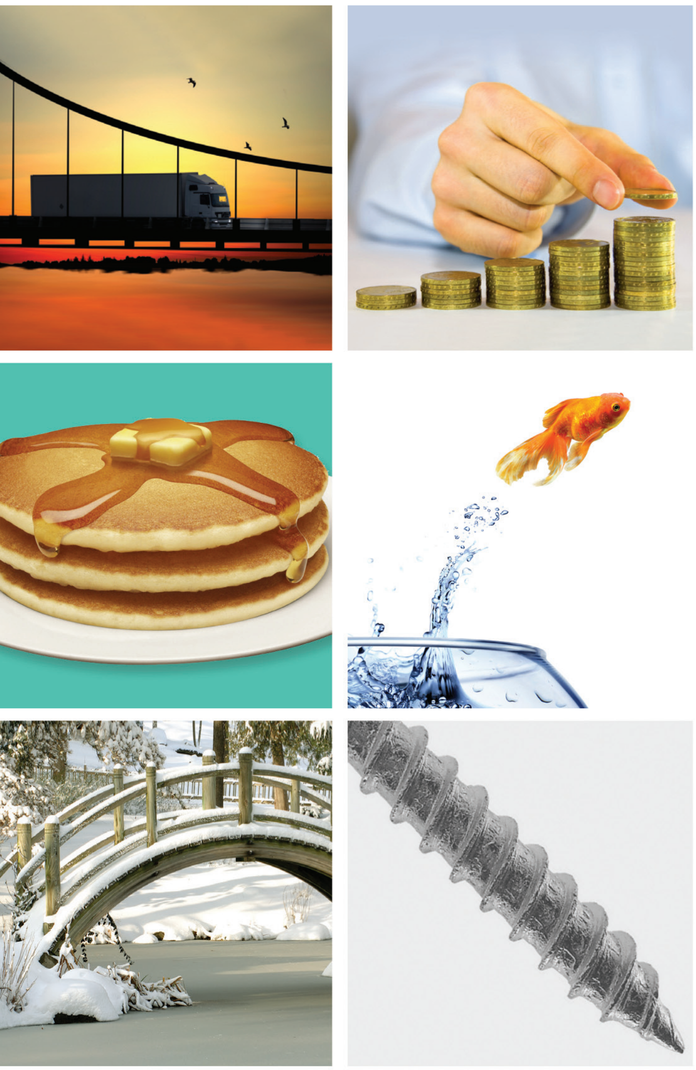
PRIZM™

Lesson Sampler



CASIO®

Simply Calculate The Difference!

GET TO KNOW... YOUR PRIZM™

Select the desired icon by highlighting it and pressing EXE or just press the number or letter in the lower right corner.

The status bar will display messages and current status like battery level, angle mode, fraction results, complex mode, or input/output settings.

The function keys allow you to access the tab (soft key) menus that will come up at the bottom of the screen. When an (>) appears above the F6 key, selecting F6 will offer more on-screen choices.

The MENU key displays every mode the calculator has. To select a mode, you may ▶ ▼ to the desired icon and press EXE or press the number or letter in the lower right hand corner of the icon.

The EXIT key operates like the back arrow on a web browser; it will take you back one screen each time you select it. The EXIT key will not take you to the icon menu.

The SHIFT key activates any function displayed on or above the calculator buttons tht is yellow. For example, to find the square root of number, you would need to press SHIFT, then x^2. SHIFT 5 gives you access to on-screen color formatting.

The AC/ON key will power the unit on. To turn the unit off, press the yellow SHIFT key, then AC/ON key.

The EXE key executes operations. When data is entered, the EXE button must be pressed to store the data.

The ALPHA key activates any function displayed on or above the calculator buttons that is in red. For example, to type the letter A, press ALPHA, then X,θ,T.

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This workbook is a product of the:

CASIO TEACHER ADVISORY COUNCIL (CTAC)

CASIO believes that instructional materials are most effective when they're written "By Teachers, For Teachers". CTAC represents an embodiment of this philosophy and the foundation for which we generate all CASIO support materials. For more information on CTAC, our ACE training program or how to get involved – email us at calcworkshops@casio.com.

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Dover, NJ 07801

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CASIO®
Simply Calculate The Difference!

Table of Contents

DISCOVER THROUGH PRIZM™

Area Under A Bridge

PRIZM WORKSHEET #101

CASIO. EDUCATION

TOPIC AREA: Curve fitting to image

NCTM STANDARDS:

- Create and use representations to organize, record, and communicate mathematical ideas.
- Use Mathematical models to represent and understand quantitative relationships.

OBJECTIVE

Given a photo file, students will be able to fit an equation on to it. Using their knowledge of polynomial functions and integral calculus, students will find the area under the curve using different methods, including Riemann sums, trapezoidal and midpoint rule.

GETTING STARTED

Have students work in pairs to help determine what points should be plotted and to facilitate discussion of the questions.

PRIOR TO USING THIS ACTIVITY:

- Students should have a basic understanding of what regression is and what it does.
- Students should understand the meaning of the Riemann sums and the trapezoidal rule.
- Students should be able to find the area of a rectangle and trapezoid.
- Students should be able calculate an output value given an input value and calculate the input value given an output value.

WAYS STUDENTS CAN PROVIDE EVIDENCE OF LEARNING:

- Students will be able to use their knowledge of geometry to find areas.
- Fill in the table to find areas.
- Find y values for midpoint values of x.

COMMON MISTAKES TO BE ON THE LOOKOUT FOR:

- Students may be careless in the placement of points.
- Students often have difficulty with the midpoint method.
- Students have difficulty with finding and using the correct y value or use the wrong value.

DEFINITIONS

- Trapezoidal rule
- Riemann sum
- Integral
- Regression

HOW TO **Area Under A Bridge**

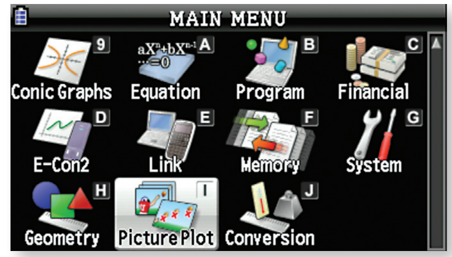
The following will walk you through the keystrokes and menus required to successfully complete the Area Under A Bridge activity.

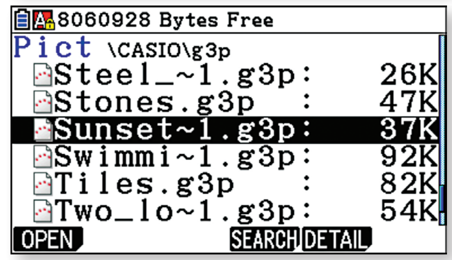
TO OPEN A BACKGROUND IMAGE IN PICTURE PLOT:

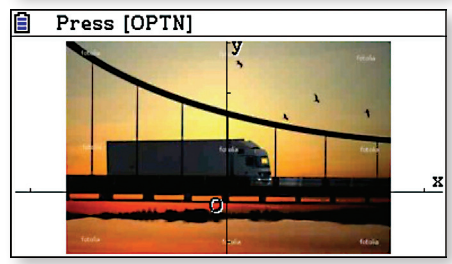
1. From the Main Menu, highlight the Picture Plot icon and press EXE or press (.

2. Press F1 (OPEN) to open the CASIO folder.

3. The g3p folder contains 47 background images. Press ▼ F1 (OPEN) to open the folder. Scroll down the list of images and highlight the desired image. You will be using the "Sunset~1" image in this activity. Press F1 (OPEN).







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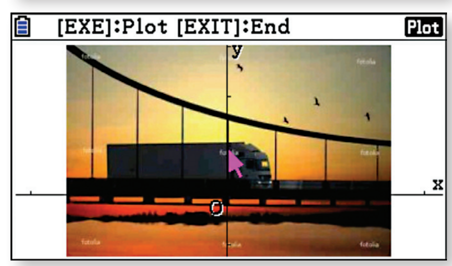
TO PLOT POINTS ON THE IMAGE AND CREATE A LIST OF POINTS:

1. The status bar at the top of the screen prompts what buttons you have to choose from. For this picture, you will need to press OPTN .

2. To plot points on the picture, press F2 (Plot). A pink arrow will appear; use ◀ ▶ ▲ ▼ to move the arrow to where you would like for it to plot a point. (Any of the number keys can also be used to jump to different areas on the screen).Press EXE to plot the point on the picture.

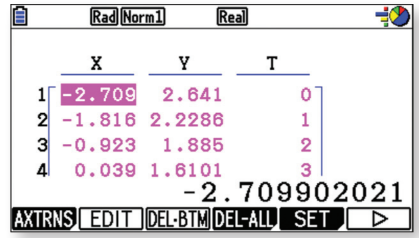
3. Continue moving the arrow and pressing EXE until you have all the points you want (one on the end of each suspension cable). To stop plotting, press EXIT .



