CASIO

CASIO TEACHING MATERIALS

REAL-LIFE PROBLEMS

with fx-991CW

DIGEST EDITION

CASIO Teaching Materials

Real-Life Problems with fx-991CW

Introduction

These teaching materials were created with the hope of conveying to many teachers and students the appeal of scientific calculators.

(1) Change awareness (emphasizing the thinking process) and boost efficiency in learning mathematics

- By reducing the time spent on manual calculations, we can have learning with a focus on the thinking process that is more efficient.
- This reduces the aversion to mathematics caused by complicated calculations, and allows students to experience the joy of thinking, which is the essence of mathematics.

(2) Diversification of learning materials and problem-solving methods

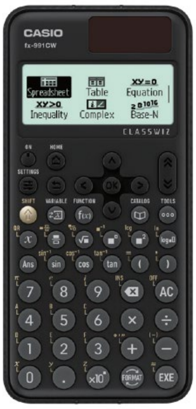
- Making it possible to do difficult calculations manually allows for diversity in learning materials and problem-solving methods.

(3) Promoting understanding of mathematical concepts

- By using the various functions of the scientific calculator in creative ways, students are able to deepen their understanding of mathematical concepts through calculations and discussions from different perspectives than before.
- This allows for exploratory learning through easy trial and error of questions.
- Listing and graphing of numerical values by means of tables allows students to discover laws and to understand visually.

Features of these teaching materials

- Makes classes more interesting by using scientific calculators
- Includes a variety of real-life problems in each unit
- Allows a deeper understanding of mathematics
- Enables students to utilize the scientific calculator's functions more skillfully
- Three degrees of difficulty settings (levels 1 to 3)



Better Mathematics Learning with Scientific Calculator

Installation angle of solar panels

Level 1

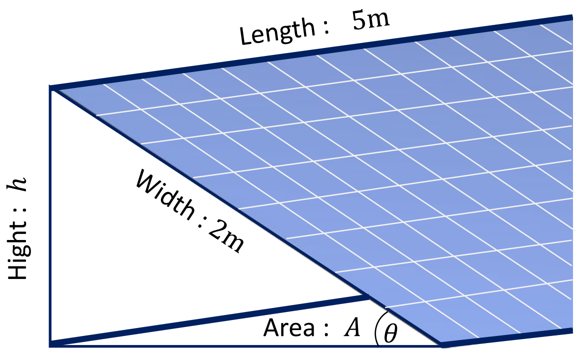


Fig. 1

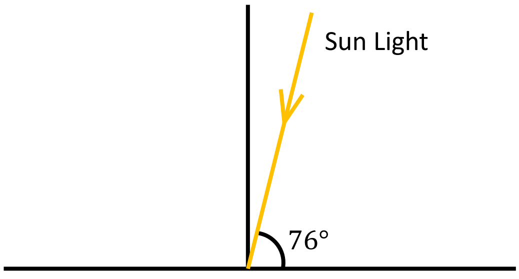


Fig. 2

(1) When installing solar panels as shown in Fig. 1, the power generation efficiency is highest when the sunlight is perpendicular to the panel. When the angle of incidence of the sun is 76° as shown in Fig. 2, find the angle θ at which the panel is perpendicular to the sunlight.

$\theta = 90 - 76 = 14°$

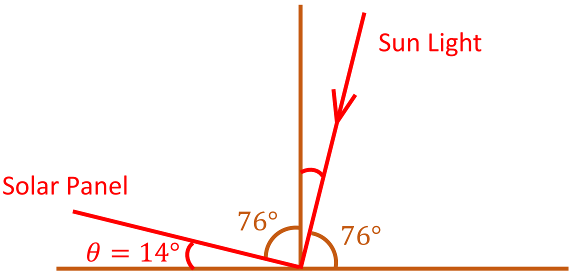
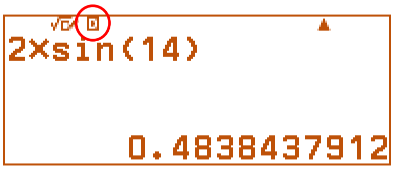


Fig. 3 (For explanation)

(2) In (1), what should the height h [m] of the support column be?

(The width of the solar panels is 2m as shown in Fig. 1.)

$h = 2 \times \sin 14° \approx 0.48\mathrm{m}$



```
2×sin(14)
0.4838437912
```

(3) In (1), what is the required footprint of the solar panels?

(The length of the solar panels is 5m as shown in Fig. 1.)

$A = 5 \times 2\cos 14° \approx 9.7\mathrm{m}^2$

```
5×2×cos(14)
9.702957263
```

Range in which a tower is visible

Level 2

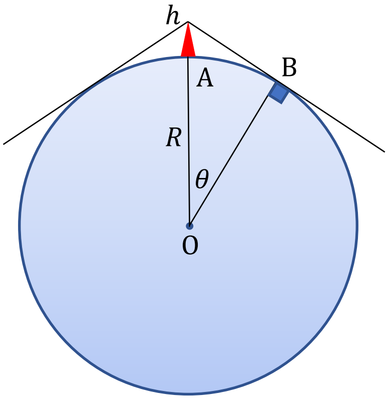


Fig. 1

(1) Express the distance between points A and B(length of arc AB) on the Earth in Fig. 1 using the Earth's radius R [km], θ [Rad], and π.

$$Arc\ \mathrm{AB} = 2\pi R \times \frac{\theta}{2\pi} = R\theta\ [\mathrm{km}]$$

(2) If the height of the tower at point A in Fig. 1 is h [m], express θ using R and h.

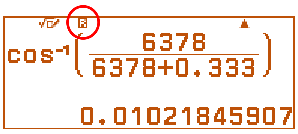
$$\cos\theta = \frac{R\ [\mathrm{km}]}{R\ [\mathrm{km}] + \frac{h}{1000}\ [\mathrm{km}]}$$

$$\theta = \cos^{-1}\frac{R}{R + \frac{h}{1000}}\ [\mathrm{Rad}]$$

(3) When the height of the tower at point A is $h = 333$ [m], calculate the distance at which the tower is visible (arc AB). The radius of the Earth is $R = 6378$ [km].

$$\theta = \cos^{-1}\frac{6378}{6378 + 0.333} \approx 0.0102\ [\mathrm{Rad}]$$

$$\mathrm{AB} = R\theta = 6378 \times \theta \approx 65.17\ [\mathrm{km}]$$

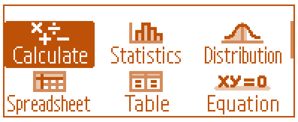


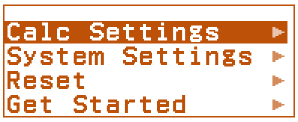
```
cos⁻¹(6378/(6378+0.333))
                0.01021845907
```

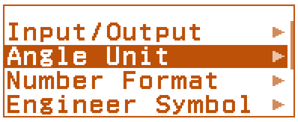
```
6378×Ans
                  65.17333192
```

Note the angular unit.

Therefore, the tower can be seen up to a range of approximately 65km from point A.







● Degree
○ Radian
○ Gradian

Tower and mountain height surveying

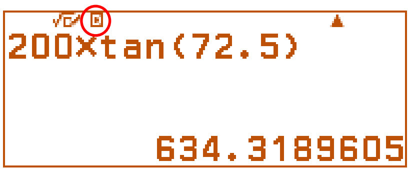
Level 2

(1) Tower height

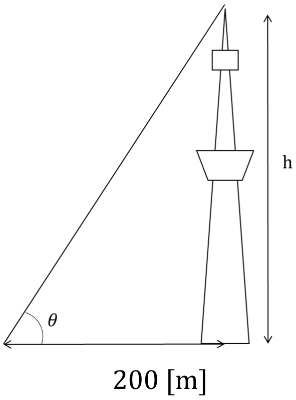
At a point 200 [m] from the tower, the elevation angle θ was 72.5°.

Find the height h [m] of the tower.

$h = 200 \times \tan 72.5 \approx 634$ [m]

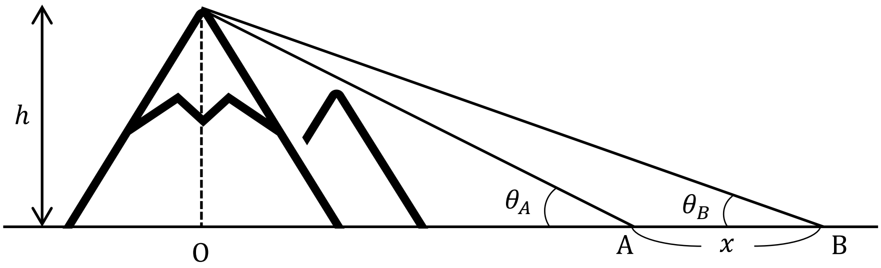


```
200×tan(72.5)
                634.3189605
```



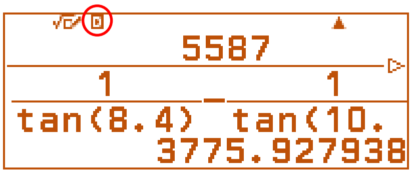
(2) Mountain Height

Unlike with the tower, it is not possible to measure the horizontal distance from the mountain, so the elevation angles were measured at two points A and B. The elevation angle at point A was $\theta_A = 10.7°$ and at point B was $\theta_B = 8.4°$. The distance between A and B was $x = 5{,}587\mathrm{m}$. Find the mountain height h [m].



$$x = \mathrm{OB} - \mathrm{OA} = \frac{h}{\tan\theta_B} - \frac{h}{\tan\theta_A} = h\left(\frac{1}{\tan\theta_B} - \frac{1}{\tan\theta_A}\right)$$

$$h = \frac{x}{\frac{1}{\tan\theta_B} - \frac{1}{\tan\theta_A}} = \frac{5{,}587}{\frac{1}{\tan 8.4°} - \frac{1}{\tan 10.7°}} \approx 3{,}776 \text{ [m]}$$



```
          5587
  ─────────────────────
      1          1
  ───────── - ─────────
  tan(8.4)    tan(10.
            3775.927938
```

In (2), we may introduce another solution that uses the sine theorem.

Relating Speed and Stopping Distance

Level 1

Assume that the relationship between the speed x [km/h] at which a certain car is traveling at the time when the brakes are applied and the stopping distance y [m] can be expressed by a quadratic function ($y = ax^2 + bx + c$)*.
The table below shows the stopping distances for the car when traveling at various speeds.

	①	②	③
Speed	20 [km/h]	30 [km/h]	40 [km/h]
Stopping Distance	9 [m]	14 [m]	21 [m]

* Assume that the relationship expressed by the quadratic function holds for speeds of 0 to 80 km/h.

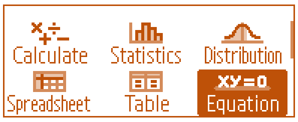
(1) Find the equation for the quadratic function that relates speed (x) and stopping distance (y).

