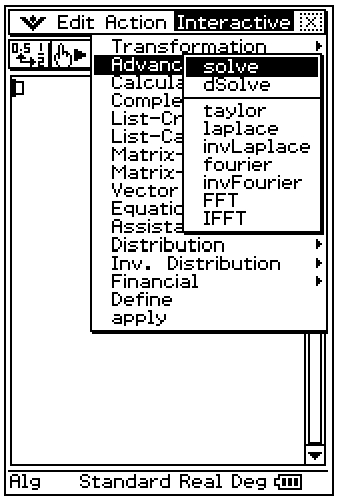
Name:__________________________________ **Date:**_________________

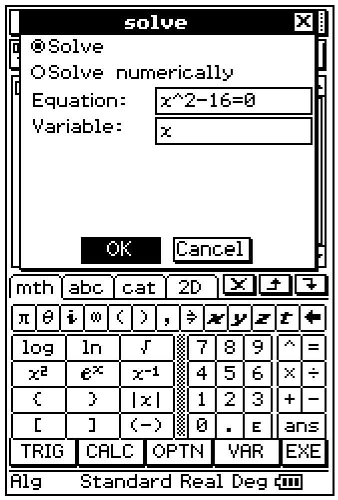
Complex Numbers

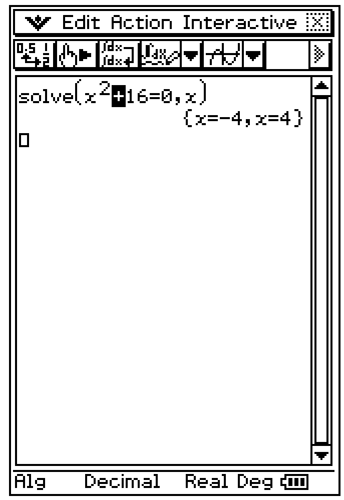
Solve the following equation $x^2 - 16 = 0$.

1. From the start menu (m) select J. Select **Edit/Clear All** if needed.
2. Select **Interactive/Advanced/Solve.**
3. Enter the above equation using the k and tap **OK**.

What are the two solutions? $x =$ $x =$







Now solve the equation $x^2 + 16 = 0$

4. Alter the previous expression by tapping to the right of the minus sign, press K then press +.
5. Tap 7 to re-execute the line.

What is the result? Why? What number, when you multiply it by itself gives you -16?

When we rewrite the expression using properties of roots we see that the new type of number that we haven't met yet is $\sqrt{-1}$.

$$x = \pm\sqrt{-16} = \pm\sqrt{(-1)\cdot 16} = \pm\sqrt{(-1)}\sqrt{16} = \pm 4\cdot\sqrt{(-1)}$$

There is no *real number* such that when squared, results in -1. Hence we have to define another type of number called an imaginary number i. When you square this number you get -1. So the definition of the imaginary unit is:

$$i^2 = -1 \text{ or } i = \sqrt{-1}$$

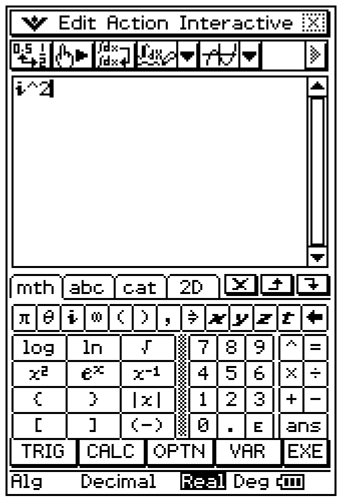
Notice now that if $i^2 = -1$ then $i^3 = i^2 \cdot i = -1i = -i$.

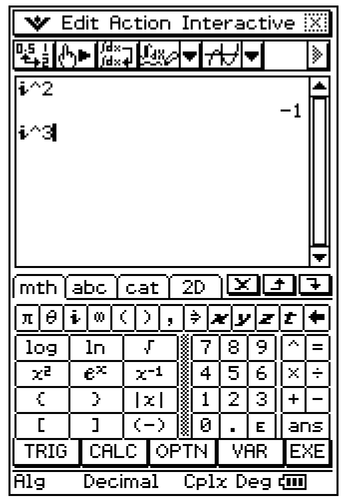
What is $i^4 = ?$ $\quad i^5 = ?$ $\quad i^6 = ?$

<u>Change from Real to Complex number system</u>

1. Change the set of numbers you are working with from Real to Complex by tapping once on the **Real** setting on the status bar at the bottom.

