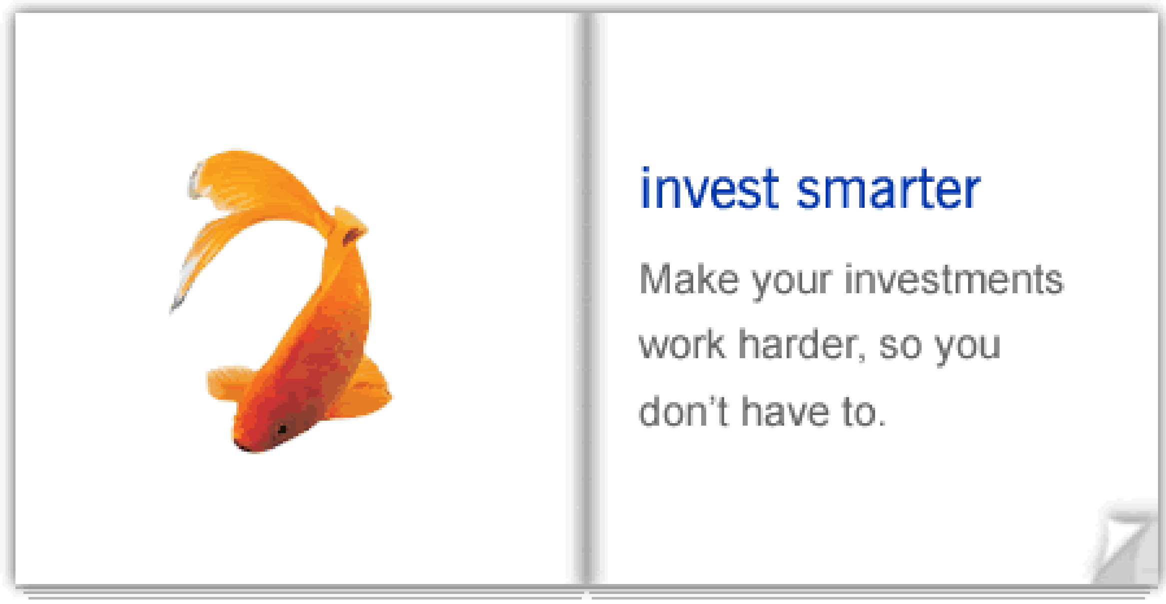
Want to turn $1000 into $47000?

- Activities dealing with compound interest -



invest smarter

Thanks to the miracle of compound interest and the wide range of opportunities available today, anyone can be a successful investor if they make the right choices.

"Compound interest" refers to the potential for your money to grow faster as you earn interest on your interest. It explains why $1000 invested at a rate of 8% pa after tax can grow to $47,000 over 50 years. No wonder Albert Einstein described compound interest as one of the greatest human discoveries.

These words of wisdom come from the web site of the financial advice company ipac

Do you want to be a smart investor? Sounds like you need to know about compound interest!

Activity 1: Growing $1000 into $47 000 - the compounding way.

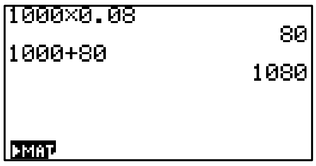
Hang on a minute, 8% of a $1000 investment is $80. If you earned this every year for 50 years you would have $80×50 = $4000 interest, plus your $1000 investment,
a measly total of $5000. So how do you get the other $42 000?

The key idea is that you earn *interest on your interest*. It works like this,

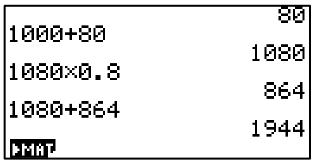
- An investor places an amount of money (the starting **principal**) into an account
- The bank agrees to pay the investor a set percentage of the investment (the interest **rate**) at regular intervals (the interest **period**).
- The interest period might be daily, monthly, yearly or whatever is agreed to.
- ***When paid, the interest is added to the starting principal to form a 'new starting principal' for the next interest period.***

Let's watch our $1000 grow through the 'miracle' of compound interest!

To calculate the value of the investment after the first year we multiply the $1000 by 0.08 to find the interest, and then add this to the $1000 principal, giving us $1080.

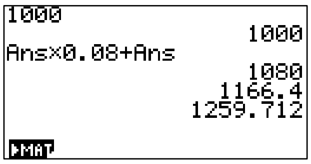


For the second year this process is repeated, but now we multiply $1080 by 0.08 to find the interest and then add it to the $1080.



Try this smarter process on a *CASIO fx 9860G AU*:

- Enter [RUN·MAT] and type in 1000
- Commit it to the calculator's answer memory by pressing [EXE].
- Multiply the answer by 0.08 and add the answer, i.e. [Ans] [×] 0.08+[Ans]. [Ans] is obtained by pressing [SHIFT] then [(–)].
- Press [EXE] once to see the $1080 value after the first year.
- Pressing [EXE] will show the value after 2 years, 3 years …



Can you see that in the first year $80 interest was earned but in the second year $86.40 was earned, and in the third year $1259.71-$1166.40 = $93.31 was earned? The magic of compound interest is at work already!

Activity 2: Computing it differently.

Instead of multiplying $1000 by 0.08 and then adding this result to $1000, we can calculate the value after the first year by multiplying $1000 by 1.08 (increasing $1000 by 8%)

1. Use a 9860G AU and the command [Ans] [×] 1.08 to find the value of the investment after 5 years (hint: [EXE] the $1000 principal before you start)
2. Show that more than $100 interest is earned in the fifth year.