TO VIEW THE LIST OF DATA POINTS:

1. Press [OPTN] [F3] to view the list of points plotted.

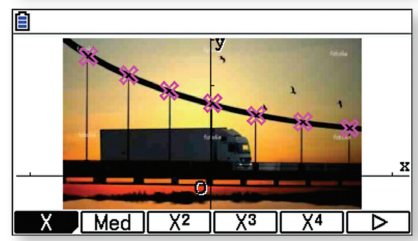
Press [EXIT] to go back to the picture and points.

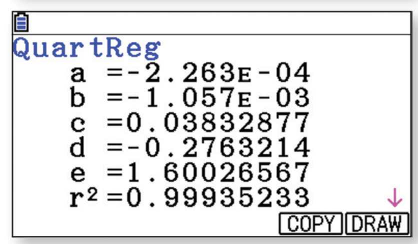


TO CREATE A BEST FIT LINE OR CURVE OF BEST FIT:

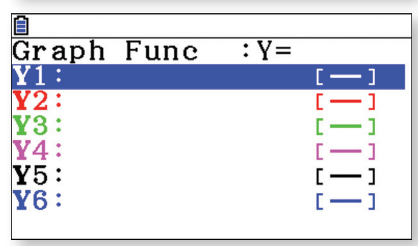
1. Press [F6] (▷) and [F2] (REG).

2. Choose the appropriate regression model. In this case, it will be X^4 so press [F5].

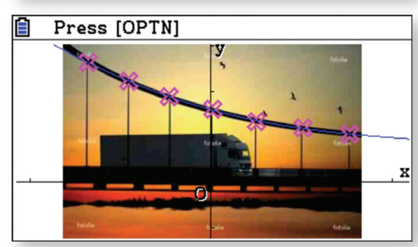




3. Press [F5] (Copy) and [EXE] to copy the equation to the Graph menu.

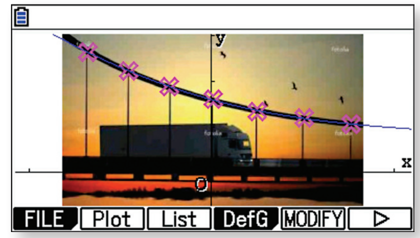


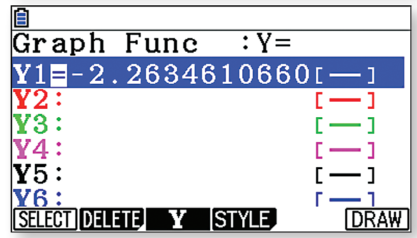
4. Press [F6] (DRAW) to see the regression curve and the points.



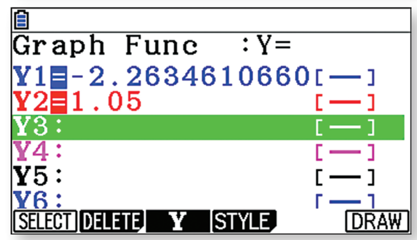
TO FIND THE INTERSECTION WITH THE TOP OF THE TRUCK:

1. To enter another equation, press OPTN F4 (DefG).

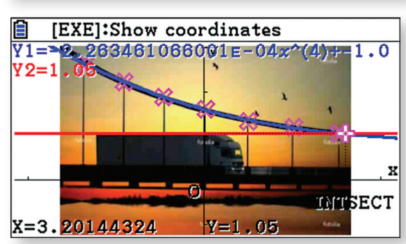




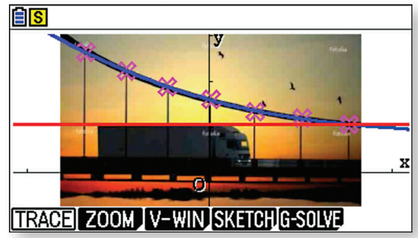
2. Press ▼ and enter 1 • 0 5 EXE F6 (DRAW).

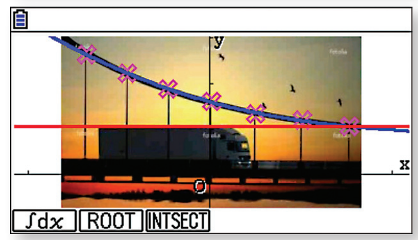


3. To find the intersection, press SHIFT F5 (G-SOLVE) F5 (INTSECT). Select curve and line, pressing EXE after each.

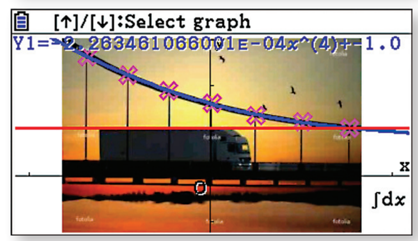


TO FIND THE INTEGRAL:

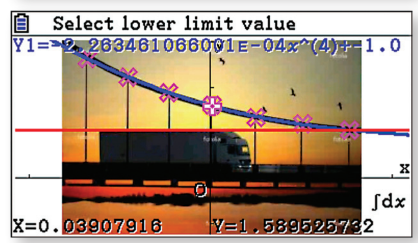


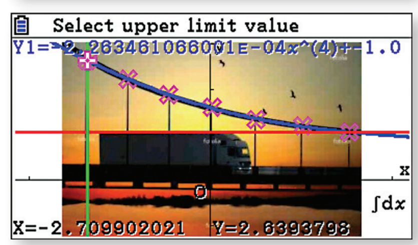


1. Press SHIFT F5 (G-Solv) F6 (▷) F3 (∫dx) F1 .



2. If there is more than one equation graphed, the calculator will ask you to specify which graph to find the integral of. Use the ▲ ▼ keys and press EXE to choose the desired graph.





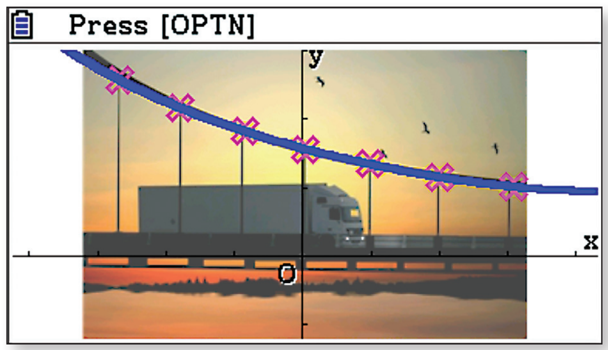
3. The calculator will prompt you to select the lower limit value and the upper limit value. Use the ◀ ▶ keys or input a value and press EXE .

Communities are trying to improve noise pollution on highways by installing noise reducing barriers. These barriers block sounds form traffic, but the barriers are expensive and must be custom made. In the picture below, you will need to find the area under the bridge cables to estimate cost and reduce waste.

Questions

1. List the coordinates of the support cables

X COORDINATE	Y COORDINATE
1	1
2	2
3	3
4	4
5	5
6	6
7	7

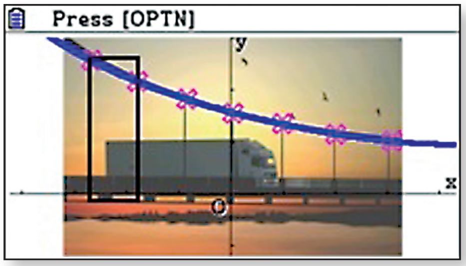


2. What is the equation of the best fit line to the curved bridge cable?

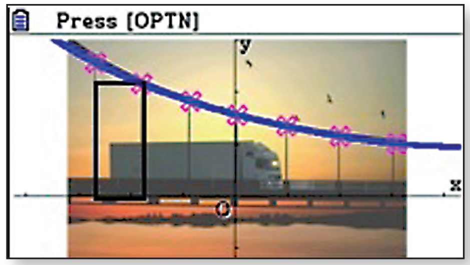
____

3. Using the coordinates find the right sided, left sided, midpoint and trapezoidal estimate of the area under the curve. (Remember–the width is the difference in x coordinates and the length is the height or the y coordinates.)