2. Enter an expression into the ClassPad using the k and tap E. Do the same for the other expressions.





Complete:

a) $i^4 =$

b) $i^5 =$

c) $i^6 =$

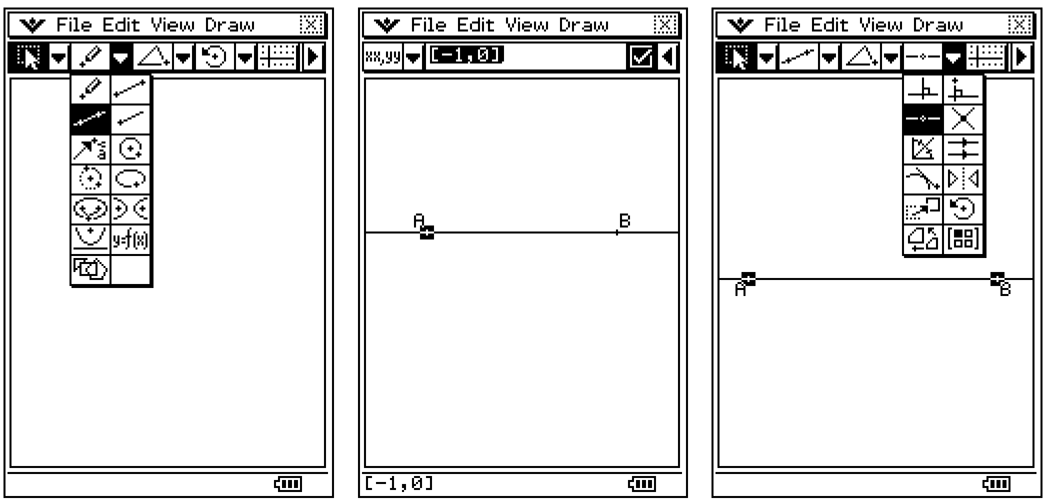
d) Without using the ClassPad, evaluate: $i^{60} =$
Now check using the ClassPad.

<u>Visualize the imaginary numbers</u>

You might think that imaginary numbers are strange and don't exist, but this is the same kind of thinking the ancient civilizations had about the negative numbers and the number zero.
(How can *something* represent *nothing* or less than nothing?)

Create a number line:

1. From the start menu (m) select G. Select **Edit/Clear All** if needed.
2. Tap the second n from the left and choose w. Or, from the Draw menu select Infinite Line.
3. Tap the left side of the screen, then tap on the right side. Try to make a perfectly horizontal line!
4. Tap u to advance the toolbar to see the Measurement Box.
5. Select point A. Edit the point to be [-1,0], if needed. **Tap** E to set the point.
6. Tap any clear space to deselect all. Tap B and edit the point to be [1,0]. **Tap** E.
7. Tap w. Use  to navigate the screen. Also, press the + or – keys to zoom the window quickly.
8. Tap both points A and B to select them, and then **tap** right most n and select u.
9. Tap in clear space to deselect.



When we look at the number line we understand that there are 4 basic operations that we can do to get from one point to the other (Addition, Multiplication, Subtraction, and Division). To get from 2 to -2 we multiply 2 by -1. Now consider the equation:

$$1 \cdot x^2 = -1$$

What must I do to the number 1 (point B) *twice* to end up at the number -1 on the number line?

Consider doing the following: (Screen shots are shown below for your aid)
1. Tap point B, then select the right most n and select F.

2. Tap on point C and then select **OK** (Angle 90)**.**

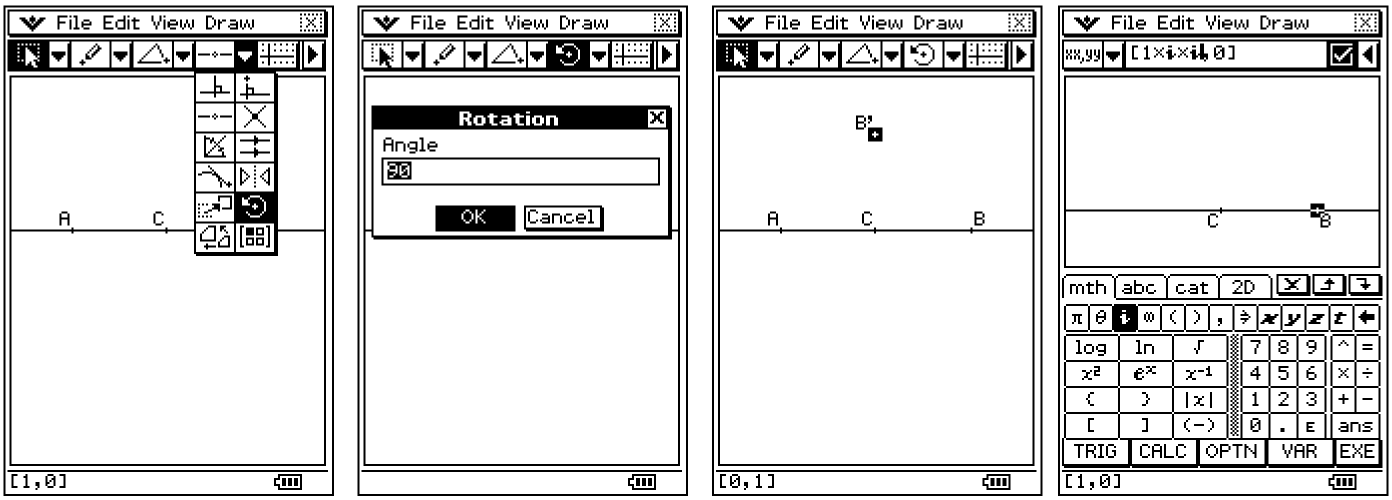
3. Deselect point B and select point B'.

