

DESIGNING PATTERNS USING EQUATIONS IN THE POLAR COORDINATE SYSTEM

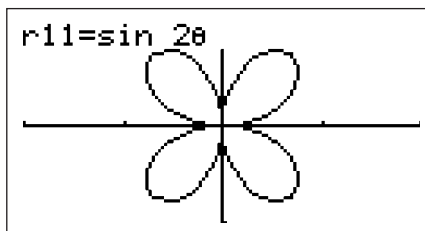
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1. THE FLOW OF GUIDANCE

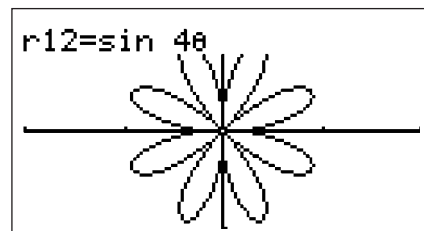
There is a limit to the level of patterns that we can design with only quadric, exponential, and logarithmic functions. Learning the polar coordinate system additionally, however, makes it possible to design them more intricately. In the introductory stage, we start with the spiral of Archimedes $r=\theta$, and introduce the trigonometric functions of sine and cosine to the class, so that we can give the students a problem on designing a flower pattern. Then, we are supposed to lead up to the next question while having them think over “which factor affects the number of flower petals?”

“What is the relationship between “ a ” in $r=\sin a\theta$ and the number of flower petals b ?”

We can simply find out the specific regularity by plotting the several graphs as shown in Screens 1 and 2.



Screen 1



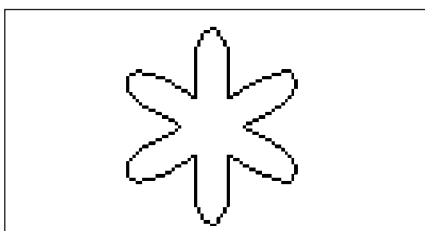
Screen 2

Here, we see the rule that b is equal to $2a$ when the “ a ” is an even number, and b is equal to a when it is an odd number. Then, have the students think over the reason and this will be a new problem for them.

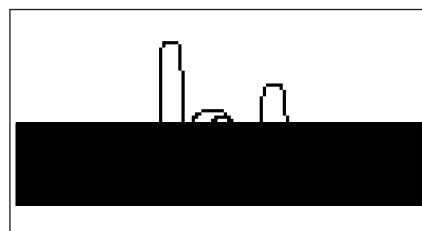
Furthermore, after substituting a with a fractional number such as $1/3$ and graphing the function ($r = \sin \frac{\theta}{3}$), have the students try to plot any figures they like. Here we make one condition that the students are allowed to use any functions to design them, but they have to include at least one function in the form of $r=f(\theta)$.

In the meantime, they imagine their own patterns and work on them some more, based on the printed pattern, and finally they have come to design really intricate patterns. For example, here are the graphs the students actually created. By the way, the sixth pattern is my work designed during the class.

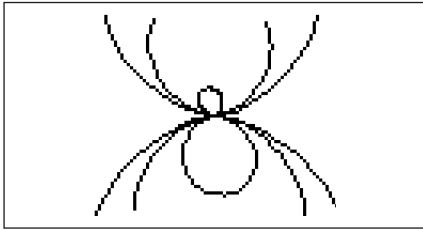
John: Snowflake



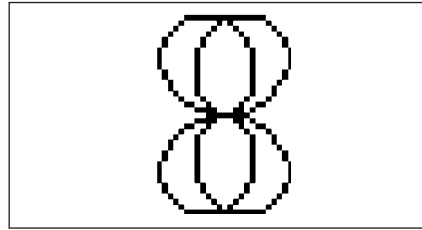
Paul: Synchronized Swimming



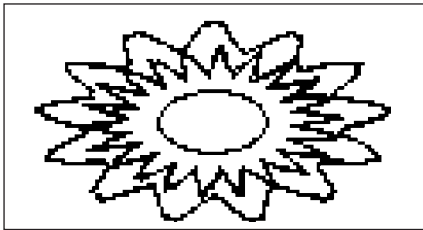
George: Spider



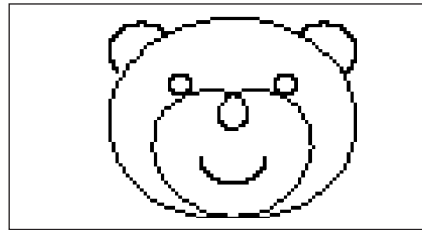
Ringo: Number 8



Yoko: Hibiscus



Author: Bear



Each function and the Range are shown below.

John:

$$r = 3 - \cos 6\theta$$

$$-6.3 \leq x \leq 6.3, \quad -3.1 \leq y \leq 3.1$$

$$0 \leq \theta \leq 2\pi$$

Paul:

$$y \leq 0, \quad r = \theta, \quad r = 1, \quad y = -(x-5)^6 + 5$$

$$y = -(x+5)^{10} + 10$$

$$-20 \leq x \leq 20, \quad -10 \leq y \leq 15$$

$$0 \leq \theta \leq \pi$$

George

$$r = \theta \sin^2 \theta$$

$$-10 \leq x \leq 10, \quad -6 \leq y \leq 6$$

$$0 \leq \theta \leq 19, \quad \text{pitch} : 0.06$$

Ringo

$$r = \cos 0.5\theta \sin \theta$$

$$-2.5 \leq x \leq 2.5, \quad -1.26 \leq y \leq 1.25$$

$$0 \leq \theta \leq 4\pi$$

Yoko

$$r = 2, \quad r = \sin 13\theta + 5 + \log 5$$

$$r = \sin 22\theta + 3 + \log 9$$

$$r = \sin 3\theta + 36 + \log 2$$

$$-8 \leq x \leq 8, \quad -7 \leq x \leq 7$$

$$0 \leq \theta \leq 19$$

Author

$$r = 4 \sin \frac{\theta}{5}$$

$$x = \cos\left(\frac{T}{5} + \pi\right), \quad y = \sin\left(\frac{T}{5} + \pi\right) - 1$$

$$x = \frac{1}{3} \cos T + 1.6, \quad y = \frac{1}{3} \sin T + 1.6$$

$$x = \frac{1}{3} \cos T - 1.6, \quad y = \frac{1}{3} \sin T + 1.6$$

$$x = \cos\left(\frac{T}{5} - \frac{\pi}{4}\right) + 2.8, \quad y = \sin\left(\frac{T}{5} - \frac{\pi}{4}\right) + 2.8$$

$$x = \cos\left(\frac{T}{5} + \frac{\pi}{4}\right) - 2.8, \quad y = \sin\left(\frac{T}{5} + \frac{\pi}{4}\right) + 2.8$$

$$-6.8 \leq x \leq 6.8, \quad -3.5 \leq y \leq 4.5$$

$$0 \leq \theta \leq 5\pi$$