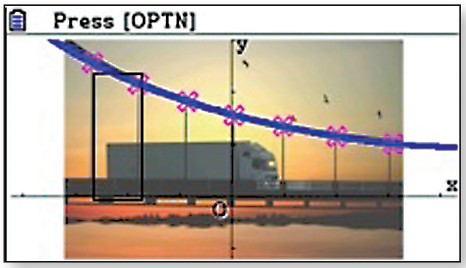
WIDTH	LENGTH	RIGHT SIDED AREA	LEFT SIDED AREA	MIDPOINT AREA	TRAPEZOIDAL AREA
		SUM:	SUM:	SUM:	SUM:



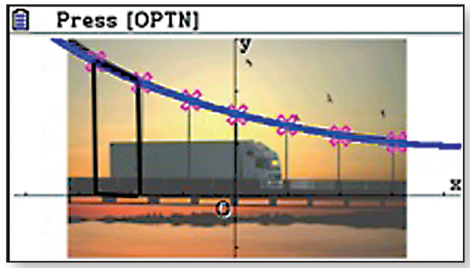
Right sided



Left sided



Midpoint



Trapezoidal

4. Using the integral feature, find the area under the curve

____

5. Which estimate is the best? Explain.

____

____

Extension

1. What is the maximum height a truck can be to make it under the cable?

____

2. Will the truck make it under the cable? If not at what point will it make contact with the cable?

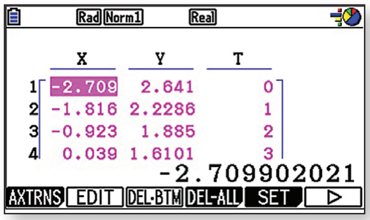
____

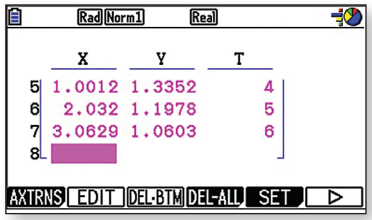
____

3. Find an equation that connects all four of the birds in the picture.

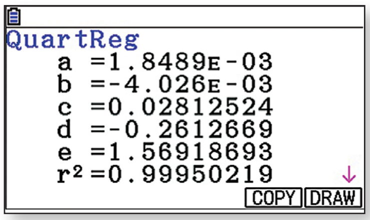
____

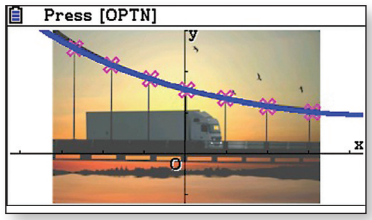
1. Answers may vary, depending on the plotted points.





2. Answers may vary, depending on the plotted points.





3. Width = x-values

Length = y-values

Right Sided Area = $(x_2 - x_1) \cdot y_1$

Left Sided Area = $(x_2 - x_1) \cdot y_2$

Midpoint Area =$(x_2 - x_1) \cdot Y1(\frac{x_1 + x_2}{2})$

Trapezoidal Area = $\frac{y_1 + y_2}{2} \cdot (x_2 - x_1)w$

WIDTH	LENGTH	RIGHT SIDED AREA	LEFT SIDED AREA	MIDPOINT AREA	TRAPEZOIDAL AREA
-2.71	2.64	2.35	1.98	2.16	2.17
-1.82	2.23	2.01	1.70	1.85	1.85
-0.92	1.89	1.66	1.42	1.53	1.54
-0.04	1.61	1.67	1.39	1.54	1.53
1.00	1.34	1.38	1.24	1.30	1.31
2.03	1.20	1.24	1.09	1.15	1.16
3.06	1.06	SUM: 10.31	SUM: 8.82	SUM: 9.53	SUM: 9.56

Right Sided Area = $(-1.82 - -2.71) \cdot 2.64 = 2.3496$

Left Sided Area = $(-1.82 - -2.71) \cdot 2.23 = 1.9847$

Midpoint Area = $(-1.8 - -2.67) \cdot Y1(\frac{-2.67 + -1.80}{2}) = (0.89) \cdot 2.4291 = 2.1619$

$Y1(-2.265) = 2.4291$

Trapezoidal Area = $\frac{2.64 + 2.23}{2} \cdot (-1.82 - -2.71) = 2.1672$

4. Area under the curve is 9.55.

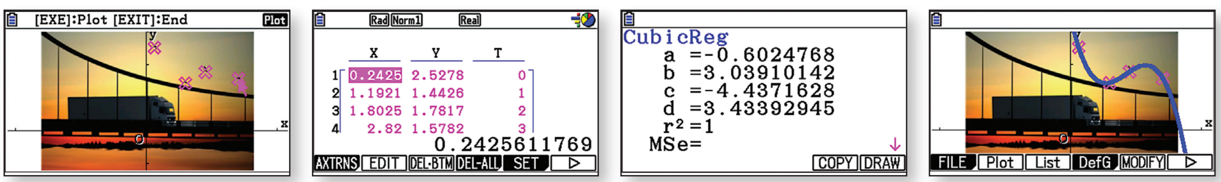
5. The Trapezoidal rule is the best estimator for area under the curve.
It was off by 0.01 units². The Midpoint rule was a close second, off by 0.02 units².

Extension Solutions

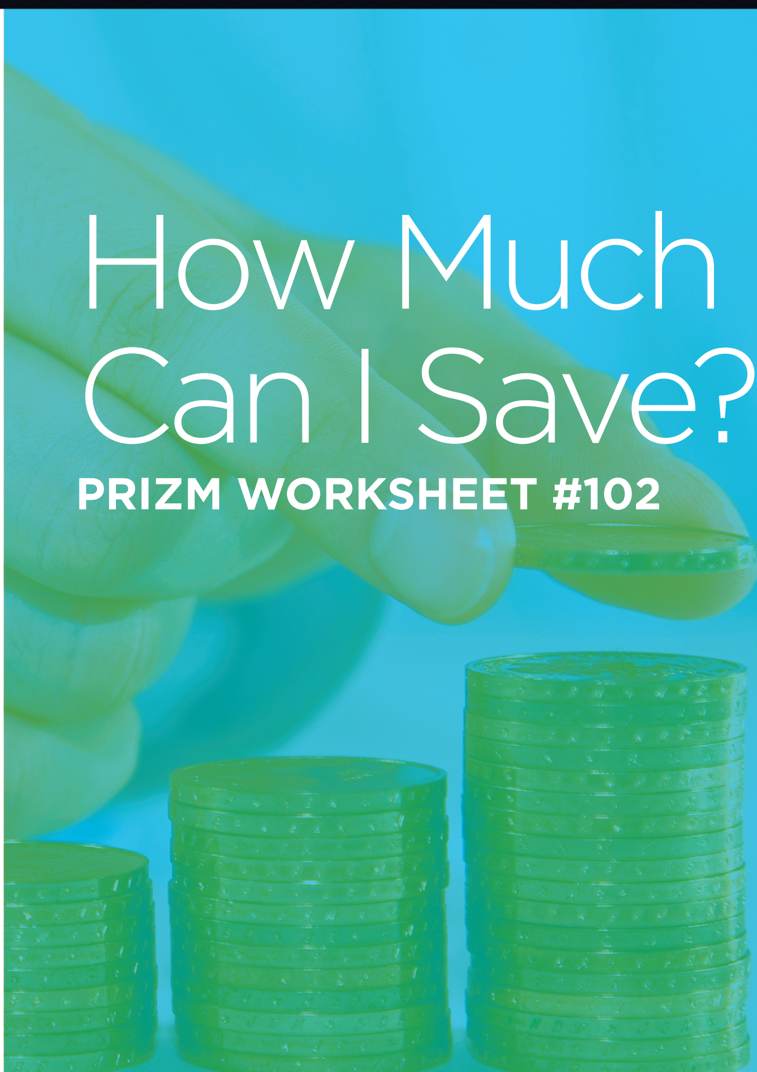
1. The maximum height for a truck is the minimum value of the function at y = 1.08.

2. Answers will vary, as long as the truck's estimated height is below the minimum of 1.08 it will. A good estimate is 1.10 for the height. In that case, the truck will hit the wire at x = 2.51.

3. One possible solution:



DISCOVER THROUGH PRIZM™



How Much Can I Save?

PRIZM WORKSHEET #102

CASIO EDUCATION

TOPIC AREA: **Quadratic Regression**

NCTM STANDARDS:

- Students will be able to select and use appropriate statistical methods to analyze data.

OBJECTIVE

Given a photo file, students will be able to analyze data to determine the quadratic regression equation for saving gold coins for retirement income.

GETTING STARTED

Have students work in pairs or small groups to use statistical and algebraic representations to examine the results of savings.

PRIOR TO USING THIS ACTIVITY:

- Students should be able to produce and manipulate graphs and tables of values manually and with the graphing calculator.
- Students should have a basic understanding of the regression equations.
- Students should be able to use regression equations to predict future outcomes.

WAYS STUDENTS CAN PROVIDE EVIDENCE OF LEARNING:

- Given a picture, students can plot points to be used to analyze data to find a graphical representation of a regression equation.
- Given an algebraic representation of a regression equation, the student can produce a table of values and determine the cumulative sum of a column of values.
- The student can state and explain how the results are interpreted to answer questions.

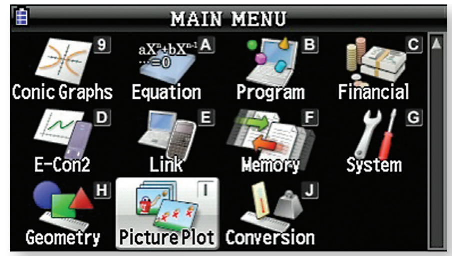
COMMON MISTAKES TO BE ON THE LOOKOUT FOR:

- Students may not follow the correct procedures to plot the points, determine the regression, constructing a table or finding a cumulative sum.
- Students may try to use a linear regression for the data instead of a quadratic regression.