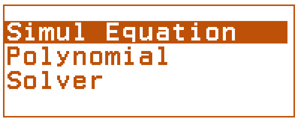
Letting the equation we want be of the form $y = ax^2 + bx + c$,

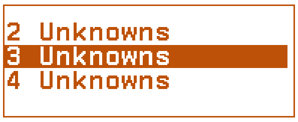
From ①, $9 = 20^2a + 20b + c \quad \rightarrow \quad 400a + 20b + c = 9$

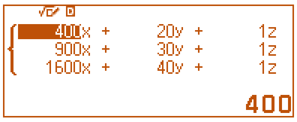
From ②, $14 = 30^2a + 30b + c \quad \rightarrow \quad 900a + 30b + c = 14$

From ③, $21 = 40^2a + 40b + c \quad \rightarrow \quad 1600a + 40b + c = 21$









400x + 20y + 1z
900x + 30y + 1z
1600x + 40y + 1z
400

$x = \frac{1}{100}$

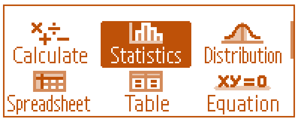
$y = 0$

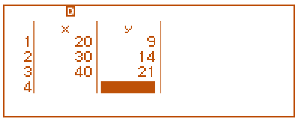
$z = 5$

Solving the system of simultaneous equations using a scientific calculator gives: $a = \frac{1}{100}$, $b = 0$, and $c = 5$

$\therefore y = \frac{1}{100}x^2 + 5$

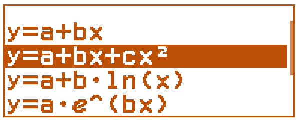
Alternate Solution: We can also find the function using the regression calculator under [Statistics].

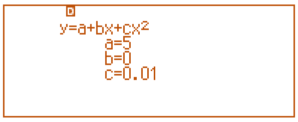




	x	y
1	20	9
2	30	14
3	40	21
4		

OK



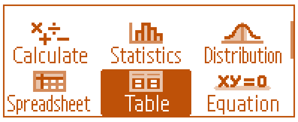


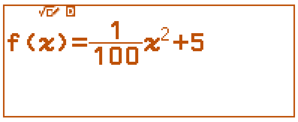
y=a+bx+cx²
a=5
b=0
c=0.01

*Note that the a and c displayed by the calculator are the reverse of those used in the calculations above.

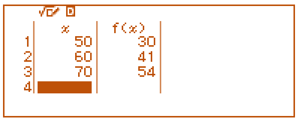
(2) Find the stopping distance when the speed is $50\ \mathrm{km/h}$, $60\ \mathrm{km/h}$, and $70\ \mathrm{km/h}$.

Substitute $x = 50,\ 60,\ \text{and } 70$ into the equation we found in (1) $y = \frac{1}{100}x^2 + 5$.





$f(x)=\frac{1}{100}x^2+5$



	x	f(x)
1	50	30
2	60	41
3	70	54
4		

Input the values directly in the x column.

Thus,

The stopping distance at $50\ \mathrm{km/h}$ is: $30\ \mathrm{m}$

The stopping distance at $60\ \mathrm{km/h}$ is: $41\ \mathrm{m}$

The stopping distance at $70\ \mathrm{km/h}$ is: $54\ \mathrm{m}$

(3) While driving at a certain speed, the driver of the car noticed an obstacle 65 m ahead and applied the brakes. What is the lowest driving speed at which the car will be unable to stop in time and hit the obstacle?

Substitute $y = 65$ into the equation we found in (1) $y = \frac{1}{100}x^2 + 5$.

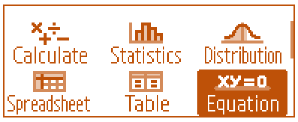
Since $65 = \frac{1}{100}x^2 + 5$,

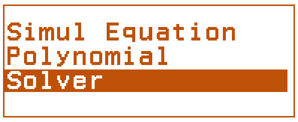
$\frac{1}{100}x^2 - 60 = 0$

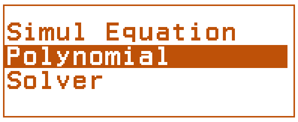
$x = \pm 20\sqrt{15}$

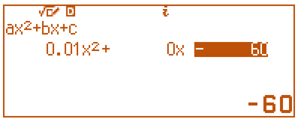
$x \approx \pm 77.46$

$\therefore\ 77.46\ \mathrm{km/h}$





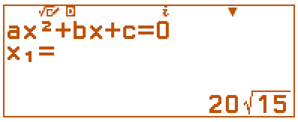




ax^2+bx+c

$0.01x^2+\quad 0x\ -60$

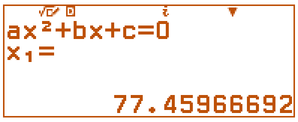
-60



$ax^2+bx+c=0$

$x_1=$

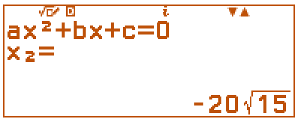
$20\sqrt{15}$



$ax^2+bx+c=0$

$x_1=$

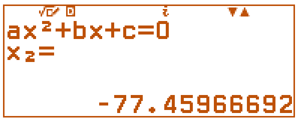
77.45966692



$ax^2+bx+c=0$

$x_2=$

$-20\sqrt{15}$

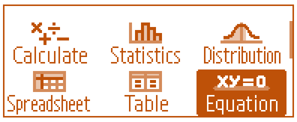


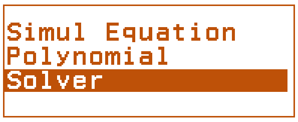
$ax^2+bx+c=0$

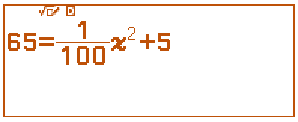
$x_2=$

-77.45966692

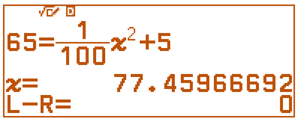
Alternate Solution







$65=\frac{1}{100}x^2+5$



$65=\frac{1}{100}x^2+5$

$x=\quad 77.45966692$

$L-R=\quad 0$

Calculating an Electric Bill

Level 3

Solve the following problems relating to the monthly electric bills for a certain apartment.

(1) To calculate the electric bill, it is necessary to calculate the energy consumption [kWh] for that month. Energy consumption [kWh] can be found by multiplying the power consumption [kW] value listed on an appliance by the number of hours the appliance is used $[\mathrm{h}]$.

Express the energy consumption $w(x)[\mathrm{kWh}]$ of a certain lamp with a listed power consumption of $60\ [\mathrm{W}]$ when used for x hours, in terms of x.

Since $60\ \mathrm{W} = 0.06\ \mathrm{kW}$,

Energy consumption: $w(x) =$ power consumption $[\mathrm{kW}] \times$ hours of use $[\mathrm{h}] = 0.06x\ [\mathrm{kWh}]$

(2) The monthly electric bill is calculated as the total of a fixed base rate plus a variable usage charge based on actual energy use. Given that the base rate is $\$14.00$ and the usage charge is calculated at the rate of $20¢$ ($\$0.20$) per kWh of energy usage, express the electric bill $f(x)$ if the lamp discussed in (1) is the only electric appliance used and is run for x hours per month.

$$f(x) = \underbrace{0.2 \times w(x)}_{\text{Usage Charge}} + \underbrace{14}_{\text{Base Rate}} = 0.2 \times 0.06x + 14 = 0.012x + 14\ \ [\text{dollars}]$$

(3) Find the electric bill (in dollars) if a lamp that draws $60\ \mathrm{W}$ of power is run for 300 hours.

$$f(x) = 0.012x + 14 = 0.012 \times 300 + 14 = 17.6\ \ [\text{dollars}]$$

0.012×300+14

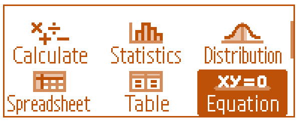
17.6

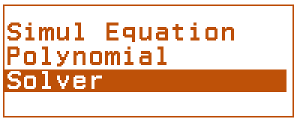
(4) What is the minimum number of hours that a 60W lamp has been run if the monthly electric bill is at least \$20.00?

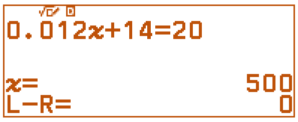
$0.012x + 14 \geq 20$

$\therefore x \geq 500$ [h]

（1 month $= 31$ days $\times 24$ h $= 744$ h > 500 h. Thus, this answer is reasonable.）







```
0.012x+14=20

x=                500
L-R=                0
```

(5) Given that if more than 30 kWh of energy is used, the usage charge for energy beyond 30 kWh is calculated at the rate of 25¢ (\$0.25) per kWh, express the electric bill $g(x)$ in terms of x.

$$g(x) = \underbrace{0.2 \times 30}_{\substack{\text{Usage Charge} \\ \text{(up to 30 kWh)}}} + \underbrace{0.25 \times (w(x) - 30)}_{\substack{\text{Usage Charge} \\ \text{(over 30 kWh)}}} + \underbrace{14}_{\text{Base Rate}} = 0.25 \times w(x) + 12.5 = 0.25 \times 0.06x + 12.5 = 0.015x + 12.5$$

(6) In a certain month, the resident of the apartment used more than 30 kWh of energy, but mistakenly calculated the payment using the equation in (2). Given that the electric company later determined that the bill was underpaid by 60¢ (\$0.60), find the number of hours that the lamp was in use during that month.

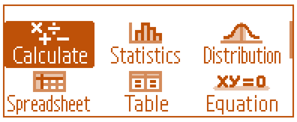
Find the value for x such that $g(x) - f(x) = 0.6$.

