

# Paying off a BIG loan for a BIG TV.

## - Activities dealing the present value of a simple annuity -

We suggest that you complete the activity called 'Understanding and working with compound interest' before starting this set of activities.

### Introduction.

Suppose I am interesting in buying a new large (45 inch) Sharp LCD TV – RRP \$11 999.

AQUOS



Also, suppose I have to take out a loan to enable the purchase. Do you understand the process by which loan repayments are calculated or how to perform calculations based on 'what if' scenarios with loans? If not, these activities will equip you with the skills to do such tasks.

Suppose you borrow \$12000 from a bank. The bank tells you that you must make regular repayments every month for 3 years until you have paid back the loan.

Unfortunately, for you, it is not as simple has repaying  $\$ \frac{1200}{36}$  because the bank has allowed you to use their money and want some money in return for the privilege (this is called interest).

The way the process works can be described as follows:

- You borrow \$12000 (**present value of the loan**) and the bank charges you (adds to the loan) a percentage of this for its use during the first month. At the end of the month you make a repayment (contribution). So at the start of the next month you owe:

**the starting present value + the interest - your repayment**

- Then the bank charges you a percentage of this month's present value for using it during this month and you make another repayment at the end of this month. So at the start of the next month you owe:

**the previous months present value + the interest - your repayment**

- This (repetitive) process continues each month until you owe nothing.

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## Activity 1: Experimenting with repayments.

Suppose the interest rate is 7% per annum compounded monthly and that you make monthly repayments of \$40.

To see the amount you would owe at the end of the first month (the present value of the loan at the start of the second month) we can do the following using the 9860G AU:

- Enter  $\overline{\text{RPN-MAT}}$
- Enter the present value at the start of the first month by pressing 12000  $\overline{\text{EXE}}$
- Calculate the present value at the start of the second month with the command

```
12000
Ans×(1+7÷1200)-40
12030
RPN-MAT
```

$\overline{\text{Ans}} \times (1 + 7 \div 1200) - 40$  ( $\overline{\text{Ans}}$  is obtained by pressing  $\overline{\text{SHIFT}}$  then  $\overline{(-)}$ )

- Pressing  $\overline{\text{EXE}}$  repeatedly will calculate the present value at the start of successive months.

1. What does this series of monthly present values mean about the consequences of making \$40 monthly payments on this loan?
2. Investigate the consequences of making different monthly repayments. Summarise your findings.
3. Is there a minimum monthly payment? Explain why this amount is minimum.



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Can you think of any other investment situations where this process is applied?

One such case is a retirement fund where a retiree has, for example, \$245 000 and invests it in an annuity and **takes out** regular installments. This is an equivalent situation to a loan, but it is the bank doing the borrowing and then making the repayments.

With cases such as these we tend to be most interested in the *present value* of the annuity – i.e. '**How much do I owe now?**' or '**How much is left in my fund now?**'

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## Activity 2: Generalising the calculation of the *present value* of an annuity.

A formula exists for computing the *present value* of an annuity. It saves us from repeatedly pressing the equal sign on the calculator and also allows for other sorts of calculations to be done. The derivation of the formula relies on knowledge of geometric series. You might like to research this.

Firstly, let's define each quantity as follows:

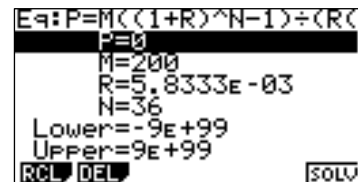
- Let the *present value* of the annuity, that consists of  $n$  compounding periods, be  $N$
- Let the regular contributions be  $M$
- Let the *percentage interest rate per compounding period* be  $r$  (expressed as a decimal)
- Let the *number of compounding periods* be  $n$ .

The formula is:

$$N = M \left( \frac{(1+r)^n - 1}{r(1+r)^n} \right)$$

The 9860G AU can be used to compute the value of  $N$ ,  $M$ ,  $r$  or  $n$  if all but one of the variables is known. In  $\frac{\text{EQUA}}{3 \times \text{M} + \text{=0}}$  mode, after choosing SOLVr  $\text{F3}$  the formula can be entered, using brackets carefully. Note that  $N$  is replaced with  $P$ .

With the formula entered, a row corresponding to each variable is created. You can enter values for the variables that are known and, **positioning the selection bar** on the variable you wish to find, press SOLV  $\text{F6}$ .



Once a result is given press REPT  $\text{F1}$  to perform another calculation with this formula.

1. Use this method to determine the size of a loan (the present value of an annuity) for which you will make 36 monthly payments of \$200 when an interest rate of 7 % p.a. ( $\frac{7}{12}\%$  per month) is applied.
2. Calculate how many monthly payments of \$200 would be needed to repay our \$12000 loan.
3. Calculate the monthly payment required if our loan is to be repaid in full in 36 months (3 years).



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## Activity 3: Other calculations using the *present value* formula.

*An annuity as an investment.*

Lets now think about the retirement fund mentioned earlier, where \$245 000 is 'loaned' to the bank and regular installments are 'repaid' to the retiree.

Suppose we want to know how long we could remove \$1500 per month from an annuity with present value \$245 000 if the interest paid was 4% compounded monthly.

This requires us to solve the equation  $245000 = 1500 \left( \frac{\left(1 + \frac{4}{1200}\right)^n - 1}{\frac{4}{1200} \left(1 + \frac{4}{1200}\right)^n} \right)$  for  $n$ .

1. Use the 9860 to solve the above equation for  $n$  and provide an answer in years.
2. If our retiree wants regular monthly payments for 25 years, how much will each payment be?
3. If he wants to receive \$2000 per month for 25 years, how much extra will he need to deposit in this annuity?

*Another loan – this time it's a new car.*

Ralph borrows \$15000 to buy a car. The conditions of the loan are that he must make repayments of \$350 each month. The interest rate associated with the loan is 12% per annum compounded monthly.

4. How long will Ralph take to pay back the loan?
5. How much interest does Ralph pay?
6. After how many periods will the amount owed first fall below \$10 000?



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