DEFINITIONS

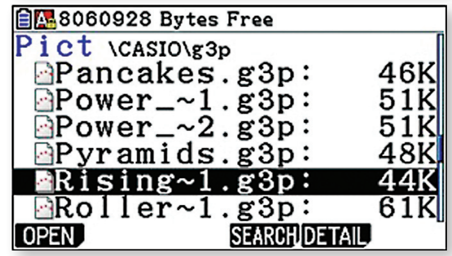
- Algebraic representation
- Cumulative sum
- Graphical representation
- Numerical representation
- Quadratic regression
- Ratios
- Regression
- Statistical representation

The following procedures will demonstrate how to retrieve an image, plot points on the picture, determine the regression equation, enter results into a table, move the table data to a list, alter data and determine the cumulative sum to answer the desired questions.



TO OPEN A BACKGROUND IMAGE IN PICTURE PLOT:

1. From the Main Menu, highlight the Picture Plot icon and press EXE or press (.
2. Press F1 (OPEN) to open the CASIO folder.



3. The g3p folder contains 47 background images. Press ▼ F1 (OPEN) to open folder. Scroll down the list of pictures and highlight the desired picture. You will be using "Rising~1" picture in this activity. Press F1 (OPEN).

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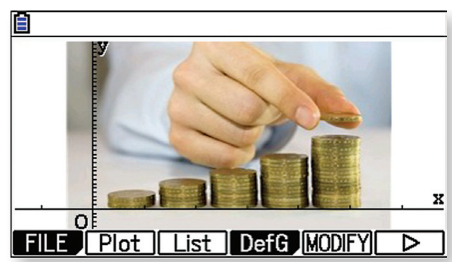
TO PLOT AND MOVE POINTS ON THE IMAGE TO DESIRED X-VALUES:

1. Press OPTN and choose F2 (PLOT).

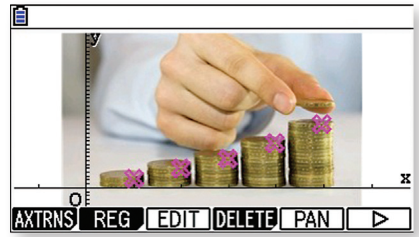


2. Move the Arrow to the desired position using ◀ ▶ ▲ ▼ and press EXE. In this activity, move the arrow to the top center of the first column of coins (to get the y-value) and then move the arrow directly to the right until it is above x = 1 and press EXE.

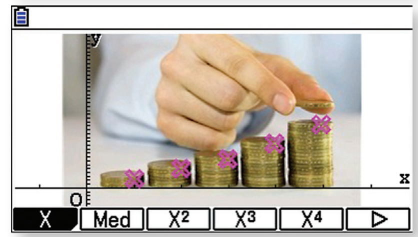
3. Continue to move the arrow and press EXE until you have all the points you want (one point for each column of coins). To stopplotting, press EXIT.

TO PLOT THE QUADRATIC REGRESSION:

1. Press [F6] (▷).

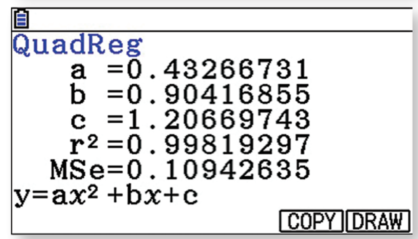


2. Press [F2] (REG).

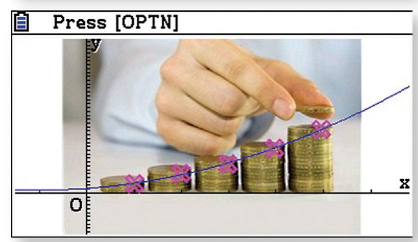


3. Press [F3] (x^2) for a quadratic regression

4. Press [F5] (COPY). This will copy the equation to other menu options such as Graph and Table, for future purposes. Press [EXE] to enter the equation for the **Y1:** equation. Keep in mind that the position of the arrow was estimated. Therefore, your equation will not be exactly the same as this equation or that of your partner.

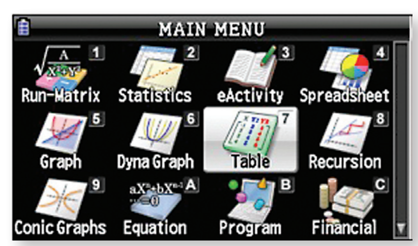


5. Press [F6] (DRAW). Notice that the quadratic regression comes very close to passing through all five plotted points.



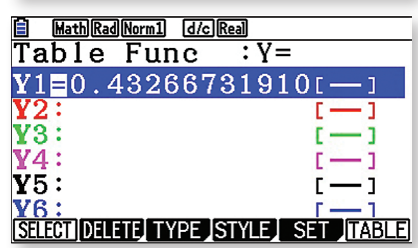
TO MOVE THE COORDINATES OF THE REGRESSION TO A TABLE:

1. From the Main Menu, highlight the Table icon and press [EXE] or press [7].

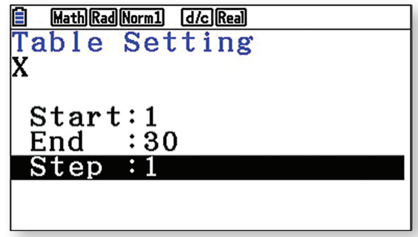


2. Press [F1] (SELECT) to select the quadratic regression equation that was stored in the last section.

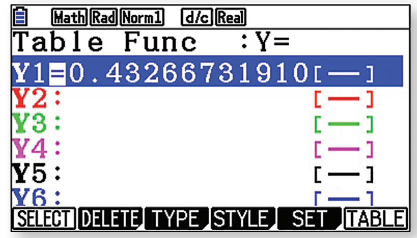
3. Press [F5] (SET). Choose the start point, end point and step units. This activity will need to know the savings for at least 30 years of working. Enter [1] for the start value and press



[EXE]. Enter 30 for the end value and press [EXE]. Enter [1] for the step value to get data for each year of working and press [EXE]. The SET screen should look like the screen on the right.



4. Press [EXIT] to return to the previous choices. Press [F6] (TABLE) to display the table that represents the amount of coins purchased each year of working. You can scroll down to see the other values.



TO CONVERT Y-VALUES USING RATIOS:

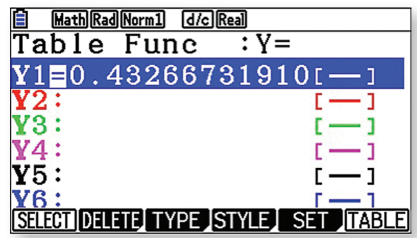
1. To convert your y-values to the actual coin quantities, you will need to use the following ratio:

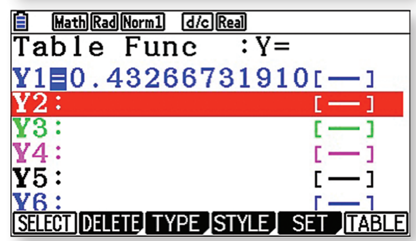
$$\frac{3 \text{ coins}}{\text{corresponding y - value}} = \frac{3 \text{ coins}}{y_1 \text{ value}}$$

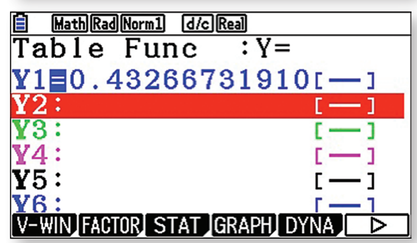
The value of the 3 coins from the table in this activity is 2.5435. Your value will probably differ.

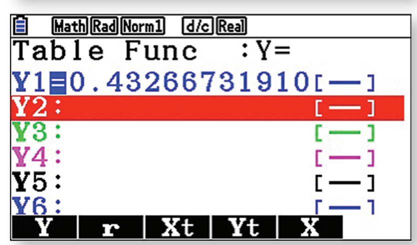
2. Press [EXIT] to return to the equation screen.

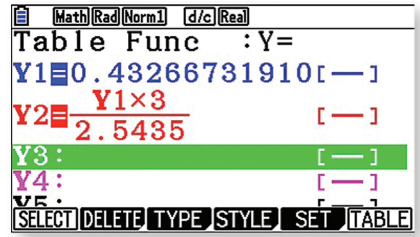
3. Enter the ratio into the **Y2:** by pressing [VARS], [F4] (GRAPH), [F1] (Y). Then, press [1] [×] [3] [÷] [2] [•] [5] [4] [3] [5] [EXE]. Notice that the fraction was entered in natural display when you press [EXE].



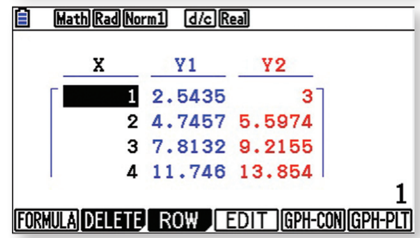






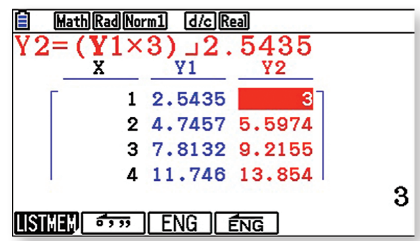


4. Press F6 (TABLE) to display the table of approximate number of coins saved each year of working. Again, you can scroll down to see how many coins were collected for each of the 30 years of working.