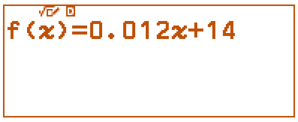
$(0.015x + 12.5) - (0.012x + 14) = 0.6$

$0.003x = 2.1$

$x = 700$ [h]

Check 1





```
f(x)=0.012x+14
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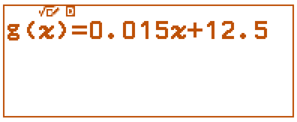
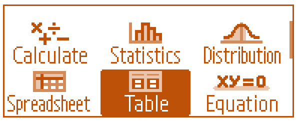


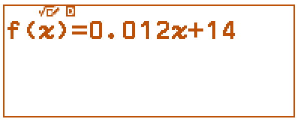
```
g(x)=0.015x+12.5
```

```
g(700)-f(700)

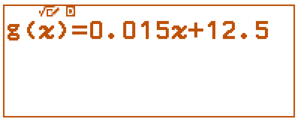
                0.6
```

Check 2

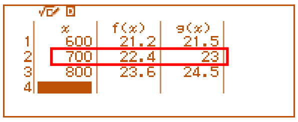




```
f(x)=0.012x+14
```



```
g(x)=0.015x+12.5
```



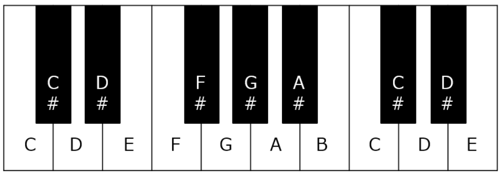
```
     x    f(x)   g(x)
1   600   21.2   21.5
2   700   22.4   23
3   800   23.6   24.5
4
```

We can see that $g(x) - f(x) = 0.6$ when x is 700 h.

Relating Musical Scales to Geometric Sequences

Level 1

The range between one tone and another tone with double the frequency is called an octave, which is made up of 12 half steps. The ratio of the frequencies of any adjacent pair of tones is always constant, and the frequencies are arranged in a geometric sequence (for equally tempered tuning systems).



Scale	C	C #	D	D #	E	F	F #	G	G #
Frequency [Hz]	261.6	277.2	293.7	311.1	329.6	349.2	370.0	392.0	415.3

A	A #	B	C	C #	D	D #	E	F	F #	G	G #	A	A #
440 Standard Pitch	466.2	493.9	523.3	554.4	587.3	622.3	659.3	698.5	740.0	784.0	830.6	880	932.3

1 octave

(1) Find the common ratio for this scale.

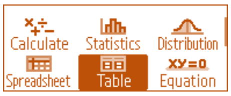
If the first term ($n = 1$) is 440, then $n = 13$ is 880. Based on the formula for the n^{th} term in a geometric sequence $a_n = a_1 r^{n-1}$

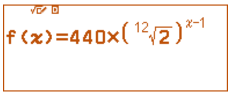
$880 = 440 \times r^{12}$

$r = \pm \sqrt[12]{2}$

It is clear from the table that $r > 0$ so, $r = \sqrt[12]{2}$

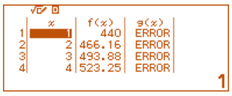
(2) The standard pitch used for tuning is an A at the frequency of 440 Hz. Find the frequencies of the other tones on the table, rounding your answer to the nearest tenth.





$f(x)=440\times(\sqrt[12]{2})^{x-1}$

```
Table Range
 Start:1
 End  :14
 Step :1
```



```
   x    f(x)    g(x)
1     1    440  ERROR
2     2 466.16  ERROR
3     3 493.88  ERROR
4     4 523.25  ERROR
                      1
```

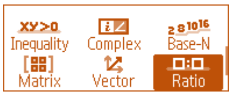
The steps in the Table can also be set to negative numbers.

```
Table Range
 Start:1
 End  :-10
 Step :-1
```

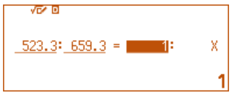
```
   x    f(x)    g(x)
1     1    440  ERROR
2     0  415.3  ERROR
3    -1 391.99  ERROR
4    -2 369.99  ERROR
                      1
```

(3) The ratios between the frequencies of the notes in a given chord are close to integers. Find the integer ratios (values very close to single-digit integers) of the frequencies in the notes C, E, and G (marked in yellow on the table).

C : E : G$= 523.3 : 659.3 : 784.0 \approx 1 : 1.259 : 1.498 \approx 100 : 125 : 150 = 4 : 5 : 6$

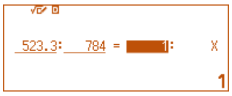


```
A:B=X:D
A:B=C:X
```



```
523.3: 659.3 = 1:   X
                    1
```

```
X=
        1.259889165
```



```
523.3:   784 = 1:   X
                    1
```

```
X=
        1.498184598
```

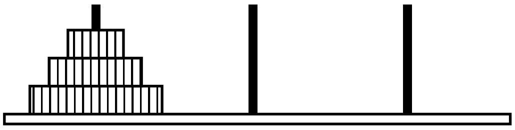
The Tower of Hanoi

Level 3

The diagram shows a Tower of Hanoi puzzle. There are three vertical pegs, and a set of n discs, each of different sizes, is placed on the left peg in descending order of size such that the smallest disc is on top.
Let a_n be the smallest number of moves it takes to move all of the discs to the right peg while following the **Rules** given below.

Rules

- Only one disc may be moved at a time.
- Discs can only be moved from the top of one stack onto another peg.
- No larger disc can be placed on top of a smaller disc.



(1) Find a_n for $n = 1$ through $n = 4$.

Number of Discs	$n = 1$	$n = 2$	$n = 3$	$n = 4$
Number of Moves a_n	1	3	7	15

*It is easy to experiment with this concept using cups or other items of differing sizes.

(2) Using the pattern you found in (1), guess the relationship between the number of moves required when there are n discs, a_n, and the number when there are $n-1$ discs, a_{n-1}. (Find the recurrence formula.)

From (1), we can see that the value of a_n is double a_{n-1} plus 1.

$\therefore a_n = 2a_{n-1} + 1$

(Proof)

The following operations are required to move a Tower of Hanoi with n discs:

1. Move all of the discs other than the largest disc ($n-1$ discs) to the center peg (requiring a_{n-1} moves).
2. Move the largest disc to the right peg (requiring 1 move)
3. Move the $n-1$ discs from the center peg to the right peg, on top of the largest disc (requiring a_{n-1} moves).

$\therefore a_n = a_{n-1} + 1 + a_{n-1} = 2a_{n-1} + 1$

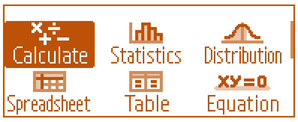
(3) Find a_n when $n = 5$ and $n = 10$.

$a_5 = 2a_4 + 1 = 2 \times 15 + 1 = 31$ (moves)

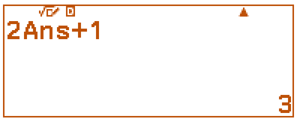
This problem is simple to solve using your scientific calculator's Answer function.

$a_{10} = 1023$ (moves)

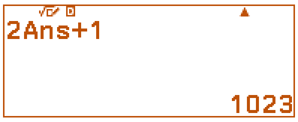
① EXE ② Ans ⊕ ① EXE EXE EXE EXE EXE EXE EXE EXE EXE



```
1
                1
```



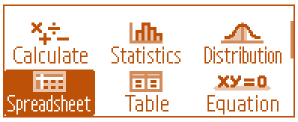
```
2Ans+1
                3
```

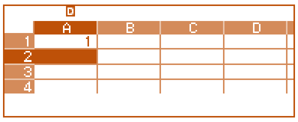