4. Select F and select point C again. Select **OK.**

What is the operation that you have completed twice?

Now consider using algebra to achieve the same result:

5. Now delete points A, B' and B" by selecting the points, and tap K or > on the soft keyboard.
6. Tap q once to toggle the axes on.
7. Select point B and tap u. **Tap** after the 1 in the coordinate and type in $\times i \times i$ using the k. Tap E.



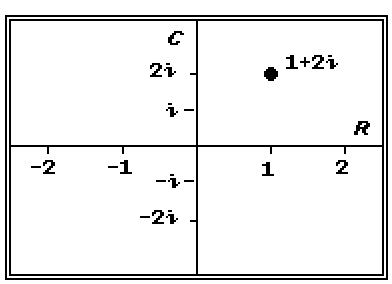
What are you doing geometrically to the real numbers on the line when you multiply by i ***? Or*** $2i, -i$ ***?***

The convention is to superimpose a vertical axis called the imaginary axis at the origin of the horizontal axis. We have just determined that numbers have 2 dimensions to them: a real part and an imaginary part.

The most general form of the number that includes the imaginary unit is:

$$x = a + bi \qquad \text{where } a, b \in \Re$$

All numbers of this form are called **Complex numbers.**
The set of complex numbers is the set of numbers that can be found as the sum of a real and an imaginary number.



Exercise:
Find the roots of the following quadratic equation: $y = x^2 - 10x + 34$

What are the two solutions?

1. From the start menu (m) select J.

$x =$

$x =$

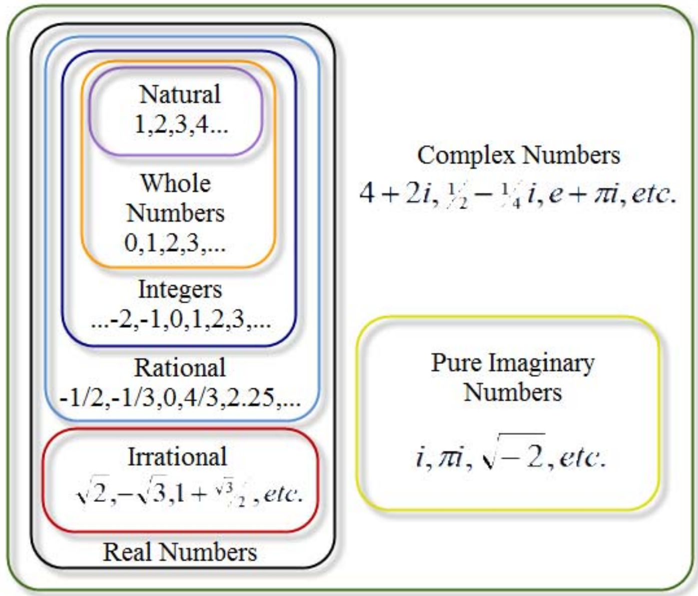
2. Make sure the status bar says **Cplx.** Tap **Real** if it doesn't.

3. Select **Edit/Clear All** if needed. Select **Interactive/Advanced/Solve.**

4. Enter the equation using the k and tap **OK.** (Remember to set $y = 0$.)

Notice that the complex roots of any polynomial always come in **conjugate pairs!**

- Over the centuries as society developed its knowledge infrastructure, we have realized that modeling phenomena in the world requires different types of numbers.
- The following diagram describes how one number set is included in the other.
- Notice that the complex numbers are all you need to find roots of all polynomials within the real number "realm".



Answer the following questions:

1. What does $i^2 = -1$ mean to you? Finish the sentence: If multiplying by -1 reflects the number about the origin then multiplying the number by i^2 ....

2. If the absolute value of a number on the number line is its distance from the origin, then geometrically show what does $|1+2i|$ mean in the complex plane? Which formula that you know of will give you this distance?

3. **BONUS**: As negative numbers model flipping, imaginary numbers **can model anything that rotates** between two dimensions “X” and “Y”. Or anything with a **cyclic, circular relationship —** have anything in mind?
Write a formula that includes i, for a person riding a 100 foot diameter ferris wheel. Hint: use $\cos(\theta)$ & $\sin(\theta)$.