Asymptotes of Rational Functions

Name:_____

Per:_____

Date:_____

GETTING READY

1) Open the Main Application (J).

2) Select **Edit** and then **Clear All**.

3) Select **OK** when prompted with the **Clear All** menu.

An asymptote is a line that a curve approaches but never reaches.

INVESTIGATION: Degree of Numerator is *Less* than Degree of Denominator

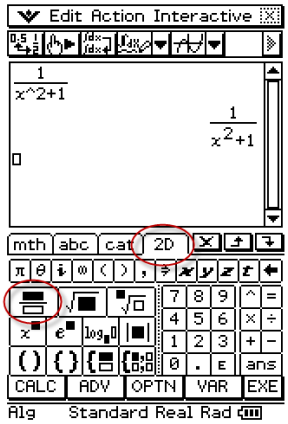
1) Make a conjecture about what asymptotes would be present in this case. Think about what would happen with the function $f(x)=\frac{1}{x^2+1}$. What will happen to $f(x)$ as x gets very large? What will happen to $f(x)$ as x gets very small?

2) On your ClassPad, in J which should already be open, enter $\frac{1}{x^2+1}$.

   a) Open your k to enter the function. Once your keyboard is open, tap the **2D** tab. Now tap on the selection for a fraction (N). You can now close your k.

   b) The cursor will automatically be in the numerator. Enter **1**. Use your cursor button to arrow down to the denominator to enter **x^2+1** using your keypad. Press **EXE**.

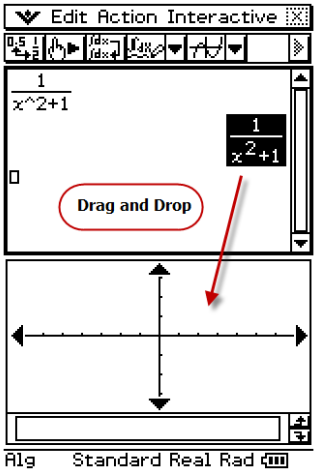
Result:



3) Graph the function expression you just entered.

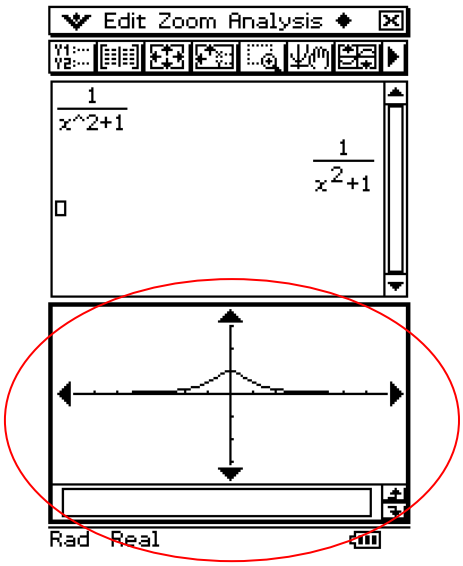
a) Open the graph window by clicking on the toolbar icon ($) in the upper right.

b) Select what you entered by highlighting, pressing, dragging, and releasing onto the graph window:



c) The function expression will graph in the graph window.

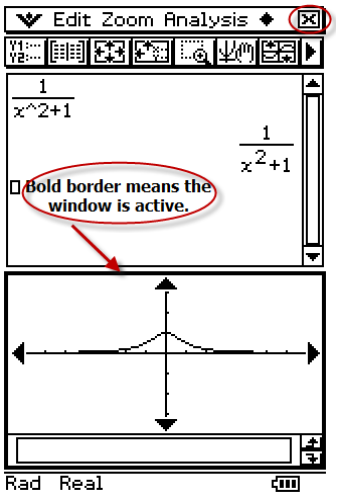
Result:



Where does it look like the asymptote lies?

d) Now close the graphing window. Make sure the window is active and close it using the S in the upper right of the menu bar.

