TEACHING PROPORTIONS BY CATEGORIZING FUNCTIONS

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1. PROBLEM

The following formulas (①~⑫) are the equations of a function.

① $y=-2x$ ② $y=2x-1$ ③ $y=x^2$ ④ $y=x+2$ ⑤ $y=-\frac{1}{2}x$

⑥ $y=2-x$ ⑦ $y=\frac{1}{2}x+1$ ⑧ $y=\frac{2}{x}$ ⑨ $y=2x$ ⑩ $y=-2x+1$

⑪ $y=-\frac{3}{x}$ ⑫ $y=2x+\frac{1}{2}$

Let's graph each equation using a graphic calculator, and classify them according to their nature viewed from various standpoints.

2. GOAL

(1) Aim of Guidance

From the standpoint of the concept guidance, it seems that merely showing some examples concerned with the concept may not give the students a profound understanding. For example, let's consider when we teach them the concept of direct proportion. In this case, clarifying the concept of direct proportion by contrasting it with liner functions or inverse proportions seems to lead the students to further understanding rather than by telling them that "When a number doubles or triples, the corresponding number also doubles or triples." or "The graph is a straight line containing the origin." From this point view, we aim to have the students achieve the following objectives in the guidance plan stated here.

① They can take a subjective approach in examining each feature of the graph by utilizing a graphic calculator.

② Having the students draw several graphs and examine each feature of the graphs, they can find out that the direct proportions, liner functions, and inverse proportions have some different and common points.

(2) Role of the Graphic Calculator

Drawing graphs is not a piece of cake for the students. It will take a considerable time to have them complete this series of tasks. In this problem, it is necessary to draw at least 12 graphs. Therefore it is expected to take a considerable amount of time to draw them by manual procedures, and we have come to the conclusion that we should have the students draw each graph with the help of a graphic calculator. Having them do so, they can spend more time finding out the different and common points of the direct proportions, liner functions, and inverse proportions, etc., by examining each graph they have drawn using the graphic calculator. It is also possible to take a view of the graph, not statically but dynamically, by moving the graph screen up, down, left, or right, whereby we can expect the students may make new discoveries which may not be recognized by just taking a look at the graph drawn on paper.

3. TARGETED GRADES AND MATHEMATICAL CONCEPT

(1) Targeted Grades

The first or later year of junior high school

(2) Mathematical Concept Used

① Direct proportion, inverse proportion (the first year of junior high school)

② Liner function (the second year of junior high school)

③ The graph of the equation *y = x2* (the third year of junior high school)

4. SUMMARY TO SOLVE PROBLEMS

(1) Guidance Plan of the Unit

First

Variation and Function (two hours)

① Variable ② Domain

Second

Direct Proportion and Inverse Proportion (three hours)

① Direct proportion ② Inverse proportion

Third

Coordinates and Graph (seven hours)

① Coordinates ② The graph of the equation of $y = ax$

③ The graphs of the direct and inverse proportions, and the graph of the linear function (five hours out of seven hours)

Fourth

Summary of the Unit (two hours)

(2) Teaching Guideline

Learning activity	Major questions and the students' reactions expected	Points on teaching
1. Presentation of the problem 15 minutes	1-1. The following formulas (①~⑫) are the equations of a function. ① $y=-2x$ ② $y=2x-1$ ③ $y=x^2$ ④ $y=x+2$ ⑤ $y=-\frac{1}{2}x$ ⑥ $y=2-x$ ⑦ $y=\frac{1}{2}x+1$ ⑧ $y=\frac{2}{x}$ ⑨ $y=2x$ ⑩ $y=-2x+1$ ⑪ $y=-\frac{3}{x}$ ⑫ $y=2x+\frac{1}{2}$ Let's graph each equation using a graphic calculator. 1-2. Firstly, let's input the formula ①. ① Is the term -2*x* inputted as $-2\times x$? 1-3. Then, let's input the remaining formulas, and draw each graph with a graphic calculator. ① How is the input of a square or fraction? ② How can an incorrect formula be deleted? ③ Is there any given order when inputting the twelve formulas? 1-4. Let's graph all these twelve inputted equations with your graphic calculator. 1-5. After drawing all graphs, let's classify the functions that have common points by observing the displayed graphs from various standpoints.	1-1-1. Have the students understand that all equations (①~⑫) prove to be functions because in each equation a value of *x* determines a unique value of *y*. 1-1-2. Arrange the vellum on which the twelve formulas are written and attach it on the blackboard to present the exercise. 1-2. Confirm the key operations, then have all the students input the formula ①. 1-3. Teach them how to input a square and friction in advance, and cope with the questions such as ① and ② on a one-to-one basis. In the case of the question ③, get them to understand that there is no specific input order. 1-4. Have all the students operate their calculators while having them confirm a series of key operations. 1-5. Confirm whether or not they understood the meaning of the exercise.