```
2Ans+1
             1023
```

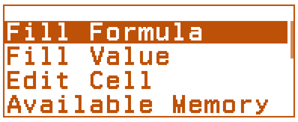
(4) Find a_n when $n = 30$.

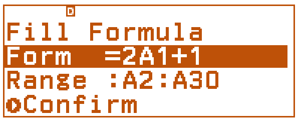
This problem is simple to solve using a spreadsheet on your scientific calculator.



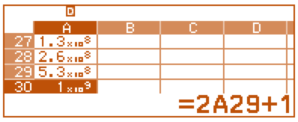


	A	B	C	D
1	1			
2				
3				
4				



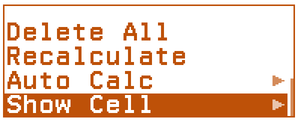


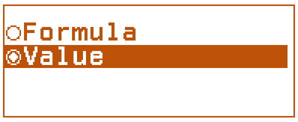
Fill Formula
Form =2A1+1
Range :A2:A30
Confirm



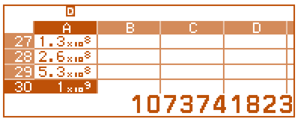
	A	B	C	D
27	1.3×10^{8}			
28	2.6×10^{8}			
29	5.3×10^{8}			
30	1×10^{9}			

=2A29+1





○Formula
●Value



	A	B	C	D
27	1.3×10^{8}			
28	2.6×10^{8}			
29	5.3×10^{8}			
30	1×10^{9}			

1073741823

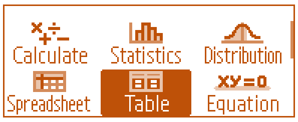
When $n = 30$, $a_{30} = 1073741823$ (moves)

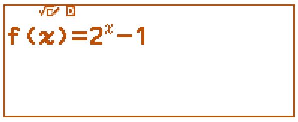
(5) Find the formula for the n^{th} term for moves required, a_n, when there are n discs. (Express a_n in terms of n.)

Focus on the fact that each term is equal to 2 to the n^{th} power minus 1.

$a_n = 2^n - 1$

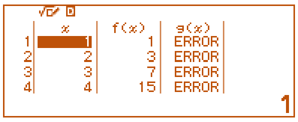
Check using your scientific calculator's Table function.





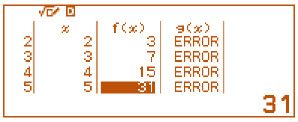
$f(x)=2^x-1$

Table Range
Start:1
End :30
Step :1



	x	f(x)	g(x)
1	1	1	ERROR
2	2	3	ERROR
3	3	7	ERROR
4	4	15	ERROR

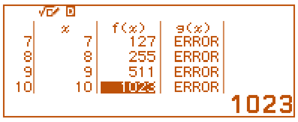
1



	x	f(x)	g(x)
2	2	3	ERROR
3	3	7	ERROR
4	4	15	ERROR
5	5	31	ERROR

31

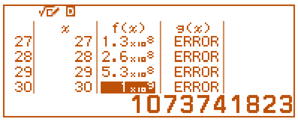
Check (3)



	x	f(x)	g(x)
7	7	127	ERROR
8	8	255	ERROR
9	9	511	ERROR
10	10	1023	ERROR

1023

Check (3)



	x	f(x)	g(x)
27	27	1.3×10^{8}	ERROR
28	28	2.6×10^{8}	ERROR
29	29	5.3×10^{8}	ERROR
30	30	1×10^{9}	ERROR

1073741823

Check (4)

(6) French mathematician Édouard Lucas purportedly recounted a legend stating that the world would end when a certain Tower of Hanoi with 64 discs had been completed. Assuming that the puzzle is solved according to the rules, and that one move is made every second, approximately how many years would it take to complete a Tower of Hanoi with 64 discs ($n = 64$)?

$2^{64}-1$

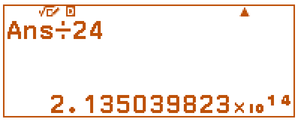
1.844674407×10^{19}

Seconds

Ans÷3600

5.124095576×10^{15}

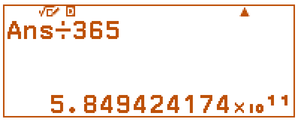
Hours



Ans÷24

2.135039823×10^{14}

Days

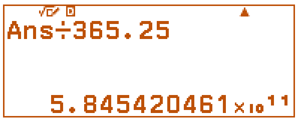


Ans÷365

5.849424174×10^{11}

Years

(Ignoring leap years)



Ans÷365.25

5.845420461×10^{11}

Years

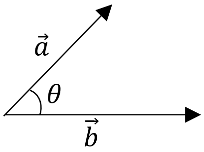
(Accounting for leap years)

∴ It would take about 584.5 billion years.

Dot Products & Angles Between Vectors

Level 1

(1) Find the angle between each pair of vectors $\vec{a} \cdot \vec{b}$ below. (Use your scientific calculator to calculate the angle.)



$\vec{a}$	$\vec{b}$	$\vec{a} \cdot \vec{b}$	Angle θ $(0 \leq \theta \leq 180)$	scientific calculator
$(1,1,1)$	$(2,2,2)$	6	0°	 VctA= 1 1 1 VctB= 2 2 2 Angle(VctA, VctB) 0
$(6,4,3)$	$(5,3,-2)$	36	41.6°	 VctC= 6 4 3 VctD= 5 3 -2 Angle(VctC, VctD) 41.60573025
$(-1,2,3)$	$(1,-2,-3)$	-14	180°	 VctA= -1 2 3 VctB= 1 -2 -3 Angle(VctA, VctB) 180
$(2,-1,3)$	$(-1,2,-3)$	-13	158.2°	 VctC= 2 -1 3 VctD= -1 2 -3 Angle(VctC, VctD) 158.2132107
$(2,0,-1)$	$(1,3,2)$	0	90°	 VctA= 2 0 -1 VctB= 1 3 2 Angle(VctA, VctB) 90
$(-3,6,5)$	$(2,7,-8)$	-4	92. 5°	 VctC= -3 6 5 VctD= 2 7 -8 Angle(VctC, VctD) 92.53327409
$(1,5,-9)$	$(2,5,3)$	0	90°	 VctA= 1 5 -9 VctB= 2 5 3 Angle(VctA, VctB) 90
$(5,-1,-1)$	$(6,-2,-2)$	34	9.4°	 VctC= 5 -1 -1 VctD= 6 -2 -2 Angle(VctC, VctD) 9.446232772

(2) What is the angle between two vectors when their dot product is 0? Explain using the dot product formula.

(Let $|\vec{a}|, |\vec{b}| \neq 0$.)

When the dot product is 0, the angle is 90°.

From $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$, we can see that when $\vec{a} \cdot \vec{b} = 0$, $\cos\theta = 0$, and thus $\theta = 90°$.

(3) What type of angle is formed when the dot product has a positive value? Explain using the dot product formula.

The angle between the vectors is $0° \leq \theta < 90°$.

From $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$, we can see that when $\vec{a} \cdot \vec{b} > 0$, $\cos\theta > 0$, and thus $0° \leq \theta < 90°$.

(4) What type of angle is formed when the dot product has a negative value? Explain using the dot product formula.

The angle between the vectors is $90° < \theta \leq 180°$.

From $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$, we can see that when $\vec{a} \cdot \vec{b} < 0$, $\cos\theta < 0$, and thus $90° < \theta \leq 180°$.

(5) Using the dot product formula, explain the relationship between the dot product and the product of the magnitudes of the vectors when the angle between them is $0°$ or $80°$.

From $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$, we can see that when $\theta = 0°$, $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|$, and when $\theta = 180°$, $\vec{a} \cdot \vec{b} = -|\vec{a}||\vec{b}|$.

In (1), we saw an instance of $\theta = 0°$:

Since $\vec{a} \cdot \vec{b} = 6$, and $|\vec{a}||\vec{b}| = \sqrt{1^2 + 1^2 + 1^2} \times \sqrt{2^2 + 2^2 + 2^2} = \sqrt{3} \times \sqrt{12} = 6$,

we can see that $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|$.

We also saw an instance of $\theta = 180°$:

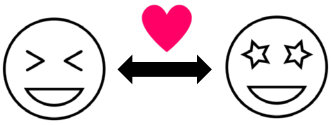
Since $\vec{a} \cdot \vec{b} = -14$, and $|\vec{a}||\vec{b}| = \sqrt{(-1)^2 + 2^2 + 3^2} \times \sqrt{1^2 + (-2)^2 + (-3)^2} = \sqrt{14} \times \sqrt{14} = 14$,

we can see that $\vec{a} \cdot \vec{b} = -|\vec{a}||\vec{b}|$.

Matching Based on Similarity

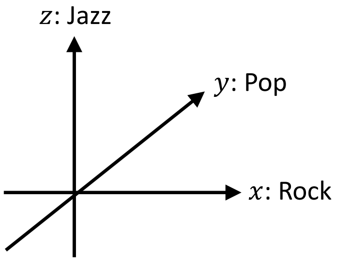
Level 2

To find classmates with similar music preferences, the top 100 songs in the playlists of students A, B, C, and D were categorized into three genres: rock, pop, and jazz. The results are as follows. Identify the pair with the most similar preferences and the pair with the most different preferences. Explain using vector analysis.

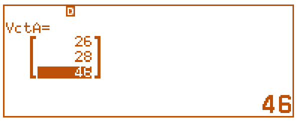


Students	Rock	Pop	Jazz
A	26	28	46
B	53	34	13
C	31	46	23
D	45	27	28

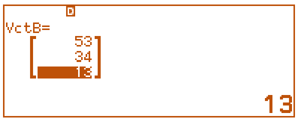
Hint: By representing each value in the vector space shown on the right, you can evaluate their preferences.



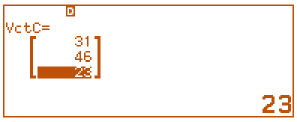
As shown on the right, plot rock on the x-axis, pop on the y-axis, and jazz on the z-axis. Smaller angles between the 3D vectors indicate more similar preferences.



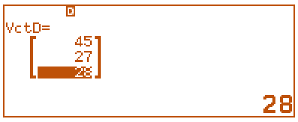
VctA= $\begin{bmatrix}26\\28\\46\end{bmatrix}$ 46



VctB= $\begin{bmatrix}53\\34\\13\end{bmatrix}$ 13



VctC= $\begin{bmatrix}31\\46\\23\end{bmatrix}$ 23



VctD= $\begin{bmatrix}45\\27\\28\end{bmatrix}$ 28

Pair	AB (The most different preferences)	AC	AD
Angle	 Angle(VctA,VctB) 40.40072442	 Angle(VctA,VctC) 28.62696552	 Angle(VctA,VctD) 25.36684975
Pair	BC	BD (The most similar preferences)	CD
Angle	 Angle(VctB,VctC) 24.76158941	 Angle(VctB,VctD) 16.49563676	 Angle(VctC,VctD) 23.28141328

Finding the Shortest Flight Path for a Drone

Level 2

Four points, A $(3,\ 6,\ 7)$, B $(-8,\ 5,\ -4)$, C $(-7,\ -5,\ 3)$, and D $(5,\ 3, -6)$, lie in the same three-dimensional space. A drone will be launched from A and make deliveries at B, C, and D, but the deliveries can be made in any order. From among the possible flight paths including B, C, and D and returning to A, let's consider the flight path with the shortest total travel distance.

(1) Assuming the flight path consists only of straight lines connecting the various points, give all possible flight paths.

E.g.) Write ABCDA if the path is A → B → C → D → A.

ABCDA ABDCA ACBDA ACDBA ADBCA ADCBA

(Check)

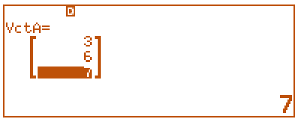
The total number of permutations for ordering B, C, and D is 3!

```
3!
                6
```

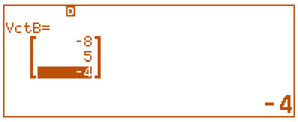
(2) Which of the flight paths from (1) has the shortest total travel distance?

Input the following values into your scientific calculator.

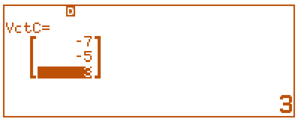
Vct A $= (3,\ 6,\ 7)$, Vct B $= (-8,\ 5,\ -4)$, Vct C $= (-7,\ -5,\ 3)$, Vct D $= (5, 3, -6)$



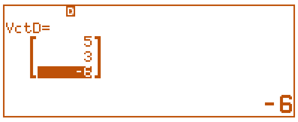
```
VctA=
 [3;6;7]
                7
```



```
VctB=
 [-8;5;-4]
               -4
```



```
VctC=
 [-7;-5;3]
                3
```

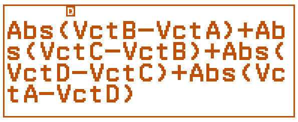


```
VctD=
 [5;3;-6]
               -6
```

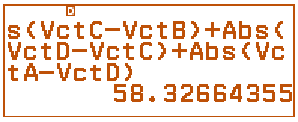
Since reversed paths have the same total travel distance,

ABCDA $=$ ADCBA, ABDCA $=$ ACDBA, and ACBDA $=$ ADBCA, so

$$\mathrm{ABCDA} = |\overrightarrow{\mathrm{AB}}| + |\overrightarrow{\mathrm{BC}}| + |\overrightarrow{\mathrm{CD}}| + |\overrightarrow{\mathrm{DA}}|$$
$$= |\vec{\mathrm{B}} - \vec{\mathrm{A}}| + |\vec{\mathrm{C}} - \vec{\mathrm{B}}| + |\vec{\mathrm{D}} - \vec{\mathrm{C}}| + |\vec{\mathrm{A}} - \vec{\mathrm{D}}|$$
$$\approx 58.3$$

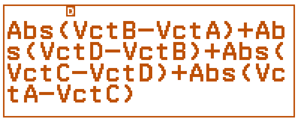


```
Abs(VctB-VctA)+Ab
s(VctC-VctB)+Abs(
VctD-VctC)+Abs(Vc
tA-VctD)
```

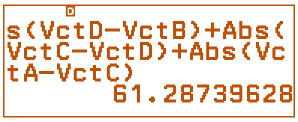


```
s(VctC-VctB)+Abs(
VctD-VctC)+Abs(Vc
tA-VctD)
      58.32664355
```

$$\mathrm{ABDCA} = |\overrightarrow{\mathrm{AB}}| + |\overrightarrow{\mathrm{BD}}| + |\overrightarrow{\mathrm{DC}}| + |\overrightarrow{\mathrm{CA}}|$$
$$= |\vec{\mathrm{B}} - \vec{\mathrm{A}}| + |\vec{\mathrm{D}} - \vec{\mathrm{B}}| + |\vec{\mathrm{C}} - \vec{\mathrm{D}}| + |\vec{\mathrm{A}} - \vec{\mathrm{C}}|$$
$$\approx 61.3$$

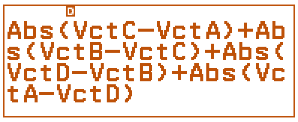


```
Abs(VctB-VctA)+Ab
s(VctD-VctB)+Abs(
VctC-VctD)+Abs(Vc
tA-VctC)
```

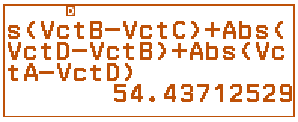


