

# HOW THICK IS THE VIDEOTAPE?

Keika Commercial High School  
Masayoshi Oura

## 1. PROBLEM

---

The videocassettes today have achieved the recording time longer than before. However, seeing the inside from the transparent plastic cover of a videocassette, we can hardly tell the difference by appearance about the quantity of each tape of 120, 160 and 180 minutes. This shows that a thinner tape is used for the videocassette for longer recording hours. Here, we try to find the thickness of the tape in each videocassette, and also consider to what extent the recording time can be lengthened.

## 2. GOAL

---

### (1) Aim of Guidance

As the tape is rolled up, the tape length per round gradually becomes longer. Therefore, we will find the tape length per round when the tape is rolled up by “ $n$ ” round(s), formulize it, and then find their sums.

### (2) Role of the Graphic Calculator

In this problem, it is not so difficult to find a general term by formularizing it. However, the use of the calculator is recommended when actually substituting values and calculating them since there is quite a difference between each value of the tape’s full length and the tape’s thickness. And also, we see the tape thickness variation in proportion to the length of the tape with the Table feature of the graphic calculator.

## 3. TARGETED GRADES AND MATHEMATICAL CONCEPT USED

---

### (1) Targeted Grades

Students who learned mathematics in the first or higher grades of high school

### (2) Mathematical Concept Used

Sequences and series, and  $\Sigma$  sign

## 4. SUMMARY TO SOLVE PROBLEMS

---

When solving this problem, the values such as the reel diameter, the thickness of the whole tape currently rolled, etc., are required to perform the series of calculations, therefore, you had better prepare three kinds of videocassettes (of 120, 160, and 180 minutes) beforehand so that each approximate value can be measured by a ruler.

### (1) Find the thickness of three kinds of videotapes.

In the case of the standard 120 minutes videocassette, the diameter of a reel is about 26mm and the thickness of the whole roll of a tape is about 27mm. Assuming that the thickness of a tape is  $x$  mm and the length of the tape per round when the reel makes “ $n$ ” revolution(s) is “ $a_n$ ” mm, the equation,  $a_n = \{26 + 2(n-1)x\}\pi$  is set up since each time a tape is rolled up by 1 round, the diameter increases by  $2x$  mm. Hereby, to find the full length of the tape when the tape is rolled up by “ $n$ ” round(s), the equation,  $\Sigma a_k = \{26n + n(n-1)x\}\pi$  mm is formulated.

Since the tape's running speed is 33.35mm /sec. in the standard mode, we can find the full length of 240,120 mm by calculating  $33.35 \times 7200$ . Therefore, the following equation is formularized.

$$\{26n + n(n-1)x\}\pi = 240120$$

Solving the above equation with  $nx$  -thickness of whole roll of a tape - as 27mm, it turns out that the value of "n" is about 1,443, and also the thickness of the whole roll of a tape "x" is about 0.019mm (equal to about 19micorns) which is derived from dividing 27 by 1,443. In this process of calculation, we can use the calculator if needed.

The same solving technique applies to the 160 and 180minutes videocassettes, however, you should pay attention to the thickness of the whole roll of a tape being about 30mm. In this case, the thickness of the whole roll of a tape of 160 and 180 minutes videocassettes are calculated at about 16 and 15microns respectively.

## (2) Generalizing

Taking a look at the inside of a videocassette, we find out the ceiling of the thickness of the whole tape rolled around a reel is about 30mm. This means each tape for 160 and 180 minutes is rolled at the full capacity. When the thickness of the whole tape is set to 30mm, it is served as the function of the recording time. When replacing the recording time with "t" minutes, and the tape thickness at that time with "x" mm ("n" rounds rolled), the equation,  $\{26n + n(n-1)x\}\pi = 33.35 \times 60t$  is formulated. Since  $nx$  is equal to 30, finally, we arrive at the following formula by finding the value of "x" with "n" eliminated, although this may be a little troublesome task.

$$x = \frac{56\pi}{33.35 \times 2t + \pi}$$

Then, we see the tape thickness variation by making a table of "t" variations from 120 to 250 in 10 minutes unit intervals with the Table feature of the graphic calculator used.

X	Y1
180	0.0146
190	0.0138
200	0.0131
210	0.0125

0.01464960566

FORM DEL 200 G-COM G-PLT

As seeing in the table, we notice that the tape becomes thinner than 13 microns, in which case the value of "t" (minutes) is 200 or more.

The tape with too little thickness is of no use because it becomes impossible to fully secure the tape tension - the tension required for stable recording and playing back - since the tape durability becomes low although it is a technical subject. Therefore, we consider that around 200 minutes are the tape's limit at the longest.