VISUAL UNDERSTANDING OF DEFINITIONS AND THEOREMS RELATING TO DIFFERENTIATION

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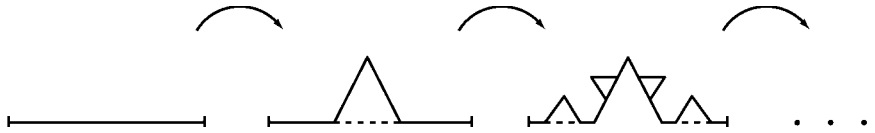
1. THE AIM OF GUIDANCE AND THE USE OF THE GRAPHIC CALCULATOR

When the students study functions, sequences, limits, differentiation, etc., it is difficult for them to get a visual understanding of the shape of a graph, how it changes, or how a limit is approached. Because definitions and theorems of functions, sequences, limits, differentiation, etc., are given to them without any preliminary knowledge, it is likely that the students can understand them only superficially. Here, we give them a visual image at the same time or prior to giving them its expression by using a graphic calculator. Then, we have the students try to derive theorems relating to functions, sequences, limits, differentiation, etc., and give them not only formulary but also visual understanding.

2. HOW LONG?

Trisect a segment 1 meter in length and bisect and fold the middle segment as shown below. If we repeat this step many times, how long is the total length?



Give the students this problem and have them guess the answer. It is obvious that the total length becomes $\frac{4}{3}$ times the original length in a single step. Then we calculate the length using the graphic calculator. Select the Table feature, enter the expression $y=(\frac{4}{3})^x$, and examine the Table. Set the Range as follows.

Start: 1
End: 100
Pitch: 1

Scroll the Table with the arrow key and you can find the total length after 100 steps. Surprisingly, it is more than 3.1 billion kilometers! Just a segment that is 1 meter in length becomes about 80,000 times the circumference of the earth (about 40,000 km).

Then, we change $\frac{4}{3}$ to 0.9 and examine how the total length changes using the graphic calculator. Finally we have students guess the following theorem.

If $r>1$, then $\lim_{x\to\infty} r^x = \infty$

If $0<r<1$, $\lim_{x\to\infty} r^x = 0$

Now you can have the students understand that the x th power a figure larger than 1 becomes larger infinitely as shown above. If some students ask the reason why, have them try to prove it.

Reference

Yoshikazu Higuchi, Hiroshi Hosokawa, Toshikazu Ikeda : Fostering Student's Capacity in Mathematics, 1998, pp.123-135.

David Nelson, George Gheverghese Joseph, and Julian Williams : Multicultural Mathematics, Oxford University Press, 1993, pp.55-57.