Check point.

Activity 3: Generalising the process.

The method used in Activity 2 can be *generalised* (written as a formula), based on the idea that repeated multiplication is equivalent *finding a power* of the multiplier

Firstly, we define each quantity as follows:

- Let the final balance, after n compounding periods, be A
- Let the initial quantity be P
- Let the percentage interest rate per compounding period be r (expressed as a decimal)
- Let the number of compounding periods be n,

The compound interest formula can be thus written as:

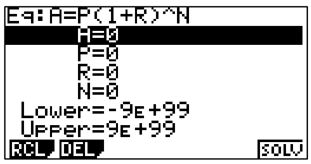
$$A = P(1 + r)^n$$

The *9860G AU* can be used to work efficiently with formulae such as this. It can calculate the value of *A, P, r* or *n* if <u>all but one</u> of them is known.

To do so go to [EQUA] and enter the `SOLVer` by pressing [F3].

The formula can be entered as $A = P(1 + R)^N$ by doing the following:

- Press the red [ALPHA] key first to be able to enter the red letters on the key pad
- The = sign can entered by pressing [SHIFT] then [•]



You can see that each variable is listed below the formula. Each can be changed to the values required. Press EXE between each change so that they are accepted by the calculator. To solve for the unknown value, put the **input bar** on the required row and press SOLV F6.
Further calculations can be made by pressing REPT F1 to return to the input screen.

1. Find the value of the $1000 investment after 5 years. Use this to check your answer to Activity 2 Question 1.
2. Find the value of the $1000 investment after 50 years.
3. Find the amount of interest earned in the 50th year.

Check point.

Activity 4: Using the generalization more broadly.

Of course we can use the formula to do more than just compute the principal after *n* interest periods.

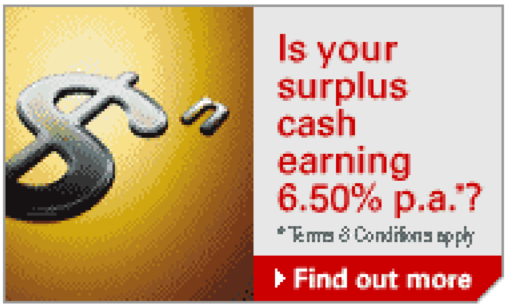
Suppose we wanted to know how many years it would take for our initial investment of
$1000 dollars would grow to $100 000 at an annual interest rate of 8%.
This would require us to solve the equation $100000 = 1000(1+0.08)^n$ for n.

In the input screen of the SOLVer, change the value in *A* and, by moving the input bar, solve for *N*

1. Complete the above calculation to find out how long it would take for our $1000 dollars would grow to $100 000 at an annual interest rate of 8%.
2. Use a similar method to find out the interest rate required if our $1000 investment is to grow to $100 000 in only 50 years (you'll be surprised!)

Check point.

Activity 5: Learning *bank speak*



Advertisements like the one seen opposite are designed to catch the eye of would-be investors.

Understanding what they offer requires a grasp of *bank speak*. Firstly, when it says the interest rate is 6.50% *p.a.* it means 6.50% *annually*. (p.a. stands for *per annum* which is Latin for per (for each) year)

The second piece of bank speak is *Terms and Conditions apply*, which means read the fine print before you get too excited. In this case the fine print says that this rate applies only to investments of $50 000 or greater, and that the interest is *compounded monthly*. This is bank speak for compound interest to be calculated monthly.

How is an annual rate compounded monthly?

The *per annum* rate will be divided by the number of compounding periods in a year to get the rate per compounding period, in this case $\frac{6.50}{12}\%$ ($\approx 0.54\%$) per month.

Activity 6: Working with compound interest

Suppose that I invested $125 000 for 3 years under these conditions. To find out how much I had at the end I could compute $125000(1+\frac{0.065}{12})^{36}$, realizing that there are 36 compounding periods (months) in this 3-year investment.

This can be done in the SOLVer by entering the rate as $0.065 \div 12$ (or $6.5 \div 100 \div 12$).

1. Find out the value of a $125 000 investment after 3 years, if it attracts 6.50% p.a. interest rate which is compounded monthly.
2. Compare this to the same investment when the interest rate is compounded annually.

Check point.

Further Exercises

1) Find the value of an investment of $1800, invested at 3.8% per annum compounded monthly, after five years.

2) Compare the values of an investment of $5600 after 12 months if it earns 7.2% compounded:

a. Annually
b. Quarterly
c. Monthly
d. Daily

3) It is possible to compound interest more often than daily, for example hourly. If you were an investor would you seek out such a compounding period? Explain, with reference to your answers to question 4, or otherwise.

4) Kellie invested $5600 at 7.2% pa compounded quarterly. What was the value of her investment after five years?

5) Simon invested $5600 at 7.2% pa compounded semi-annually (i.e. twice every year). Predict whether his investment would accumulate, in five years, to more or to less than that of Kellie in the previous question. Use your calculator to check your prediction.

6) A house increased in value from $160 000 to $275 000 in $2\frac{1}{2}$ years. What is the equivalent rate of interest if a bank account, compounded monthly, was to show the same degree of growth.

7) How long will it take for an investment of $100 to double in value if it is invested at 6% p.a. compounded daily?

Check point.