2. Solution 20 minutes	2-1. Let's examine each graph on the screen, write out your analysis, and classify the graphs and write their equations according to your analyses. ① Straight lines ② Graphs that pass through the origin ③ Curves ④ Straight lines that pass through the origin ⑤ Parallel lines ⑥ Graphs of two disconnected parts ⑦ Straight lines that slope upwards when going from left to right ⑧ Straight lines that slope downwards when going from left to right ⑨ Graphs that cross the y-axis at the same point (y-intercepts) 2-2. Let's verify whether or not your classification is correct with your graphic calculator, and then classify them into the groups you think relevant.	2-1-1. Get worksheets distributed in advance. 2-1-2. If you find that quite a few students cannot understand the related matters, explain their obscure points using some examples. 2-1-3. For the students who can make only one analysis, have them try to find out other analyses. ..... (a) 2-2-1. Confirm that they are trying to check whether their classifications are correct or not with the graphic calculators.(b) 2-2-2. Explain a series of key operations to select the function before having them draw graphs. 2-2-3. If you find that there are still some students who cannot understand, have a certain student make his presentation about his classification for example, and explain to them using it.
3. Presentation 15 minutes	3-1. Now, have the students make their presentation about their analyses and their classification result. (Refer to 2-1 for the example of their reactions.)	3-1-1. Have them give a brief description of their analyses, and make presentations of the formulas classified according to their analyses. 3-1-2. Have the students demonstrate their presentation with the graphic calculator for OHP, and have all the students verify it.

<Valuation basis of the points on teaching>

(a) Is he/she trying to take a subjective approach by making more than one analysis?

(b) Can he/she make an induction from a nature of a graph by drawing and examining it on the graphic calculator?

(3) In class

The teaching points and the role of the graphic calculator have already been mentioned. Here, I put the followings into shape; how the students' activities and attitudes toward learning, and the contents of the problem have been influenced by the use of the graphic calculator.

① The activities and attitudes of the students

Regarding the exercise given in this class, each student has made at least four analyses, and we can get almost all the expected reactions described in the teaching guideline. However, we found the students could not think about the concept of parallelism in this exercise.

When solving the problem, all twelve graphs are drawn with a graphic calculator (see Figure 1). Therefore, the students go through the process of forecasting how each graph is categorized, selecting some formulas according to their forecasts, and verifying their forecasts by drawing the graphs with the calculators. In this way the students seem to be fully conscious of the relationship between the formulas and graphs.

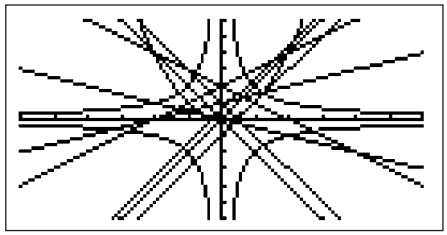

Figure 1

When they made a presentation of their analyses, the following matters were presented as analyses of classification;

Graphs that do not pass through the origin

Straight lines that do not cross the *x*-axis and the *y*-axis

Straight line that pass through the origin

Straight lines that pass through the origin and slope upwards when going from left to right

Straight lines that pass through the origin and slope downwards when going from left to right

Straight lines that slope upwards when going from left to right

Straight lines that slope downward when going from left to right

Curves

Graphs that have two disconnected parts

In addition to the above, the group of "parallel lines" has been added from the standpoints of the teacher's side. As you can see from the presentation, this approach to the exercise have been able to bring about most of the contents of direct proportions, inverse proportions, and liner functions, which we intended to have them understand.

② Attitudes towards learning and the contents of the problem

In this guidance plan, we comprehensively dealt with each graph of the direct proportions, inverse proportions, and liner functions. The following respects can characterize a learning using a graphic calculator in this exercise.

In terms of the most characteristic activity, we can make mention of the heuristic approach. As mentioned earlier, when solving the problem, we make an induction of a nature of each graph from the process of drawing the twelve graphs with a graphic calculator, forecasting how each graph is categorized, and verifying our forecasts with the help of drawn graphs. By thinking deductively the induction we have made from the relationship between the graphs and the related formulas, we could have the students carry out the thought processes of discovering each characteristic of the direct proportions, inverse proportions, and liner functions. If we were not able to utilize a graphic calculator, it would be fairly difficult to have them draw as many as twelve graphs due to the time factor. Even if they were able to draw them, we could hardly verify what we think of it immediately. Consequently, the students would not get interested in these respects. Secondarily, we can make mention of a dynamic approach in examining the graphs. Some students referred to graphs that do not cross the *x*-axis and the *y*-axis (Figure 2: graphs ⑧ and ⑪). In connection with this analysis, there was an argument whether they cross or not. A certain student expressed the opinion that the graphs

