VISUAL UNDERSTANDING OF DEFINITIONS AND THEOREMS RELATING TO DIFFERENTIATION

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1. THE AIM OF GUIDANCE AND THE USE OF THE GRAPHIC CALCULATOR

When the students study functions, sequences, limits, differentiation, etc., it is difficult for them to get a visual understanding of the shape of a graph, how it changes, or how a limit is approached. Because definitions and theorems of functions, sequences, limits, differentiation, etc., are given to them without any preliminary knowledge, it is likely that the students can understand them only superficially. Here, we give them a visual image at the same time or prior to giving them its expression by using a graphic calculator. Then, we have the students try to derive theorems relating to functions, sequences, limits, differentiation, etc., and give them not only formulary but also visual understanding.

2. SQUARE ROOT

There is two methods to find the square root of a number n; the conventional method in which a number is separated in two digits to the left and to the right of the decimal point, and the other is the divide-and-average method, which is the Alexandrine Heron's algorithm (Heron; around 62 AD). The latter method can be used to have the students understand visually using a graphic calculator why the square root of a number is found by such an algorithm. The algorithm of Heron is as follows. (David Nelson, 1993)

1. Select an estimated value.
2. Divide ***n*** by the estimated value.
3. Find the average of the quotient and the estimated value.
4. Let the average be the next estimated value.

In a class, it is advisable just to give the students the algorithm without explaining its purpose, and ask them what it is for. For example, let *n* be 5 and enter the following expression in the graphic calculator.

$$a_{n+1} = \frac{\left(\frac{5}{a_n} + a_n\right)}{2}$$

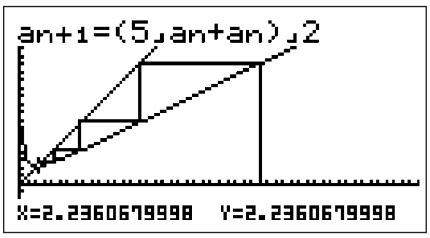
Let the initial value be 30 and examine the Table. We obtain a value of 2.236067977 in the 9th term, which is approaching to $\sqrt{5}$.

This can be explained if you solve the characteristic equation and obtain x= $\sqrt{5}$ after substituting a_{n+1} and *n* with *x*. But to the students, the reason why the characteristic equation should be solved remains unknown. Then, the Web feature of the graphic calculator is useful. In other words, the intersection of the following two graphs indicates $\sqrt{5}$.

$$y = x$$

$$y = \frac{\left(\frac{5}{x} + x\right)}{2}$$

Let the initial value be 30 and we understand visually how a_n approaches to $\sqrt{5}$.



Screen 2

Reference

Yoshikazu Higuchi, Hiroshi Hosokawa, Toshikazu Ikeda : Fostering Student's Capacity in Mathematics, 1998, pp.123-135.

David Nelson, George Gheverghese Joseph, and Julian Williams : Multicultural Mathematics, Oxford University Press, 1993, pp.55-57.