TO COPY THIS TABLE TO A LIST:

1. Since you will need to copy the **Y2:** column of the table, make sure that the calculator cursor is positioned anywhere in that column.

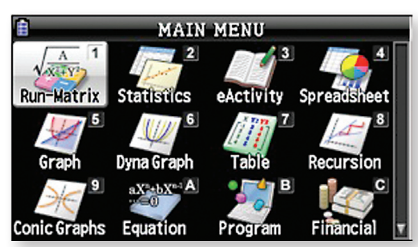


2. Press OPTN. Press F1 (LISTMEM).

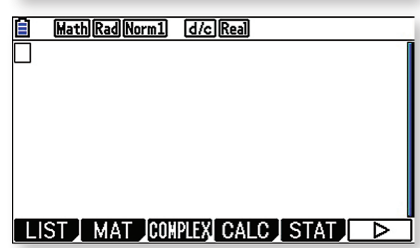
3. Input the number of the list you want to copy and press EXE. In this activity, use List 1.

TO CREATE A CUMULATIVE LIST:

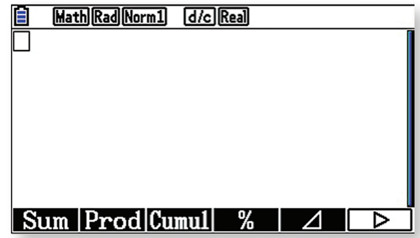
1. From the Main Menu, highlight the Run-Mat icon and press EXE or press 1.



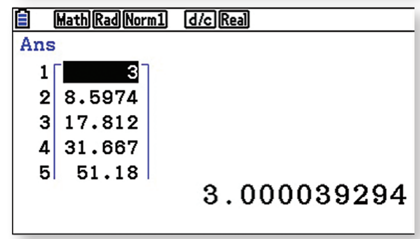
2. Press OPTN and F1 (LIST).



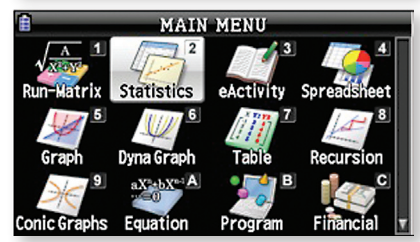
3. Press F6 twice.

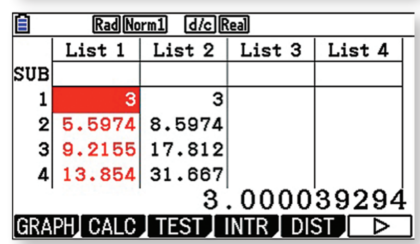


4. Press F3 (Cumul) F6 (▷) F1 (LIST) and then press the number of list that you want to create the cumulative sum. Our data was stored in List 1, therefore press 1 . Press ➡ (Store), F1 (LIST), 2 . This is because you need to store the values in List 2. Press EXE .



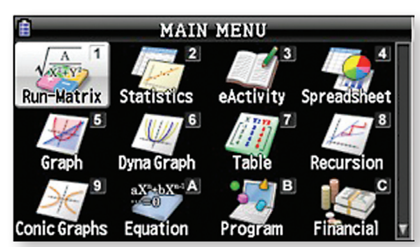
5. From the Main Menu, highlight the Statistics icon. Press EXE or press 2 to display the list of cumulative sums of the coins for each year of working.



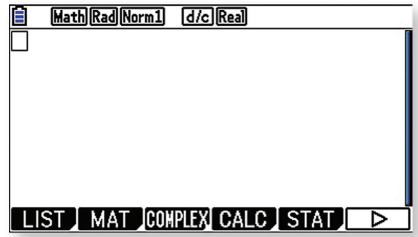


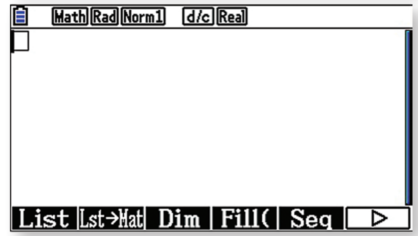
TO CREATE A LIST OF THE VALUE OF THE COINS:

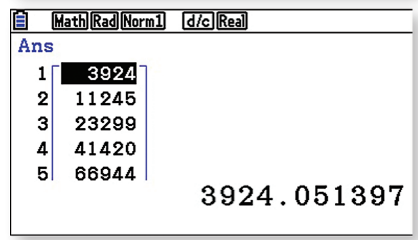
1. From the Main Menu, highlight the Run-Mat icon and press EXE or press 1 .



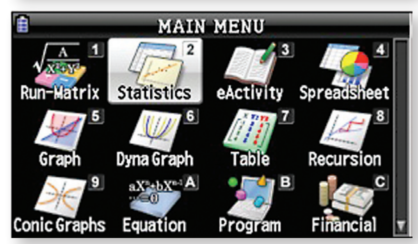
2. Press [OPTN], [F1] (LIST), [F1] (LIST) again, [2] [×] [1] [3] [0] [8] [→] (Store) [F1] (LIST) [3] [EXE]. This is because the value of the coin at the time this activity was written was $1,308, and you are storing these values into List 3.

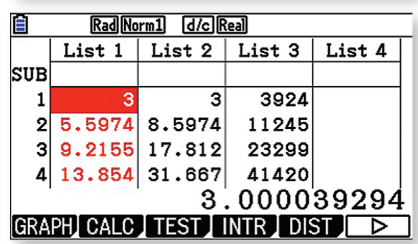






3. From the Main Menu, choose the Statistics icon. Press [EXE] to display the amount that was cumulatively saved each year of working.





Mr. and Mrs. Brinkley are hoping their nest egg that they have saved over the years of hard work will sustain their life style as they enter their golden years of retirement. When Brian and Amy first entered the workforce, they decided to save 1 oz. gold coins to reach this goal. They also decided that they would continue working until either their savings reached $4,000,000 or they worked 30 years. Finally, they have reached their goal. The price of gold is now $1,308 an ounce.

In this activity, you will be given a picture of five stacks of gold coins, shown below. The first stack represents the gold coins that were saved by the Brinkley's their first year of work. The second column represents the coins saved their second year, the third represents their third year, and so on. Brian and Amy continued to save at the same rate until they retired. You will need to find the regression to determine their future yearly and total savings. Did the Brinkley's reach their goal of saving four million dollars and retire early or did they retire after working 30 years? You will need to determine how many years they worked and how much they saved toward their goal.

Questions

1. Using the calculator, copy image "Rising~1" to the calculator screen. Plot a point at x = 1 to represent the number of coins saved the first year. Plot the second point at x = 2 to represent the number of coins saved the second year. Continue this pattern for all five stacks of coins.

2. Determine and draw the quadratic regression equation on the calculator. What was the quadratic regression equation?

3. Use a table to list the y-value of coins saved each year for 30 years. Enter the values in the table, below:

YEAR	Y-VALUE	YEAR	Y-VALUE	YEAR	Y-VALUE
1		11		21	
2		12		22	
3		13		23	
4		14		24	
5		15		25	
6		16		26	
7		17		27	
8		18		28	
9		19		29	
10		20		30	

4. Convert the y-values to the number of coins saved each year by using ratios. What is the Y_2 equation used to represent this ratio conversion?

5. Enter the number of coins saved each year in the table below:

YEAR	COINS SAVED	YEAR	COINS SAVED	YEAR	COINS SAVED
1		11		21	
2		12		22	
3		13		23	
4		14		24	
5		15		25	
6		16		26	
7		17		27	
8		18		28	
9		19		29	
10		20		30	

6. Copy the Y_2 column of the table to a list. Use list 1.
Create a cumulative list of the number of coins saved in list 2. Write your answers for list 2.

YEAR	TOTAL COINS SAVED	YEAR	TOTAL COINS SAVED	YEAR	TOTAL COINS SAVED
1		11		21	
2		12		22	
3		13		23	
4		14		24	
5		15		25	
6		16		26	
7		17		27	
8		18		28	
9		19		29	
10		20		30	

7. Create a list of the cumulative value of the coins to list 3.
What equation did you use to determine the value of the coins for list 3?

8. Enter the values for list 3 into the following table.

YEAR	COINS SAVED	YEAR	COINS SAVED	YEAR	COINS SAVED
1		11		21	
2		12		22	
3		13		23	
4		14		24	
5		15		25	
6		16		26	
7		17		27	
8		18		28	
9		19		29	
10		20		30	

9. Did the Brinkley's earn 4 million dollars? If so, what year did they reach that goal?

10. How much do the Brinkley's earn in 30 years?

11. How many years do the Brinkley's work before retiring? Did they get to retire early?

Extensions

1. Explain why your answers do not exactly match the answers on this activity or the answers of other students in your class.

