Infinite Sequences

Name______________________

A sequence can be thought of as a list of numbers written in order:

$$a_1, a_2, a_3, a_4, \ldots, a_n, a_{n+1}, \ldots$$

The number a_1 is called the *first term*, a_2 is the *second term*, and in general a_n is the *n*th *term*. There are infinitely many numbers in the list, so after each term there will always be another, a_{n+1}. Take note that not all sequences start with *n=1*.

The sequence $\{a_1, a_2, a_3, \ldots\}$ is often denoted by

$\{a_n\}$ or $\{a_n\}_{n=1}^{\infty}$

We often define sequences by a formula for the nth term.

a) $\left\{\dfrac{1}{n}\right\}$ $\qquad a_n = \dfrac{1}{n}$ $\qquad \left\{1, \dfrac{1}{2}, \dfrac{1}{3}, \dfrac{1}{4}, \ldots, \dfrac{1}{n}, \ldots\right\}$

b) $\left\{\dfrac{(-1)^n n}{n^2+1}\right\}_{n=0}^{\infty}$ $\qquad a_n = \dfrac{(-1)^n n}{n^2+1}$ $\qquad \left\{0, -\dfrac{1}{2}, \dfrac{2}{5}, -\dfrac{3}{10}, \dfrac{4}{17}, \ldots, \dfrac{(-1)^n n}{n^2+1}, \ldots\right\}$

c) $\{\cos(\pi n)\}_{n=0}^{\infty}$ $\qquad a_n = \cos(\pi n)$ $\qquad \{1, -1, 1, -1, \ldots, \cos(\pi n), \ldots\}$

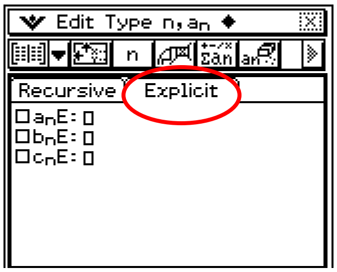
Exercise 1

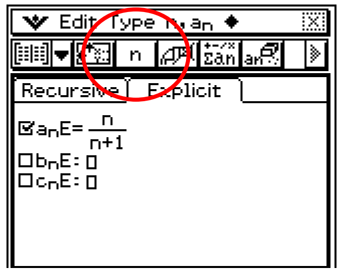
Fill in the formula for the nth term and then give the first four terms of the sequences.

a) $\left\{\dfrac{n}{n+1}\right\}_{n=1}^{\infty}$

b) $\left\{\sqrt{n-4}\right\}_{n=4}^{\infty}$

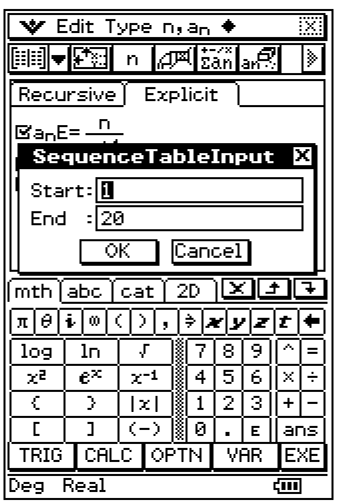
Let's explore sequences with the ClassPad! From the Application Menu, go to the Sequence Application H. Tap the Explicit tab, enter the formula $a_n E = \dfrac{n}{n+1}$, and then press EXE. (The ***n*** can be found in the toolbar. The 2D fraction button is in the 2D tab of the Keyboard)

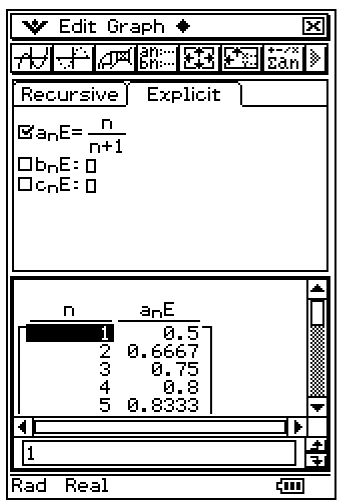


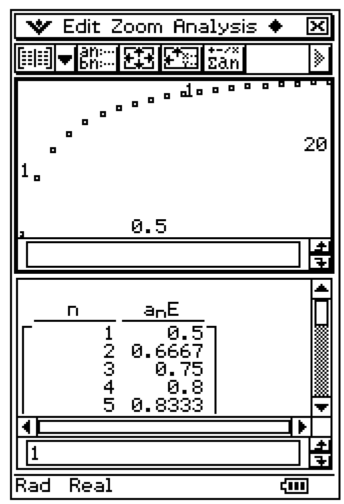


Then tap the Sequence Table Input button, 8, in the toolbar. In the dialog box start at 1 and end at 20 then tap OK. Tap the Ordered Pair Table button, #, to view a table with the first 20 values.

With the Table window active (in bold), tap the discrete graph button, !. To better see your data, select Zoom in the menu bar and select Auto.







Exercise 2

From the table and the graph of points can you determine what number the sequence $\left\{ \frac{n}{n+1} \right\}_{n=1}^{\infty}$ approaches as n increases?

Exercise 3

Can you determine what numbers (if any) the sequences $\left\{ \frac{(-1)^n n}{n^2+1} \right\}_{n=0}^{\infty}$ and $\{\cos(\pi n)\}_{n=0}^{\infty}$ approach as n increases?