Closing the Graphing Window:

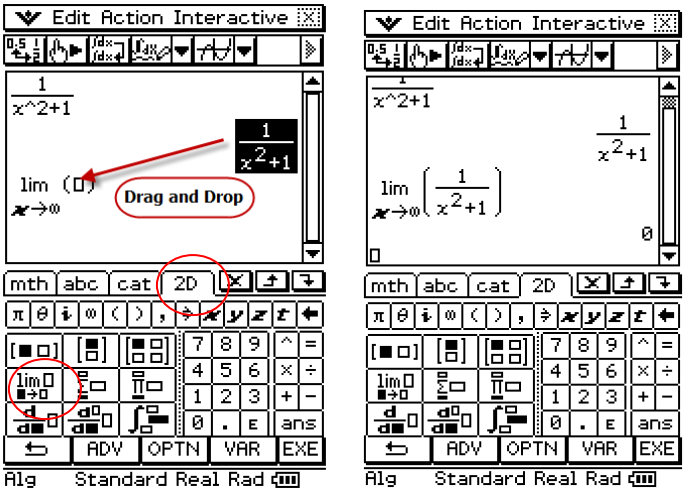


4) Find the limits of the function as x goes to positive and negative infinity. Tap in the J window.

a) Open your k to enter the limits. Once your keyboard is open, tap the **2D** tab and select the **CALC** tab at the bottom. Now tap on the selection for a limit (**;**).

b) The cursor will automatically be in the lower left box. Enter **x**. Use your cursor button to arrow over to the next box to enter ∞. Highlight, press, and drag the function expression into the box between the parenthesis of the limit. (See below.) Press **EXE**.

Entering Limits: Result:



c) Now repeat the process with **x** approaching -∞.

5) Look at your work in step 4 above. How do the limits relate to the asymptote? Do you think this would be the same for every function where the degree of the numerator is less than the degree of the denominator? Why?

6) Write an equation for the asymptote of the function $f(x) = \frac{1}{x^2 + 1}$.

7) Clear your graphing window and main window.

   a) Make the window you want to clear active by tapping in it.

   b) Choose **Edit**, **Clear All**. When prompted, select **OK**.

8) Now repeat steps 1-6 with a function of your choice where the degree of the numerator is less than the degree of the denominator. Number and answer all questions below as you encounter them in the steps above for your new function. Start by writing the function in the space provided.

FUNCTION: _____

INVESTIGATION: Degree of Numerator is *Same* as Degree of Denominator

Now that you know the process to follow from the above investigation, use your knowledge to explore the function $f(x)=\frac{2x^2+7}{3x^2+x+12}$. Follow the all the steps above for this function and make sure to answer all of the questions as you are investigating this function. You can number your answers and put them below.

After you are finished, make sure you **don't forget step 8** above for a function of your choice where the degree of the numerator is the same as the degree of the denominator.

For step 8:

FUNCTION: _____

INVESTIGATION: Degree of Numerator is *One Degree Higher* than Degree of Denominator

Now, use your knowledge to explore the function $f(x)=\frac{x^2+x+1}{x}$. Follow the all the steps from the first investigation for this function. What happens when you complete step 4? Finish answering the questions up to and including step 4. **Stop after you complete step 4**. In this case, there is a special way to find the asymptote. You must use polynomial division.

1) Go ahead and do the following division:

$$x \overline{)\ x^2 + x + 1}$$

2) What did you get as your quotient?

3) On your ClassPad, enter this quotient in J and press **EXE**. Now drag it in to the graph window. Make sure your original function is graphed in this window as well. What do you notice?

4) Write the equation for the asymptote for this function.

5) Repeat steps 1-4 in this investigation with a function of your choice where the degree of the numerator is one higher than the degree of the denominator. Don't forget to answer the questions!

Choose your own:

FUNCTION: _____

INVESTIGATION: Degree of Numerator is *More Than One Degree Higher* than Degree of Denominator

1) Graph the following functions on your ClassPad. After graphing each function, answer the following question(s): Does it look like the graph has an asymptote? If, so where?

a) $f(x) = \frac{2x^4}{3x^2 + 1}$

b) $f(x) = \frac{x^3}{x + 1}$

c) $f(x) = \frac{x^5 + 3}{3x^2 + 2}$

2) Make a conjecture about asymptotes of rational functions where the degree of the numerator is more than one degree higher than the degree of the denominator.

PUTTING IT ALL TOGETHER...

Use what you've learned in the investigations above to fill out the following table (if a certain category does not have asymptotes present, just write NONE):

Rational Function	**Equation of Asymptote (In General)**	**Example of Function and its Asymptote**
Degree of Numerator is *Less* Than Degree of Denominator	y=	
Degree of Numerator is *Same* as Degree of Denominator	y=	
Degree of Numerator is *One Degree Higher* than Degree of Denominator	y=	
Degree of Numerator is *More Than One Degree Higher* than Degree of Denominator	y=	