```
s(VctD-VctB)+Abs(
VctC-VctD)+Abs(Vc
tA-VctC)
      61.28739628
```

$$\mathrm{ACBDA} = |\overrightarrow{\mathrm{AC}}| + |\overrightarrow{\mathrm{CB}}| + |\overrightarrow{\mathrm{BD}}| + |\overrightarrow{\mathrm{DA}}|$$
$$= |\vec{\mathrm{C}} - \vec{\mathrm{A}}| + |\vec{\mathrm{B}} - \vec{\mathrm{C}}| + |\vec{\mathrm{D}} - \vec{\mathrm{B}}| + |\vec{\mathrm{A}} - \vec{\mathrm{D}}|$$
$$\approx 54.4$$



```
Abs(VctC-VctA)+Ab
s(VctB-VctC)+Abs(
VctD-VctB)+Abs(Vc
tA-VctD)
```



```
s(VctB-VctC)+Abs(
VctD-VctB)+Abs(Vc
tA-VctD)
      54.43712529
```

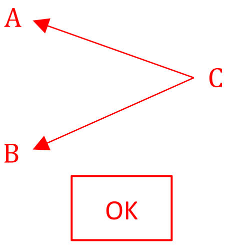
Thus, the path with the shortest total travel distance is ACBDA or ADBCA.

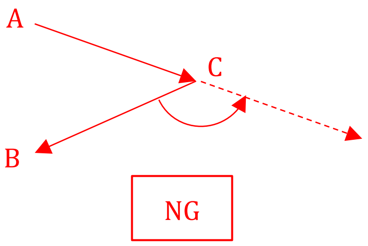
(3) For the shortest flight path found in (2), find the angle for each of the turns along the path.

First, let the angle of the turn at C in path $A \to C \to B \to D \to A$ be $\angle ACB$.

To find the measure of $\angle ACB$, use your scientific calculator to find the angle between $\overrightarrow{CA}$ and $\overrightarrow{CB}$, which have the same initial point, as shown in the diagram.

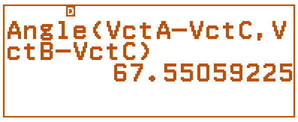
(*Be careful not to find the angle between $\overrightarrow{AC}$ and $\overrightarrow{CB}$ by following the route, as shown.)





- $\angle ACB$: angle between $\overrightarrow{CA}$ and $\overrightarrow{CB}$ = angle between $(\vec{A} - \vec{C})$ and $(\vec{B} - \vec{C})$

Calculating with your scientific calculator gives $\angle ACB = 67.6°$.

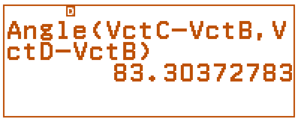


```
Angle(VctA−VctC,V
ctB−VctC)
       67.55059225
```

Calculating the measures of the other angles in the same manner gives:

- $\angle CBD$: angle between $\overrightarrow{BC}$ and $\overrightarrow{BD}$ = angle between $(\vec{C} - \vec{B})$ and $(\vec{D} - \vec{B})$

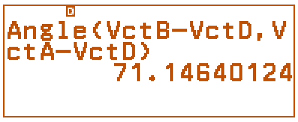
$\therefore \angle CBD = 83.3°$



```
Angle(VctC−VctB,V
ctD−VctB)
       83.30372783
```

- $\angle BDA$: angle between $\overrightarrow{DB}$ and $\overrightarrow{DA}$ = angle between $(\vec{B} - \vec{D})$ and $(\vec{A} - \vec{D})$

$\therefore \angle BDA = 71.1°$

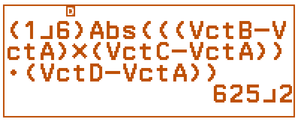


```
Angle(VctB−VctD,V
ctA−VctD)
       71.14640124
```

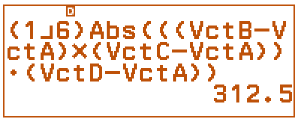
(4) Find the volume V of the air space through which the drone flies (the volume of tetrahedron ABCD).

Formula for the volume of a tetrahedron:

$$V = \frac{1}{6} \times \left|\left(\overrightarrow{AB} \times \overrightarrow{AC}\right) \cdot \overrightarrow{AD}\right| = \frac{1}{6} \times \left|\left((\vec{B} - \vec{A}) \times (\vec{C} - \vec{A})\right) \cdot (\vec{D} - \vec{A})\right| = 312.5$$



```
(1⌟6)Abs(((VctB−V
ctA)×(VctC−VctA))
•(VctD−VctA))
             625⌟2
```



```
(1⌟6)Abs(((VctB−V
ctA)×(VctC−VctA))
•(VctD−VctA))
             312.5
```

Total score of university entrance exams

Level 1

(1) Table 1 shows the raw scores of high school students X, Y, and Z in the university entrance exams for three subjects: mathematics, English, and science. Find the total raw scores of X, Y, and Z for each of the three subjects.

Table 1. Raw score table

	Mathematics	English	Science
X	80	80	80
Y	90	60	90
Z	90	70	80

Total raw score of X $= 80 + 80 + 80 = 240$

Total raw score of Y$= 90 + 60 + 90 = 240$

Total raw score of Z$= 90 + 70 + 80 = 240$

(2) Table 2 shows the weighting of subjects used to evaluate high school students' grades at three universities, P, Q, and R. Each university multiplies the raw score of each student's subject by the corresponding value in Table 2, and the sum of the three subjects is the final total score. At this point, use matrix calculations to find each student's total score and complete Table 3. Also, which of the three students has the highest final total score at each university?

Table 2. Subject weighting

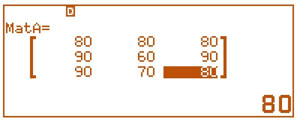
	P Uni.	Q Uni.	R Uni.
Mathematics	10	9	8
English	9	8	10
Science	8	10	9

Table 3. Final Total Score

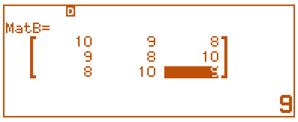
	P Uni.	Q Uni.	R Uni.
X	2160	2160	2160
Y	2160	2190	2130
Z	2170	2170	2140

By multiplying the matrices in Table 1 and Table 2, we can find each student's final total score at each university.

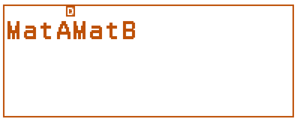
$$\begin{bmatrix} 80 & 80 & 80 \\ 90 & 60 & 90 \\ 90 & 70 & 80 \end{bmatrix}\begin{bmatrix} 10 & 9 & 8 \\ 9 & 8 & 10 \\ 8 & 10 & 9 \end{bmatrix} = \begin{bmatrix} 2160 & 2160 & 2160 \\ 2160 & 2190 & 2130 \\ 2170 & 2170 & 2140 \end{bmatrix}$$



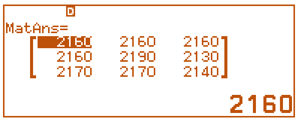
```
MatA=
[ 80   80   80 ]
[ 90   60   90 ]
[ 90   70   80 ]
                 80
```



```
MatB=
[ 10    9    8 ]
[  9    8   10 ]
[  8   10    9 ]
                  9
```



```
MatAMatB
```



```
MatAns=
[ 2160  2160  2160 ]
[ 2160  2190  2130 ]
[ 2170  2170  2140 ]
                 2160
```

Therefore, the students with the highest final total scores at each university are as follows.

University P: Z, University Q: Y, University R: X

Cryptography using matrices

Level 2

There is a method of transmitting information by encrypting (matrix calculation) numbers assigned to letters, as shown below.

Letter	A	B	C	D	E	F	G	H	I	J
Number	11	12	13	14	15	16	17	18	19	20

K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36

$$\underset{\textit{Letter}}{\begin{bmatrix} M & A \\ T & H \end{bmatrix}} \xrightarrow{\text{Number Allocation}} \underset{\textit{Number}\ \text{(matrix)}}{\begin{bmatrix} 23 & 11 \\ 30 & 18 \end{bmatrix}} \xrightarrow{\text{Coding}} \underset{\textit{Code}\ \text{(matrix)}}{\begin{bmatrix} 311 & 167 \\ 182 & 98 \end{bmatrix}}$$

Use the table above

Multiply a *Key* (matrix) by the front of the *Number* to obtain the *Code*.

$\boxed{Key \times Number = Code}$ ……①

$$\begin{bmatrix} 7 & 5 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} 23 & 11 \\ 30 & 18 \end{bmatrix} = \begin{bmatrix} 311 & 167 \\ 182 & 98 \end{bmatrix}$$

※ In this example, the *Key* is $\begin{bmatrix} 7 & 5 \\ 4 & 3 \end{bmatrix}$

(1) Decode the *Code* $\begin{bmatrix} 236 & 235 \\ 138 & 138 \end{bmatrix}$ by using the *Key* $\begin{bmatrix} 7 & 5 \\ 4 & 3 \end{bmatrix}$.

① can be transformed as follows:

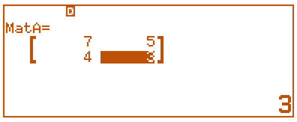
$\boxed{[Key][Number] = [Code]}$ ……①

$$[Key]^{-1}[Key][Number]=[Key]^{-1}[Code]$$

$$[Number]=[Key]^{-1}[Code]$$

Therefore, to find the *Number*, we can multiply the inverse of the *Key* from the front of the *Code*.

$$[Number] = \begin{bmatrix} 7 & 5 \\ 4 & 3 \end{bmatrix}^{-1} \begin{bmatrix} 236 & 235 \\ 138 & 138 \end{bmatrix} = \begin{bmatrix} 18 & 15 \\ 22 & 26 \end{bmatrix}$$

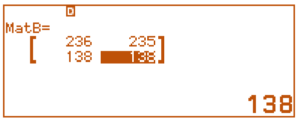


MatA=

$\begin{bmatrix} 7 & 5 \\ 4 & 3 \end{bmatrix}$

3

Input the *Key* to Mat *A*



MatB=

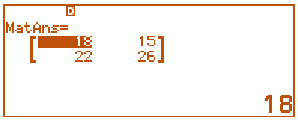
$\begin{bmatrix} 236 & 235 \\ 138 & 138 \end{bmatrix}$

138

Input the *Code* to Mat *B*

MatA⁻¹MatB

Enter the formula



MatAns=

$\begin{bmatrix} 18 & 15 \\ 22 & 26 \end{bmatrix}$

18

Number

Assigning *Letters* from the table $\begin{bmatrix} 18 & 15 \\ 22 & 26 \end{bmatrix} = \begin{bmatrix} \text{H} & \text{E} \\ \text{L} & \text{P} \end{bmatrix}$ → HELP

(2) Information was leaked that when the $Code$ $\begin{bmatrix} 328 & 133 \\ 209 & 85 \end{bmatrix}$ is decoded, the letters are "SAVE". Find the Key (matrix) in this case.

① can be transformed as follows.

$\boxed{[Key][Number] = [Code]}$ ……①

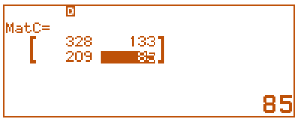
$[Key][Number]\ [Number]^{-1} = [Code][Number]^{-1}$

$[Key] = [Code][Number]^{-1}$

Therefore, to obtain the Key, we can multiply the inverse of the $Number$ from the back of the $Code$.

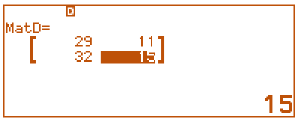
Since the $Number$ of SAVE is $\begin{bmatrix} 29 & 11 \\ 32 & 15 \end{bmatrix}$ from the table,

$$[Key] = \begin{bmatrix} 328 & 133 \\ 209 & 85 \end{bmatrix}\begin{bmatrix} 29 & 11 \\ 32 & 15 \end{bmatrix}^{-1} = \begin{bmatrix} 8 & 3 \\ 5 & 2 \end{bmatrix}$$