____

____

____

____

____

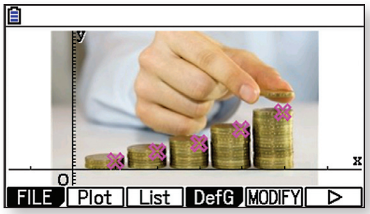
2. What is the regression equation if the Brinkley's would have saved 3 coins the first year, 5 coins the second, 7 coins the third, 10 coins the fourth and 16 coins the fifth year?

____

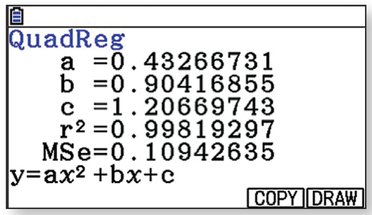
____

____

1.

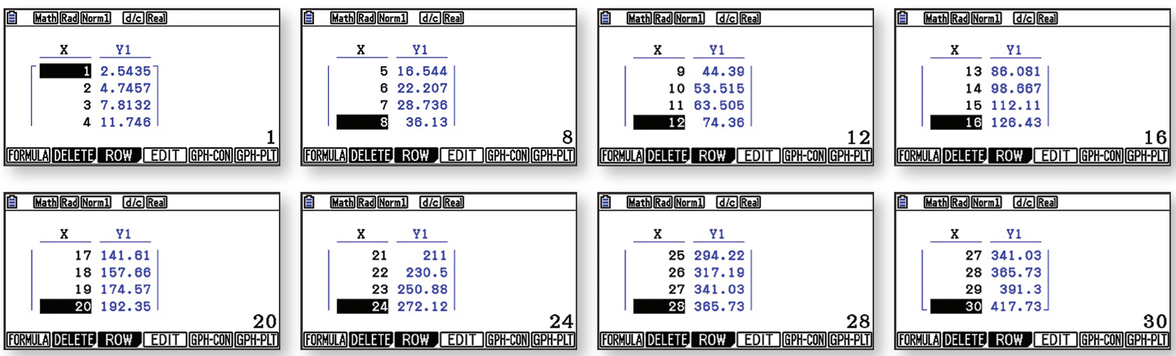


2.

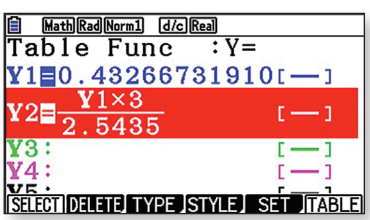


Answers may vary,
y = .43266731x² + 0.90416855x + 1.20669743

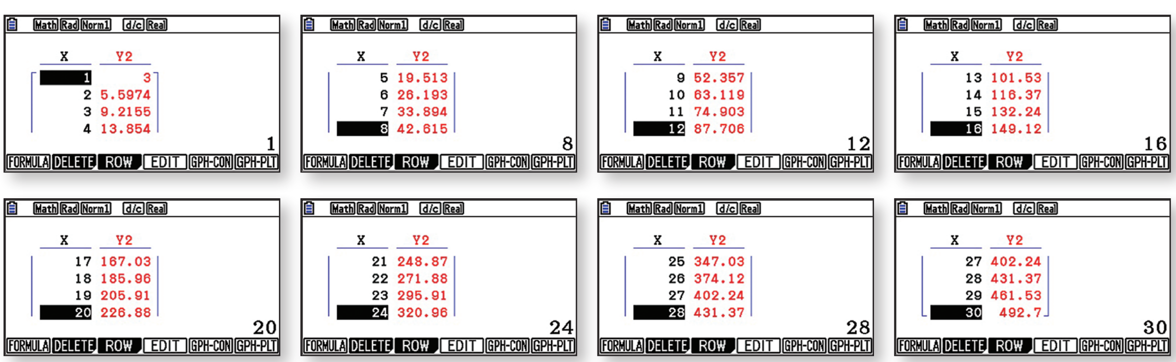
3. Answers may vary.



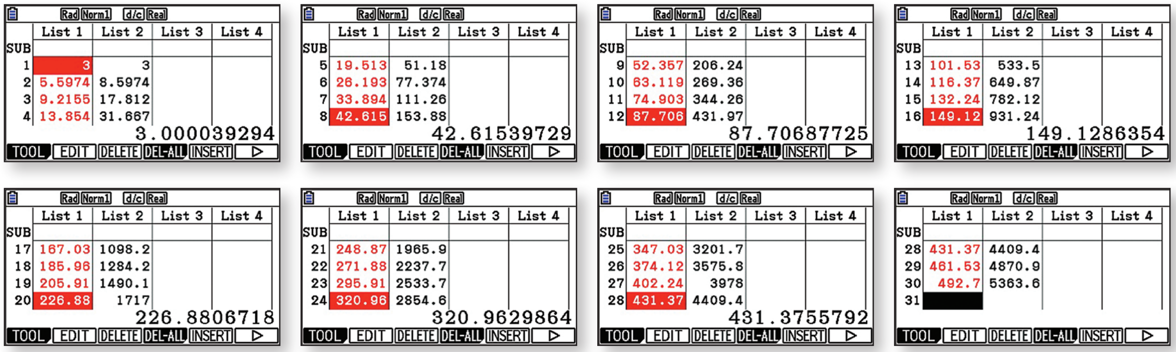
4. Answers may vary.



5. Answers may vary.

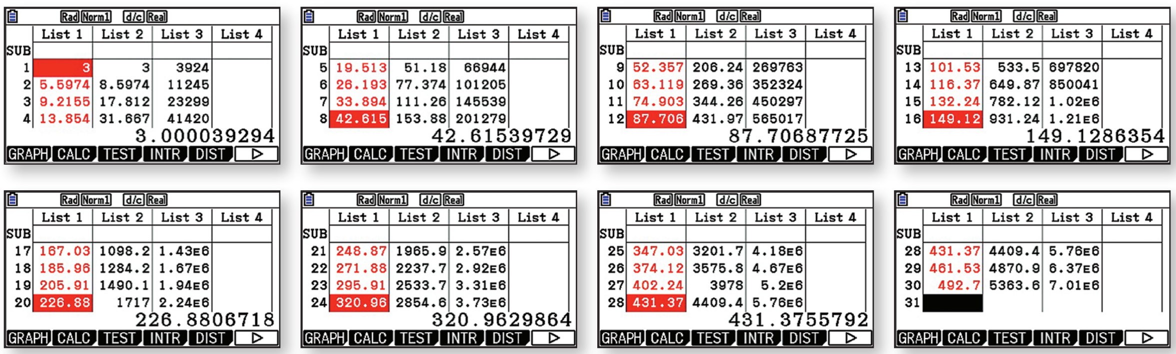


6. Answers may vary.



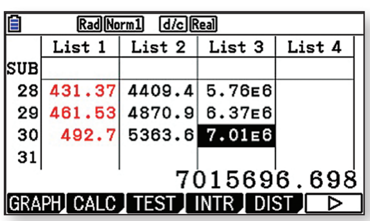
7. Answers may vary. List 2 x 1308 is stored to List 3.

8.



9. Yes. In the 24th year.

10. Answers may vary. $7,015,696.70

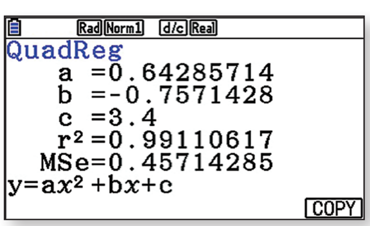


11. 24 years. Yes.

Extension Solutions

1. Because when plotting the points at the beginning of the activity, the points were approximated visually.

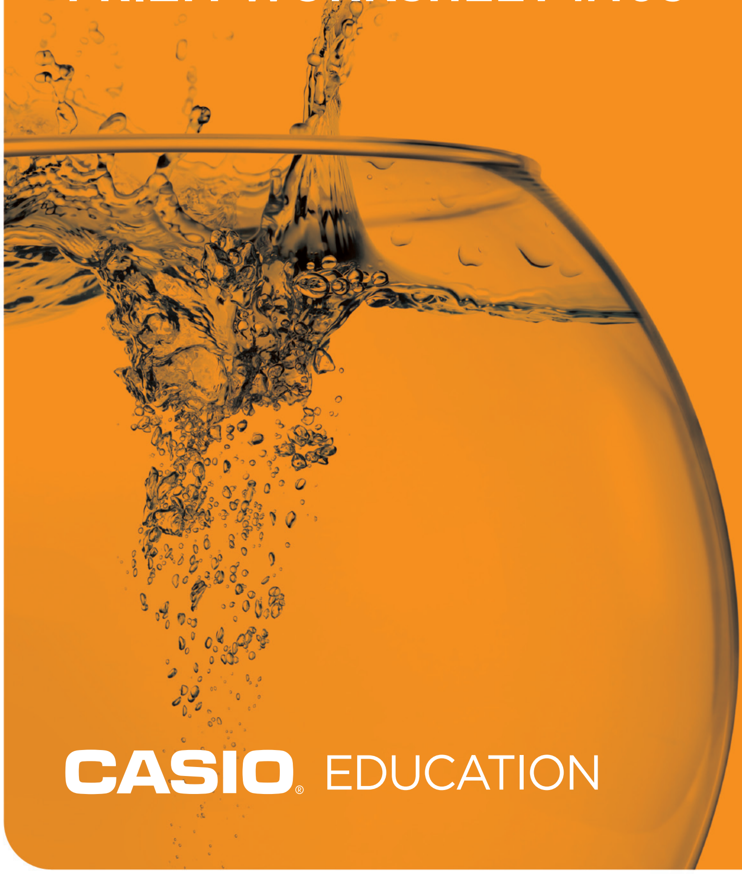
2. $y = 0.64285714x^2 - 0.7571428x + 3.4$



DISCOVER THROUGH PRIZM™

Flying Fish

PRIZM WORKSHEET #103



CASIO EDUCATION

TOPIC AREA:

Curve fitting quadratics to a .gb3 file

NCTM STANDARDS:

- Create and use representations to organize, record, and communicate mathematical ideas.
- Use Mathematical models to represent and understand quantitative relationships.

