2 A3). First, input 5 for A1.

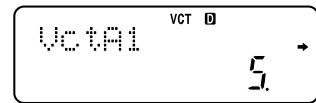
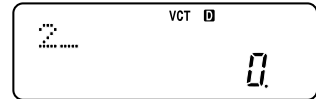
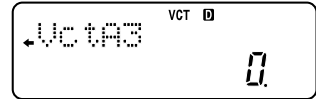


- Next, input 1 into A2 and 2 into A3.

1 =



2 =



- The display returns to the initial element (A1) after input of values for all elements is complete.

AC

Use the same procedure to input elements for Vector B and Vector C.

SHIFT **VCT** **1** (Dim) **2** (B) **3** =

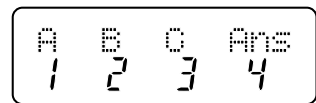
1 = **0** = **1** = **AC**

SHIFT **VCT** **1** (Dim) **3** (C) **3** =

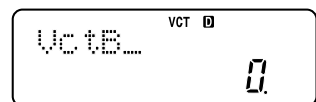
2 = **4** = **2** = **AC**

3. Calculate $\overrightarrow{AB} = \vec{b} - \vec{a}$.

SHIFT **VCT** **3** (Vct)

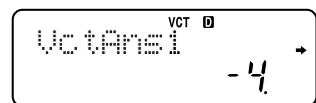


2 (B)



-

SHIFT **VCT** **3** (Vct) **1** (A) =



4. Calculate the absolute value of \overrightarrow{AB} .

The value of \overrightarrow{AB} is stored in Ans memory by step 3, so you can use Vct Ans as shown below.

SHIFT Abs SHIFT VCT 3 (Vct) 4 (Ans) =

VCT D
Abs VctAns
4242640687

5. Use the same procedure to obtain absolute values for \overrightarrow{BC} and \overrightarrow{CA} .

SHIFT VCT 3 (Vct) 3 (C) =

SHIFT VCT 3 (Vct) 2 (B)

VCT D
VctC-VctB
4242640687

=

VCT D
VctAns1
1

SHIFT Abs SHIFT VCT 3 (Vct) 4 (Ans) =

VCT D
Abs VctAns
4242640687

SHIFT VCT 3 (Vct) 1 (A) =

SHIFT VCT 3 (Vct) 3 (C)

VCT D
←VctA-VctC
4242640687

=

VCT D
VctAns1
3

SHIFT Abs SHIFT VCT 3 (Vct) 4 (Ans) =

VCT D
Abs VctAns
4242640687

The above tells us that the sides of triangle ABC are of equal length, which means it is a regular triangle.

● MEMO ●

● MEMO ●