```
MatC=
[ 328    133]
  209     85
                85
```

Input $Code$ to Mat C

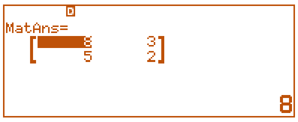


```
MatD=
[ 29     11]
  32     15
                15
```

Input $Number$ to Mat D

```
MatCMatD⁻¹
```

Enter the formula



```
MatAns=
[ 8      3]
  5      2
                 8
```

Key

(3) Decode the $Code$ $\begin{bmatrix} 224 & 197 \\ 142 & 125 \end{bmatrix}$ using the Key obtained in (2).

As in (1), to obtain the $Number$, we can multiply the inverse of the Key from the front of the $Code$.

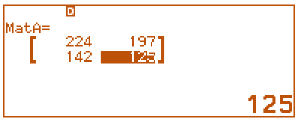
$$[Number] = \begin{bmatrix} 8 & 3 \\ 5 & 2 \end{bmatrix}^{-1}\begin{bmatrix} 224 & 197 \\ 142 & 125 \end{bmatrix} = \begin{bmatrix} 22 & 19 \\ 16 & 15 \end{bmatrix}$$

```
MatAns
```

Enter Mat Ans for Key in (2)

```
MatAns⁻¹
```

Inverse matrix

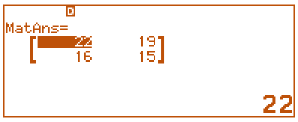


```
MatA=
[ 224    197]
  142    125
               125
```

Enter $Code$ in Mat A

```
MatAns⁻¹MatA
```

Enter the formula



```
MatAns=
[ 22     19]
  16     15
                22
```

$Number$

Assigning $Letters$ from the table, $\begin{bmatrix} \mathrm{L} & \mathrm{I} \\ \mathrm{F} & \mathrm{E} \end{bmatrix}$ → LIFE

(4) Decode the $Code$ $\begin{bmatrix} 66 & 82 & 64 & 30 \\ 14 & 45 & 16 & 17 \\ 11 & 21 & 34 & 28 \\ 53 & 175 & 96 & 96 \end{bmatrix}$ by using the Key $\begin{bmatrix} 1 & 3 & 2 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 3 & 1 & 5 \end{bmatrix}$.

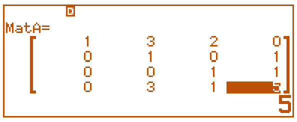
However, the following table should be used for this question.

Letter	A	B	C	D	E	F	G	H	I	J	K	L	M
Number	0	1	2	3	4	5	6	7	8	9	10	11	12

N	O	P	Q	R	S	T	U	V	W	X	Y	Z	_
13	14	15	16	17	18	19	20	21	22	23	24	25	26

As in (1), to obtain the $Number$, we can multiply the inverse of the Key from the front of the $Code$.

$$Number = \begin{bmatrix} 1 & 3 & 2 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 3 & 1 & 5 \end{bmatrix}^{-1} \begin{bmatrix} 66 & 82 & 64 & 30 \\ 14 & 45 & 16 & 17 \\ 11 & 21 & 34 & 28 \\ 53 & 175 & 96 & 96 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 18 & 8 \\ 14 & 26 & 2 & 0 \\ 11 & 2 & 20 & 11 \\ 0 & 19 & 14 & 17 \end{bmatrix}$$

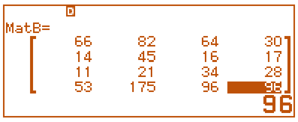


MatA=

$\begin{bmatrix} 1 & 3 & 2 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 3 & 1 & 5 \end{bmatrix}$

5

Input Key to Mat A

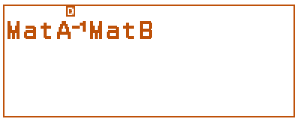


MatB=

$\begin{bmatrix} 66 & 82 & 64 & 30 \\ 14 & 45 & 16 & 17 \\ 11 & 21 & 34 & 28 \\ 53 & 175 & 96 & 96 \end{bmatrix}$

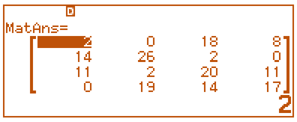
96

Enter $Code$ in Mat B



MatA⁻¹MatB

Enter the formula



MatAns=

$\begin{bmatrix} 2 & 0 & 18 & 8 \\ 14 & 26 & 2 & 0 \\ 11 & 2 & 20 & 11 \\ 0 & 19 & 14 & 17 \end{bmatrix}$

2

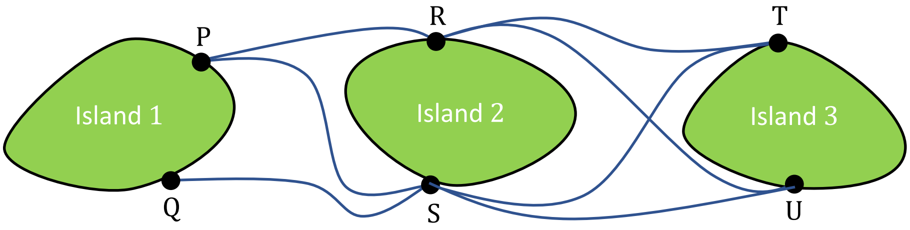
$Number$

Assigning $Letters$ from the table, $\begin{bmatrix} \mathrm{C} & \mathrm{A} & \mathrm{S} & \mathrm{I} \\ \mathrm{O} & \_ & \mathrm{C} & \mathrm{A} \\ \mathrm{L} & \mathrm{C} & \mathrm{U} & \mathrm{L} \\ \mathrm{A} & \mathrm{T} & \mathrm{O} & \mathrm{R} \end{bmatrix}$ → CASIO_CALCULATOR

Number of paths and matrices

Level 3

1



(1) When the number of routes from island 1 to island 2 in the diagram is expressed as the matrix below, what is the matrix representing the number of routes from island 2 to island 3 and what is the total number of routes? Routes from island 2 back to island 1 and from island 3 back to island 2 will not be considered.

Matrix A showing the number of routes from island 1 to 2:

Arrival

represents the number of paths from P to S

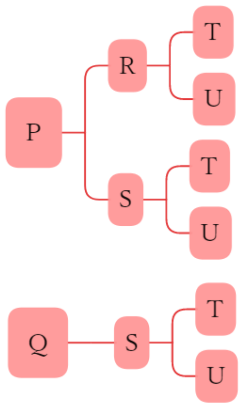
Departure $$\begin{array}{cc} & \begin{array}{cc} R & S \end{array} \\ \begin{array}{c} P \\ Q \end{array} & \begin{bmatrix} 1 & \textcircled{1} \\ 0 & 1 \end{bmatrix} \end{array} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$ → Total number of routes: 3 (sum of matrix elements)

Matrix B showing the number of routes from island 2 to 3:

Arrival

Departure $$\begin{array}{cc} & \begin{array}{cc} T & U \end{array} \\ \begin{array}{c} R \\ S \end{array} & \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \end{array} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$ → Total number of routes: 4(sum of matrix elements)

(2) Use a tree diagram to represent all combinations of routes from island 1 to 3 via island 2, and then answer the total number of routes.



Six ways based on the tree diagram

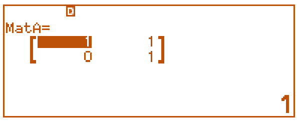
(3) Using (1) and the notation below, answer the matrix that shows the number of routes from island 1 to island 3. Also, confirm that the total number of routes is the same as the value found in (2).

Matrix C showing the number of routes from island 1 to 3:

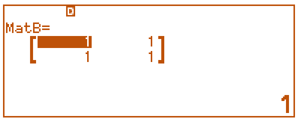
$$\text{Departure} \quad \begin{array}{c} \\ \\ \mathrm{P} \\ \mathrm{Q} \end{array} \begin{array}{c} \text{Arrival} \\ \begin{array}{cc} \mathrm{T} & \mathrm{U} \end{array} \\ \begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix} \end{array} = \begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix} \quad \rightarrow \quad \text{Total number of routes: } \text{6 (sum of matrix elements)}$$

(4) Verify that the matrices A, B, and C in (1) and (3) satisfy the relationship $AB = C$.

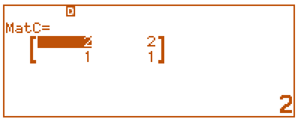
1. Input the matrices A, B, and C to Mat A, B, C.



```
MatA=
[1  1]
[0  1]
                1
```



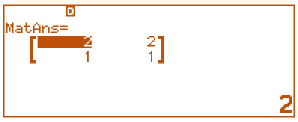
```
MatB=
[1  1]
[1  1]
                1
```