OBJECTIVE

Given a video file, students will be able to fit a quadratic to a goldfish jumping between bowls. Using their knowledge of quadratics, students will find and interrupt the meaning of the roots, maximum, and points that make up the curve.

GETTING STARTED

Have students work in pairs to help determine what points should be plotted and to facilitate discussion of the questions.

PRIOR TO USING THIS ACTIVITY:

- Students should have a basic understanding of regression and what it does.
- Students should understand the meaning of the roots of a parabola as they apply to real life situations.
- Students should be able to understand the meaning of the maximum of a parabola as it applies to a real life situation.
- Students should be able calculate an output value given an input value and calculate the x values given a y value, using quadratic formula.

WAYS STUDENTS CAN PROVIDE EVIDENCE OF LEARNING:

- Given a table of data, the student will be able to create an appropriate matrix to represent the data.
- Given two matrices, the student will be able to multiply the matrices and analyze the results.

COMMON MISTAKES TO BE ON THE LOOKOUT FOR:

- Students may be careless in the placement of points. Students should look for the center of the fish in every screen.
- Students may get confused when using X-cal and Y-cal; these are often interchanged by students.

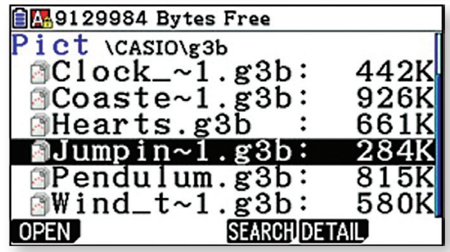
DEFINITIONS

- Quadratic
- Roots
- Maximum
- Regression

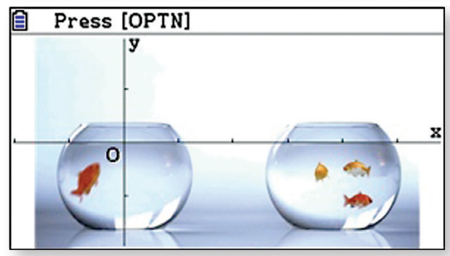
The following will walk you through the keystrokes and menus required to successfully complete the flying fish activity.

TO OPEN AN IMAGE SERIES FILE IN PICTURE PLOT:

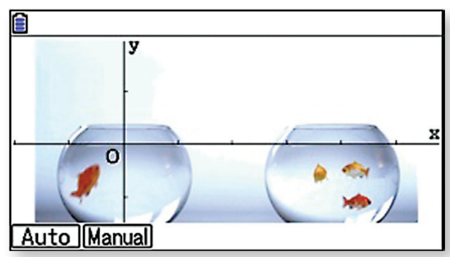
1. From the Main Menu, highlight the Picture Plot icon and press [EXE] or press [(].
2. Press [F1] (OPEN) to open the CASIO folder.
3. The g3b folder contains 8 image series files. Press [F1] (OPEN) to open the folder. Scroll down the list of image series and highlight the desired image series. You will be using the "Jumpin~1" image series in this activity. Press [F1] (OPEN).



4. To preview the image series, press [OPTN] [F6] (▷) [F6] (▷) [F2] (PLAY).
   The calculator gives you two options: [F1] (Auto) or [F2] (Manual). Auto plays through the entire sequence of images; Manual requires you to press ◀ or ▶ to move through each image.
5. Press [AC/ON] to stop the image series.

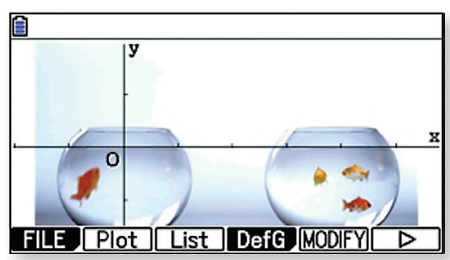


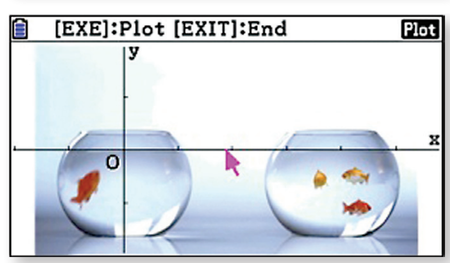
©VIZUEL—FOTOLIA.COM



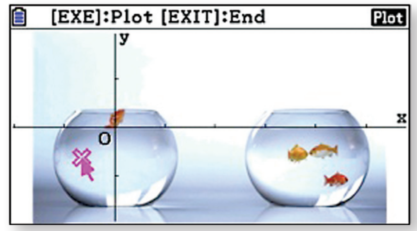
TO PLOT POINTS ON THE IMAGE SERIES AND CREATE A LIST OF POINTS:

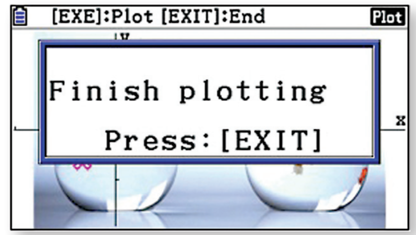
1. To plot points, press iw(PLOT). A pink arrow will appear; use ◀ ▶ ▲ ▼ to move the arrow to where you would like for it to plot a point. (Any of the number keys can also be used to jump to different areas on the screen). Press [EXE] to plot the point and advance the image series one frame.

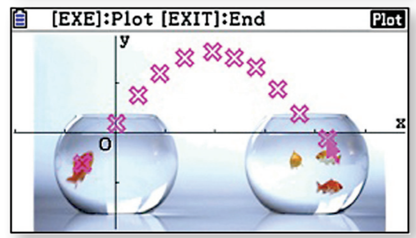




2. Continue moving the arrow and pressing EXE until the image series ends. To stop plotting before the end of the image series, press EXIT .

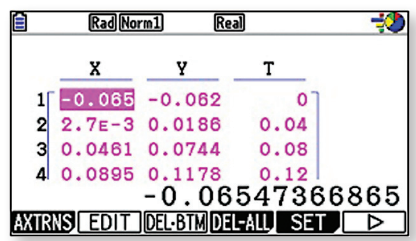






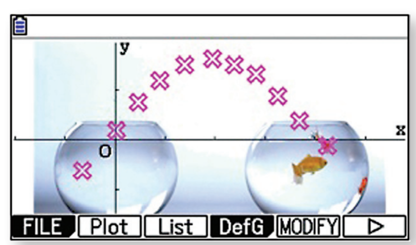
TO VIEW THE LIST OF DATA POINTS:

1. Press F3 (List) to view the list of points plotted. Press EXIT to go back to the picture and points

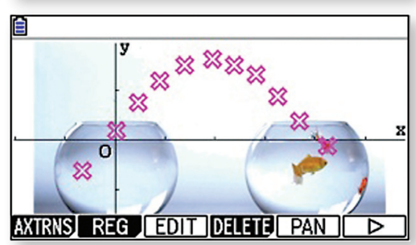


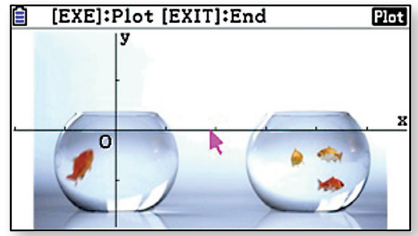
TO CREATE A BEST FIT LINE OR CURVE OF BEST FIT:

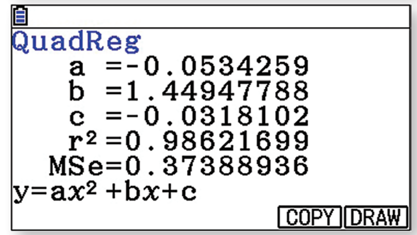
1. Press F6 (▷) F2 (REG).



