

# VISUAL UNDERSTANDING OF DEFINITIONS AND THEOREMS RELATING TO DIFFERENTIATION

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## 1. THE AIM OF GUIDANCE AND THE USE OF THE GRAPHIC CALCULATOR

When the students study functions, sequences, limits, differentiation, etc., it is difficult for them to get a visual understanding of the shape of a graph, how it changes, or how a limit is approached. Because definitions and theorems of functions, sequences, limits, differentiation, etc., are given to them without any preliminary knowledge, it is likely that the students can understand them only superficially. Here, we give them a visual image at the same time or prior to giving them its expression by using a graphic calculator. Then, we have the students try to derive theorems relating to functions, sequences, limits, differentiation, etc., and give them not only formulary but also visual understanding.

## 2. DERIVATIVES OF ELEMENTARY FUNCTIONS

Let's consider the derivatives of some familiar elementary functions using the concept of differentiation. The problem is how to calculate such elementary functions as  $f(x) = \sin x$ ,  $f(x) = \cos x$ ,  $f(x) = e^x$ ,  $f(x) = \log x$ . To calculate derivatives of these functions using the definition as in a text, we need such a theorem as the following and it will be hard to calculate.

$$\lim_{x \rightarrow \infty} \frac{\sin x}{x} = 1$$

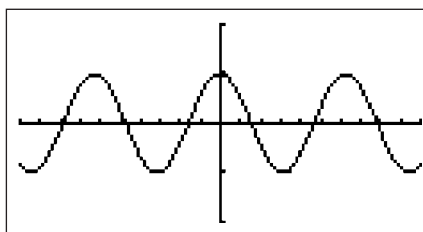
Although it is very important to calculate derivatives using a definition, not all students learn mathematics for their whole life. I think there can be such a mathematical education in which students understand differentiation visually using a graphic calculator.

Now we find the derivative of  $f(x) = \sin x$  from the definition using the graphic calculator.

$$f'(x) = \lim_{x \rightarrow \infty} \frac{\sin(x+h) - \sin x}{h}$$

Let  $h=0.01$  and put it in the graphic calculator, and we obtain the following graph immediately.

$$y = \frac{\sin(x+0.01) - \sin x}{0.01}$$



Screen 7

Looking at this graph, some students may wonder "Is the derivative of  $y = \sin x$   $y = \cos x$ ?" In this way, they can understand the derivative of  $\sin x$  visually, without troublesome calculations. It is very easy to guess derivatives of some elementary functions using the graphic calculator. If some students want to prove it, have them try. To those who are not interested in proof, it is just a burden. In other words, the aim of the class can be how to have the students feel the necessity of proof.

### Reference

Yoshikazu Higuchi, Hiroshi Hosokawa, Toshikazu Ikeda : Fostering Student's Capacity in Mathematics, 1998, pp.123-135.

David Nelson, George Gheverghese Joseph, and Julian Williams : Multicultural Mathematics, Oxford University Press, 1993, pp.55-57.