```
MatC=
[2  2]
[1  1]
                2
```

2.Calculating the value of AB

```
MatAMatB
```

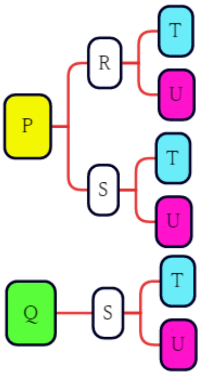


```
MatAns=
[2  2]
[1  1]
                2
```

$$\therefore \quad AB = C$$

$$\underset{A}{\begin{array}{c|cc} & \mathrm{R} & \mathrm{S} \\ \hline \mathrm{P} & 1 & 1 \\ \mathrm{Q} & 0 & 1 \end{array}} \ \underset{B}{\begin{array}{c|cc} & \mathrm{T} & \mathrm{U} \\ \hline \mathrm{R} & 1 & 1 \\ \mathrm{S} & 1 & 1 \end{array}} = \begin{bmatrix} 1\times 1(\mathrm{P}\rightarrow\mathrm{R}\rightarrow\mathrm{T}) + 1\times 1(\mathrm{P}\rightarrow\mathrm{S}\rightarrow\mathrm{T}) & 1\times 1\,(\mathrm{P}\rightarrow\mathrm{R}\rightarrow\mathrm{U}) + 1\times 1(\mathrm{P}\rightarrow\mathrm{S}\rightarrow\mathrm{U}) \\ 0\times 1(\mathrm{Q}\rightarrow\mathrm{R}\rightarrow\mathrm{T}) + 1\times 1(\mathrm{Q}\rightarrow\mathrm{S}\rightarrow\mathrm{T}) & 0\times 1(\mathrm{Q}\rightarrow\mathrm{R}\rightarrow\mathrm{U}) + 1\times 1(\mathrm{Q}\rightarrow\mathrm{S}\rightarrow\mathrm{U} \end{bmatrix}$$

$$= \underset{C}{\begin{array}{c|cc} & \mathrm{T} & \mathrm{U} \\ \hline \mathrm{P} & 2 & 2 \\ \mathrm{Q} & 1 & 1 \end{array}}$$



From the equations and the color tree diagram, we can confirm that $AB = C$

(5) Answer the following the questions to find matrix Z showing the number of routes from island 3 to 1 via island 2.

(i)Answer matrix X showing the number of routes from island 3 to 2 and matrix Y showing the number of routes from island 2 to 1.

Matrix X showing the number of routes from island 3 to 2 : Departure $\begin{array}{c|cc} & \multicolumn{2}{c}{\text{Arrival}} \\ & \mathrm{R} & \mathrm{S} \\ \mathrm{T} & 1 & 1 \\ \mathrm{U} & 1 & 1 \end{array} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

Matrix Y showing the number of routes from island 2 to 1 : Departure $\begin{array}{c|cc} & \multicolumn{2}{c}{\text{Arrival}} \\ & \mathrm{P} & \mathrm{Q} \\ \mathrm{R} & 1 & 0 \\ \mathrm{S} & 1 & 1 \end{array} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

(ii)Answer matrix Z showing the number of routes from island 3 to 1, and answer the total number of routes.

Considering the relationship in (4), XY is a matrix showing the number of routes from island 3 to 1.

$Z = XY = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix}$ → Total number of routes: 6 (Sum of matrix elements)

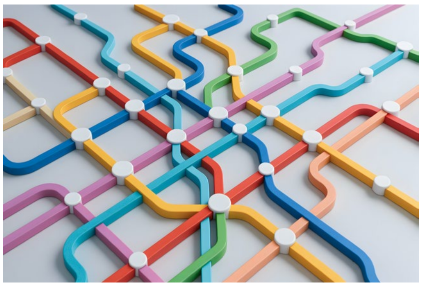
(6) What is the relationship between matrix C and matrix Z ?

$C : \begin{array}{c|cc} & \mathrm{T} & \mathrm{U} \\ \mathrm{P} & 2 & 2 \\ \mathrm{Q} & 1 & 1 \end{array}$ $\qquad Z : \begin{array}{c|cc} & \mathrm{P} & \mathrm{Q} \\ \mathrm{T} & 2 & 1 \\ \mathrm{U} & 2 & 1 \end{array}$

$C^t = Z$ (or $C = Z^t$)

The matrix C and the matrix Z are in a transpose matrix relationship.

$\boxed{2}$ As shown in the table, there are four bus terminals in each of X, Y, and Z cities, and routes connecting each terminal are indicated with an O in the table. (You must transfer at Y to go from X to Z)



		Y city			
		5	6	7	8
X city	1		O	O	O
	2	O	O		
	3	O		O	O
	4	O	O	O	O

		Z city			
		9	10	11	12
Y city	5	O			O
	6		O	O	
	7	O	O	O	
	8		O		O

(1) Answer matrix B, which represents the number of routes from City Y to City Z.

Matrix A : represents the number of routes from city X to city Y

$$\text{Departure } X \quad \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \text{Arrival Y} \\ \begin{array}{cccc} 5 & 6 & 7 & 8 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

Matrix B : represents the number of routes from city Y to city Z

$$\text{Departure } Y \quad \begin{array}{c} \\ 5 \\ 6 \\ 7 \\ 8 \end{array} \begin{array}{c} \text{Arrival } Z \\ \begin{array}{cccc} 9 & 10 & 11 & 12 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

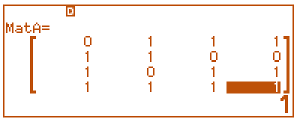
(2) Find the matrix that represents the number of routes from City X to City Z, and find the total number of routes.

Considering the relationship of $\boxed{1}$-(4), AB is a matrix that represents the number of routes from City X to City Z.

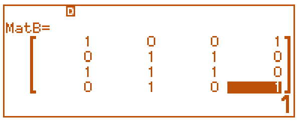
$$AB = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 2 & 1 \\ 1 & 1 & 1 & 1 \\ 2 & 2 & 1 & 2 \\ 2 & 3 & 2 & 2 \end{bmatrix}$$

→ Total number of routes: 27

(Sum of matrix elements)

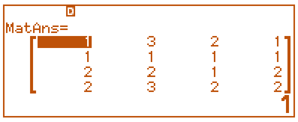


```
MatA=
[ 0  1  1  1 ]
[ 1  1  0  0 ]
[ 1  0  1  1 ]
[ 1  1  1  1 ]
              1
```



```
MatB=
[ 1  0  0  1 ]
[ 0  1  1  0 ]
[ 1  1  1  0 ]
[ 0  1  0  1 ]
              1
```

```
MatAMatB
```

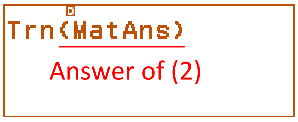


```
MatAns=
[ 1  3  2  1 ]
[ 1  1  1  1 ]
[ 2  2  1  2 ]
[ 2  3  2  2 ]
              1
```

(3) Find a matrix that represents the number of routes going from City Z to City X, and find the number of routes going from Terminal 10 in City Z to Terminal 4 in City X.

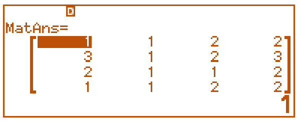
Considering the relationship of $\boxed{1}$-(6), the transpose of (2) is the matrix that represents the number of routes from City Z to City X.

$$\begin{bmatrix} 1 & 3 & 2 & 1 \\ 1 & 1 & 1 & 1 \\ 2 & 2 & 1 & 2 \\ 2 & 3 & 2 & 2 \end{bmatrix}^t = \begin{bmatrix} 1 & 1 & 2 & 2 \\ 3 & 1 & 2 & 3 \\ 2 & 1 & 1 & 2 \\ 1 & 1 & 2 & 2 \end{bmatrix}$$



```
Trn(MatAns)
```

Answer of (2)



```
MatAns=
[1  1  2  2]
[3  1  2  3]
[2  1  1  2]
[1  1  2  2]
```

$$\begin{array}{cc} & \text{Arrival X} \\ & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \text{Departure } Z \begin{array}{c} 9 \\ 10 \\ 11 \\ 12 \end{array} & \begin{bmatrix} 1 & 1 & 2 & 2 \\ 3 & 1 & 2 & 3 \\ 2 & 1 & 1 & 2 \\ 1 & 1 & 2 & 2 \end{bmatrix} \end{array}$$

From the above, the number of routes from Terminal 10 in City Z to Terminal 4 in City X is 3.

Probability of Getting a Particular Gumball Color Level 1

Answer the following questions about a gumball machine with the properties given below.

- There are 12 different colors of gumball in the machine.
- Turning the handle 1 time gives 1 color of gumball.
- No matter how many times you turn the handle, the probability of getting any given color is the same.
  (The probability of getting each color is always $\frac{1}{12}$.)

(1) Find the probability of not getting the color that you want after turning the handle 4 times.

The probability of not getting a particular color 4 times is

$$\frac{11}{12}\times\frac{11}{12}\times\frac{11}{12}\times\frac{11}{12}=\left(\frac{11}{12}\right)^4\approx 0.706$$

$\left(\frac{11}{12}\right)^4$

0.7060667438

(2) Find the probability of getting the color that you want at least 1 time after turning the handle 4 times.

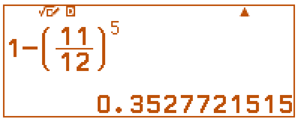
$$1-\left(\frac{11}{12}\right)^4\approx 0.294$$

1−Ans

0.2939332562

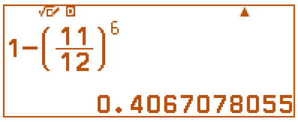
(3) Find the number of times you need to turn the handle to make the probability of getting the color that you want at least 1 time be 0.5 or higher.

Letting x be the number of turns, find the value for x for which $1-\left(\frac{11}{12}\right)^x$ first exceeds 0.5.



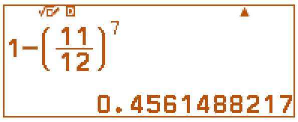
$1-\left(\frac{11}{12}\right)^5$

0.3527721515



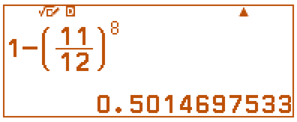
$1-\left(\frac{11}{12}\right)^6$

0.4067078055



$1-\left(\frac{11}{12}\right)^7$

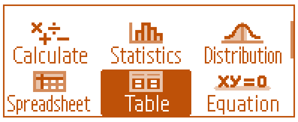
0.4561488217

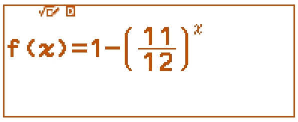


$1-\left(\frac{11}{12}\right)^8$

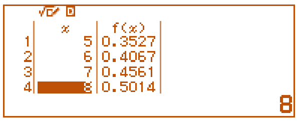
0.5014697533

Alternate Solution 1





$f(x)=1-\left(\frac{11}{12}\right)^x$



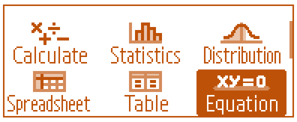
	x	f(x)
1	5	0.3527
2	6	0.4067
3	7	0.4561
4	8	0.5014

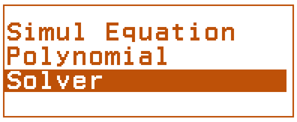
8

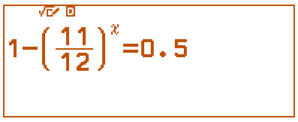
$x=5$	$x=6$	$x=7$	$x=8$
0.352	0.407	0.456	0.501

∴ At least 8 turns.

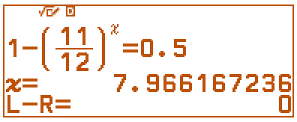
Alternate Solution 2







$1-\left(\frac{11}{12}\right)^x=0.5$



$1-\left(\frac{11}{12}\right)^x=0.5$

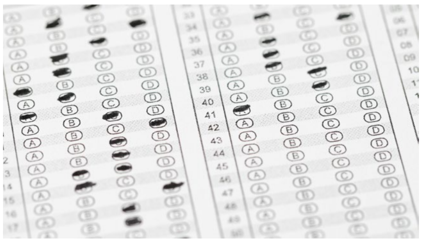
$x=$ 7.966167236

L−R= 0

∴ At least 8 turns.

Number of Correct Answers When Randomly Selecting Test Responses Level 2

Consider a set of 10 problems, each with only 1 correct answer among 4 possible answer choices.



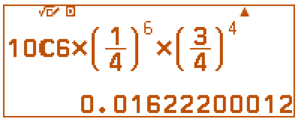
(1) Let X be the number of correct answers and $P(X)$ be the probability of X when each response is randomly selected. Complete the table.

Number of Correct Answer X	0	1	2	3	4	5	6	7	8	9	10
Probability $P(X)$	0.0563	0.1877	0.2815	0.2502	0.1459	0.0583	0.0162	3×10^{-3}	3.8×10^{-4}	2.8×10^{-5}	9.5×10^{-7}

- Method 1

Example) The probability of getting 6 correct answers is

$P(6) = {}_{10}C_6 \times (\frac{1}{4})^6 \times (\frac{3}{4})^4 \approx 0.01622$ (Approximately 1.6%)

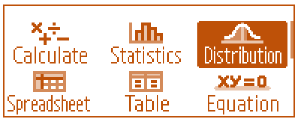


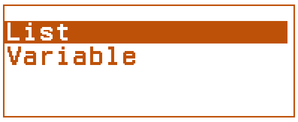
```
10C6×(1/4)^6×(3/4)^4
                0.01622200012
```

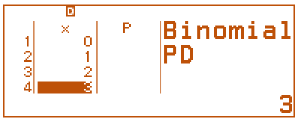
Use the same method to calculate the other probabilities $P(X) = {}_{10}C_X \times (\frac{1}{4})^X \times (\frac{3}{4})^{10-X}$.

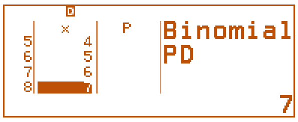
- Method 2

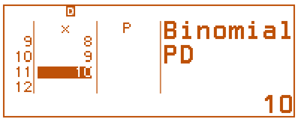
You can use the [Distribution] function to create a binomial distribution (probability distribution) table.





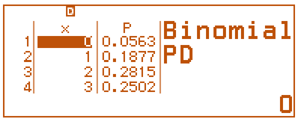




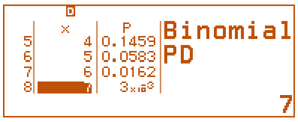


OK

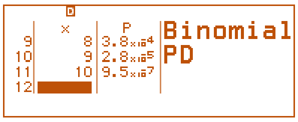
```
Binomial PD
 N     :10
 p     :0.25
Execute
```