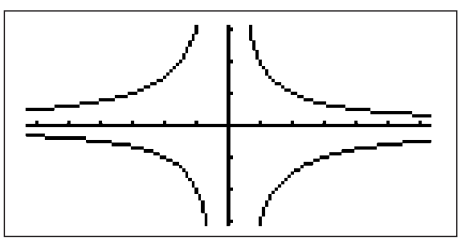

Figure 2

never cross the axes because the lines would seem not to cross the axes even if he/she moved the graph screen to the right to verify it. (Figure 3; The graph screen of ⑧ moved in the direction of the positive *x*-axis) As you can see from his/her opinion, the students generally seemed to regard the graphs as being examined not statically but dynamically. Needless to say, we cannot do like this while taking a look at the graph in print. Another student made an attempt to carry out the deductive approach in this regard; that is, the value comes close to zero boundlessly because the denominator of the formula becomes larger and larger, however, the value does not become zero because the numerator is the value other than zero, therefore, it does not cross the *x*-axis.

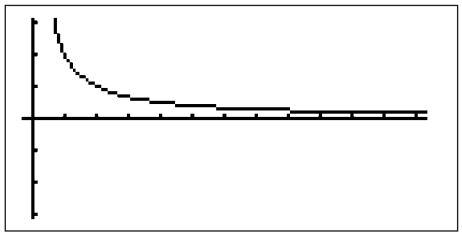

Figure 2

(4) In the second lesson

In the second lesson, we provided further insights into the contents that they presented in their first lesson. We can approach them in the following two ways; One is to deal with them individually, and the other is to make a comparison between, for example, graphs that slope upwards or downwards when going from left to right. In my class, we adopted the latter one. As a result, we were able to deal with the following contents.

① The graph of the liner function is a straight line

We dealt with this respect as making a comparison between straight lines and curves among the analyses they presented. Having the students think of the common and different points by making a comparison between these graphs, they realized that *y* is represented in terms of *x* in polynomial of degree one for all straight lines. Then we have come to conclude that we call a function linear when *y* is represented in terms of *x* in polynomial of degree one and we have also shown the general equation $y = ax+b$ for the linear function, of which the graph is a straight line. Furthermore, we have concluded that a relationship of the direct proportion is given when the straight line passes through the origin.

② Constant of proportion and slope

We dealt with this respect as making a comparison between straight lines that slope upwards or downwards when going from left to right. Having the students think of the features of the formula by making a comparison between the respective functions included. As a result, the student expressed the opinion that the negative sign is not shown in equations for straight lines that slope upwards when going from left to right. Finally they could reach a conclusion that the coefficient of *x* is positive in the case of the upward (from left to right) straight line, and concerning the direct proportion, if the constant of proportion is positive, the corresponding graph is the upward (from left to right) straight line that passes through the origin. In addition, I took notice that in the liner functions, we do not call the coefficients of *x* the constant of proportion, and I did not refer to the term of slope.

③ Hyperbolas

We dealt with this respect with the analyses which referred to the features of graph of the inverse proportion, for example, “The graph of a hyperbola does not cross the *x*-axis or the *y*-axis”, and “Its graph has two disconnected parts” rather than making a comparison. We provided further insights into whether or not the curves crosses the axes by associating this problem with the concerned formula, which was argued once when the related presentation was made. As a result, we confirmed that the curves come close to the axes boundlessly but it never crosses the axes. Also, we examined the reason why the graph of a hyperbola has two disconnected parts by associating this problem with the formula. Finally, we could reach a conclusion that there is the difference of the location of the disconnected parts in quadrants between the positive constant of proportion and the negative one, and could deal with the term of hyperbolas.

④ The slopes of two parallel lines are the same.

This analysis was added later from the standpoints of the teacher’s side. However, some students realized this rule when examining the graph and solving the exercise. The students seemed to notice that the coefficients of *x* are the same while examining the formula for the parallel lines. It is slightly regretful that I have not gone no further than teaching the line is parallel to another line. I should have taught the reason why both lines are parallel when the coefficients of *x* are the same.

In my second lesson, we could mainly deal with the abovementioned respects. This time, I taught the students in the first year of junior high school, but the some contents that were dealt with in the class are for the students in the second year. However, it did not seem to me that the students found it difficult to understand a series of exercise. This may be because they were able to utilize the graphic calculator that can appeal to their eyes. Therefore, I am sure that we can teach the proportions and graphs of linear functions at the same time, though both are taught separately under existing circumstances.