VISUAL UNDERSTANDING OF DEFINITIONS AND THEOREMS RELATING TO DIFFERENTIATION

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1. THE AIM OF GUIDANCE AND THE USE OF THE GRAPHIC CALCULATOR

When the students study functions, sequences, limits, differentiation, etc., it is difficult for them to get a visual understanding of the shape of a graph, how it changes, or how a limit is approached. Because definitions and theorems of functions, sequences, limits, differentiation, etc., are given to them without any preliminary knowledge, it is likely that the students can understand them only superficially. Here, we give them a visual image at the same time or prior to giving them its expression by using a graphic calculator. Then, we have the students try to derive theorems relating to functions, sequences, limits, differentiation, etc., and give them not only formulary but also visual understanding.

2. FROM BUNGEE JUMPING TO DIFFERENTIATION

It was written on the bungee jumping pamphlet in Australia like this.

> This platform was made just for bungee jumping. There has been no accidents since built, so your safety is 100% assured. You can try jumping for yourself or just watch the jumper's jump, it's your choice. The platform is 15 stories high (44m), the drop speed will reach 90km per hour. If you came this way you need to go all the way.

Using this pamphlet as a base, first show them a video on bungee jumping and have them anticipate the movements. By focusing on the jumper's falling speed, you will sea that the velocity increases after jumping, and the speed reaches the maximum when the jumper falls the length of the rubber. After that you will know that the rubber stretches and the speed decreases. From this you can introduce the following problem which will bring on the concept of differentiation. "At what meter does the rubber that is put on the jumper's ankle start to stretch (how long is the rubber?)?"

Based on the relationship for falling bodies derived from the Newton's law,

$$f(x) = \frac{gt^2}{2} \qquad \left(g = 9.8\text{m} / \text{s}^2\right)$$

when x is the distance of fall (m), t is the elapsed time (second) and g is the acceleration of gravity (9.8 m/s^2), we can calculate the speed at t as follows. Divide the distance fall between t and t+0.001 by 0.001.

$$y = \frac{f(x + 0.001) - f(x)}{0.001}$$

Because the maximum speed is 25m/s (90 km/hour), we find that the maximum speed is reached after about 2.5 seconds by drawing the graph using the graphic calculator and examining it with the Trace key. From this fact we know the length of the rubber is about 31 meters. Thus by using the graphic calculator, it is possible for the students to experience the idea of differentiation and solve the problem without troublesome calculations. Here all we have to do is to prove the following definition of differentiation based on the matters discussed so far.

$$f'(x) = \lim_{x \to \infty} \frac{f(x + h) - f(x)}{h}$$

Reference

Yoshikazu Higuchi, Hiroshi Hosokawa, Toshikazu Ikeda : Fostering Student's Capacity in Mathematics, 1998, pp.123-135.

David Nelson, George Gheverghese Joseph, and Julian Williams : Multicultural Mathematics, Oxford University Press, 1993, pp.55-57.