```
   x   P
1  0   0.0563
2  1   0.1877
3  2   0.2815
4  3   0.2502
                0
```



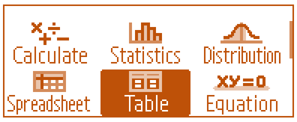
```
   x   P
5  4   0.1459
6  5   0.0583
7  6   0.0162
8  7   3×10^-3
                7
```



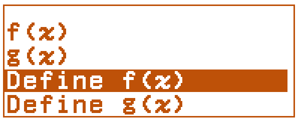
```
    x   P
9   8   3.8×10^-4
10  9   2.8×10^-5
11  10  9.5×10^-7
12
```

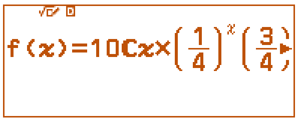
- Method 3

[Table], f(x) [Define f(x)], input $f(x) = {}_{10}C_x \times (\frac{1}{4})^x \times (\frac{3}{4})^{10-x}$, input the value of x $(0 \sim 10)$ in the table.

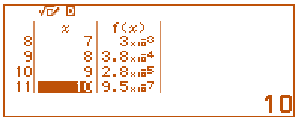


f(x)





```
f(x)=10Cx×(1/4)^x(3/4
```



```
    x   f(x)
8   7   3×10^-3
9   8   3.8×10^-4
10  9   2.8×10^-5
11  10  9.5×10^-7
                10
```

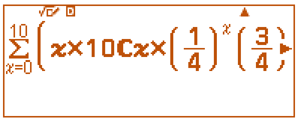
(2) Find the expected value for the number of correct answers when randomly selecting responses.

- Method 1

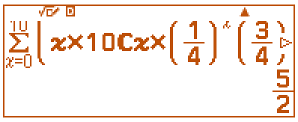
From the definition of an expected value:

$$0 \cdot P(0) + 1 \cdot P(1) + 2 \cdot P(2) + 3 \cdot P(3) + 4 \cdot P(4) + 5 \cdot P(5) + 6 \cdot P(6) + 7 \cdot P(7) + 8 \cdot P(8) + 9 \cdot P(9) + 10 \cdot P(10)$$

$$= \sum_{X=0}^{10} X \cdot P(X) = \sum_{X=0}^{10} X \cdot {}_{10}C_X \times \left(\frac{1}{4}\right)^X \times \left(\frac{3}{4}\right)^{10-X} = 2.5$$



$\sum_{x=0}^{10}\left(x\times 10Cx\times\left(\frac{1}{4}\right)^x\left(\frac{3}{4}\right)\right.$



$\sum_{x=0}^{10}\left(x\times 10Cx\times\left(\frac{1}{4}\right)^x\left(\frac{3}{4}\right)\right.$

$\frac{5}{2}$

- Method 2

The expected value of a binomial distribution (number of trials: n, probability: p) is np.

Thus, $np = 10 \times \frac{1}{4} = 2.5$.

(3) Find the standard deviation of the number of correct answers when randomly selecting responses.

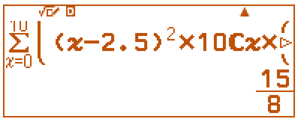
- Method 1

From the definition of a variance:

Variance$= (0-2.5)^2 \cdot \mathrm{P}(0) + (1-2.5)^2 \cdot \mathrm{P}(1) + (2-2.5)^2 \cdot \mathrm{P}(2) + (3-2.5)^2 \cdot \mathrm{P}(3) + (4-2.5)^2 \cdot \mathrm{P}(4)$

$+(5-2.5)^2 \cdot \mathrm{P}(5) + (6-2.5)^2 \cdot \mathrm{P}(6) + (7-2.5)^2 \cdot \mathrm{P}(7) + (8-2.5)^2 \cdot \mathrm{P}(8) + (9-2.5)^2 \cdot \mathrm{P}(9) + (10-2.5)^2 \cdot \mathrm{P}(10)$

$$= \sum_{X=0}^{10} (X-2.5)^2 \cdot P(X) = \sum_{X=0}^{10} (X-2.5)^2 \cdot {}_{10}C_X \times \left(\frac{1}{4}\right)^X \times \left(\frac{3}{4}\right)^{10-X} = \frac{15}{8}$$



$\sum_{x=0}^{10}\left((x-2.5)^2\times 10Cx\times\right.$

$\frac{15}{8}$

$\therefore$ Standard deviation$= \sqrt{\text{Variance}} = \sqrt{\frac{15}{8}} = \frac{\sqrt{30}}{4} \approx 1.37$

$\sqrt{\text{Ans}}$

$\frac{\sqrt{30}}{4}$

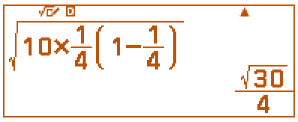
$\sqrt{\text{Ans}}$

1.369306394

- Method 2

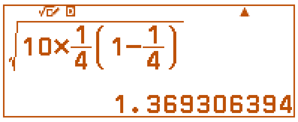
The standard deviation of a binomial distribution (number of trials: n, probability: p) is $\sqrt{np(1-p)}$.

Thus, $\sqrt{np(1-p)} = \sqrt{10 \times \frac{1}{4}(1-\frac{1}{4})} \approx 1.37$



$\sqrt{10\times\frac{1}{4}\left(1-\frac{1}{4}\right)}$

$\frac{\sqrt{30}}{4}$



$\sqrt{10\times\frac{1}{4}\left(1-\frac{1}{4}\right)}$

1.369306394

Estimating T-Shirt Production Levels Using Height Distribution Level 2

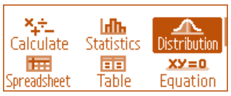
A certain clothing manufacturer is planning to produce a total of 1,000 men's T-shirts. If the number of shirts of each size that should be produced is estimated using the probability distribution for height among men in the country, find the approximate number of shirts of each size that should be produced using the information below.

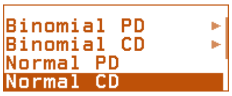
- This table shows the recommended height range for each shirt size.

Size	S	M	L	XL
Height [cm]	Up to 165	165 to 175	175 to 185	185 and up

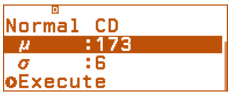
- Heights among men in the country follow a standard normal distribution, with a mean (μ) of 173 cm, and standard deviation (σ) of 6.0 cm.

Find each of the probabilities using the [Distribution (Normal CD)] function on your scientific calculator.





```
Binomial PD
Binomial CD
Normal PD
Normal CD
```



```
Normal CD
μ     :173
σ     :6
Execute
```

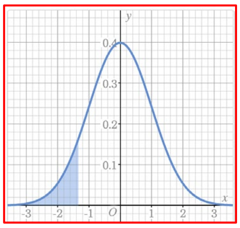
μ (mean): input 173
σ (standard deviation): input 6

*The actual upper and lower bounds of the standard normal distribution are ∞ and $-\infty$, respectively.

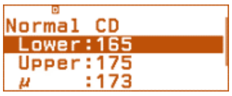
S: Lower: input 0*
Upper: input 165

```
Normal CD
 Lower:0
 Upper:165
 μ    :173
```

```
P=
      0.09121121973
```

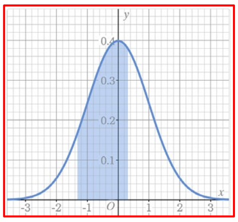


M: Lower: input 165
Upper: input 175

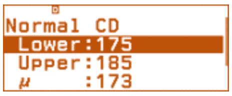


```
Normal CD
 Lower:165
 Upper:175
 μ    :173
```

```
P=
      0.5393474401
```

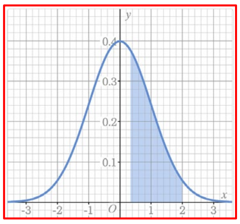


L: Lower: input 175
Upper: input 185

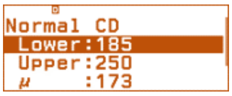


```
Normal CD
 Lower:175
 Upper:185
 μ    :173
```

```
P=
      0.3466912082
```

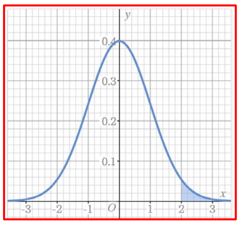


XL: Lower: input 185
Upper: input 250*



```
Normal CD
 Lower:185
 Upper:250
 μ    :173
```

```
P=
      0.022750132
```



Size	S	M	L	XL
Height [cm]	Up to 165	165 to 175	175 to 185	185 to 195
Probability	0.09121	0.53935	0.34669	0.02275
Quantity 1000 ×Probability	91.21 ≈ 91	539.35 ≈ 539	346.69 ≈ 347	22.75 ≈ 23

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