VISUAL UNDERSTANDING OF DEFINITIONS AND THEOREMS RELATING TO DIFFERENTIATION

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1. THE AIM OF GUIDANCE AND THE USE OF THE GRAPHIC CALCULATOR

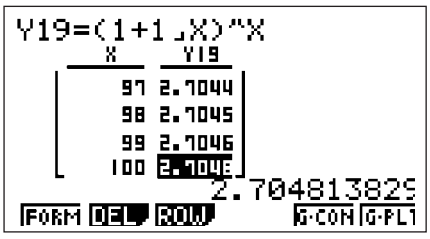
When the students study functions, sequences, limits, differentiation, etc., it is difficult for them to get a visual understanding of the shape of a graph, how it changes, or how a limit is approached. Because definitions and theorems of functions, sequences, limits, differentiation, etc., are given to them without any preliminary knowledge, it is likely that the students can understand them only superficially. Here, we give them a visual image at the same time or prior to giving them its expression by using a graphic calculator. Then, we have the students try to derive theorems relating to functions, sequences, limits, differentiation, etc., and give them not only formulary but also visual understanding.

2. NAPIERIAN NUMBER e

We now modify the problem. Let *x* be a figure larger than 1 and think the x th power of $\left(1+\frac{1}{x}\right)$. If we assign numerically larger and larger values to *x*, does the *x* th power of $\left(1+\frac{1}{x}\right)$ become larger and larger accordingly? Like before, have the students guess the result and check it by using the graphic calculator.

Select the Table feature, enter the expression $\left(1+\frac{1}{x}\right)^x$, then examine the Table. Set the Range as follows.

Start: 1
End: 100
Pitch: 1



Screen 1

Now we obtain a value of 2.704813 after 100 steps and feel that the figure is much smaller than the previous figure of 3.1 billion even though the number of steps is as large as 100. A question may arise from the students.
"Is there a value to which it is approaching? Or is it getting larger and larger infinitely?"
This question may lead to the necessity of proof and you can show the students the concept of the finite and monotonic quality. Finally the following definition can be introduced.

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e$$

It is advisable to introduce such a definition after having the students calculate using the graphic calculator and guess the result. This method is more effective than to show the definition from the start because the students will be more interested in it. Furthermore, they feel familiar to this definition if they use the graphic calculator to guess the result.

Reference

Yoshikazu Higuchi, Hiroshi Hosokawa, Toshikazu Ikeda : Fostering Student's Capacity in Mathematics, 1998, pp.123-135.
David Nelson, George Gheverghese Joseph, and Julian Williams : Multicultural Mathematics, Oxford University Press, 1993, pp.55-57.