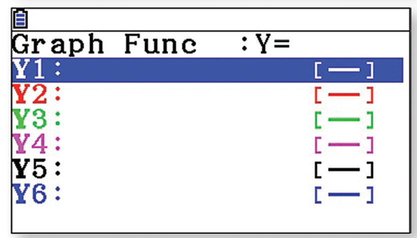
2. Choose the appropriate regression model. In this case, it will be X^2, since it is a parabolic Motion, so press F3 .



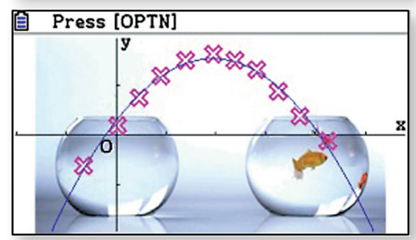


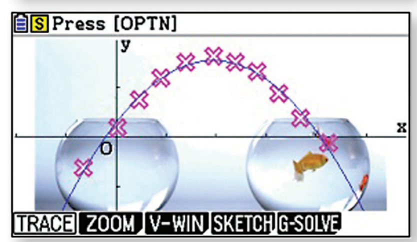


3. Press F5 (Copy) and EXE to copy the equation to the Graph menu.



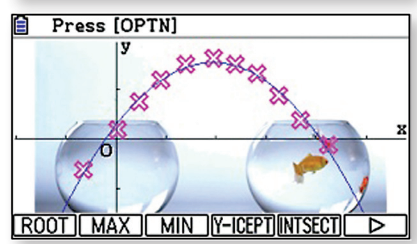
4. Press F6 (DRAW) to see the regression curve and the points.

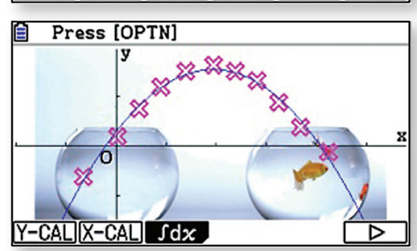




TO FIND ROOTS, MAX, AND X AND Y VALUES:

1. Press SHIFT F5 **(G-Solv)** to analyze features of the regression.
F1 (ROOT)- finds the root(s). The left-most root will always display first. Press ▶ to see the next root to the right.
F2 (MAX)- determines the maximum.
F6 (▷) F1 (Y-CAL)- finds a y-value given x.
F2 (X-CAL)- finds an x-value given y.





Questions

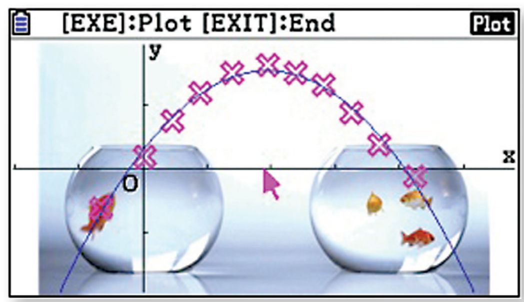
1. Play the image series and write down one question you would like to answer in this activity.

____

____

2. Plot points on the image series and write down the list of values below. Your picture should look similar to the one below.

HORIZONTAL	VERTICAL



3. Which data points represent the roots of the parabola created by the plot?

____

4. According to the data, what is the maximum height achieved by the fish?

____

5. What does the first data point represent?

6. Use the data set to find a best fit quadratic equation. What is the equation (round to the nearest hundredth)?

7. What are the roots of the quadratic equation? Think about the situation. Do the x-coordinates make sense? Explain.

8. What should they be? Why?

9. If the second bowl was placed on the floor 20 units down, how far from the x- axis should it be placed?

10. If you would like to train the fish to jump through hoops and you place the hoops at a height of 9 units up, how far along the x axis should they be placed?

11. Would the fish be able to jump over a bar placed at a height of 15 units? What is the highest the fish should be able to jump?

12. I want to place the second bowl at x = 100 how far down must it be placed?

13. The fish appears to be 6 units long. The world record high jump is 8 feet, by Javier Sotomayor Sanabria of Cuba; he is 6 foot 5 inches tall. Relative to height, who is jumping higher? Explain you answer.

14. By how much?

____

15. Who is a better long-jumper, the fish or Mike Powell, who is 6 foot 1 inch and jumped 29 feet 4.5 inches?

____

Extensions

1. Write the equation in vertex form and find the coordinates of the focus and axis of symmetry. What do these values represent in terms of the fish?

____

____

2. Show that the distance from any point to the focus is equal to the distance from that point to the directrix.

____

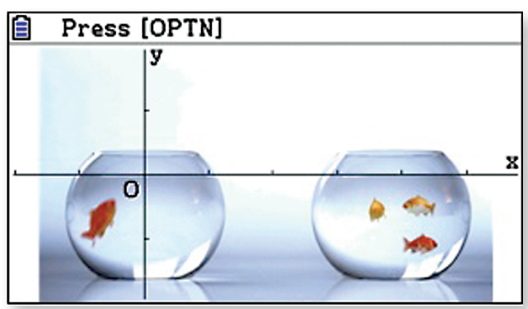
____

3. Find the equation of a line for a cat that snatches the fish out of midair at the maximum point, if the cat starts at the point (-12, -14).

____

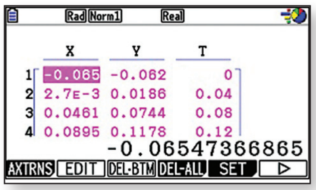
4. Find the equation of the hyperbola created by the inner curves of the bowl.

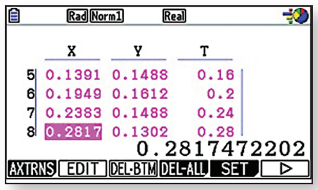
____

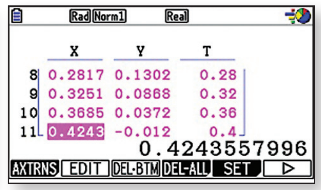


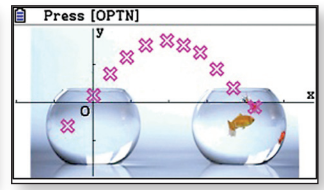
1. Answers will vary.
   Will the fish make it?
   How high does it go?
   How far does it jump?

2. Answers will vary, depending on points plotted. Hint: students should try to plot points through the thickest part of the fish in every frame (center of gravity).



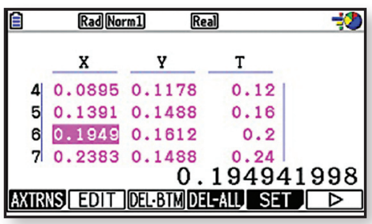






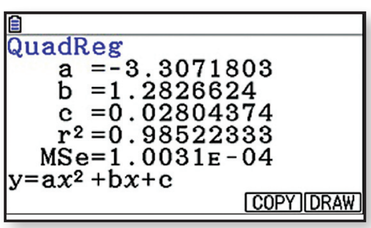
3. The second data point and the second to last data point. The fish is underwater on the first and last data point.

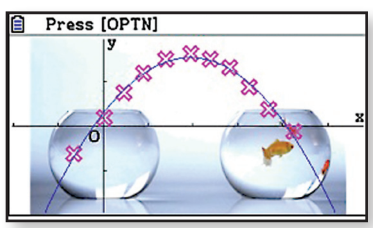
4. Answers will vary, depending on the plotted points. Approximately (14.857, 10.052)



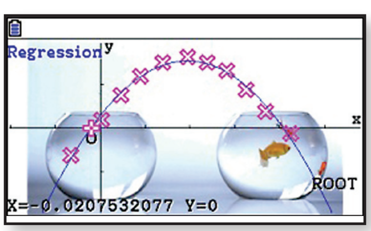
5. The initial take off point or launch point.

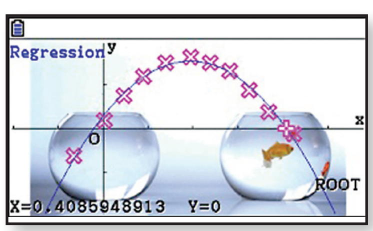
6. Answers will vary, depending on the points plotted.
   $y = -0.053x^2 + 1.45x - 0.3$





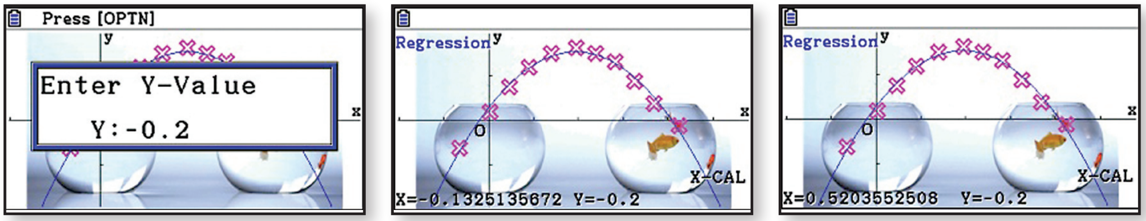
7. Roots (0.02, 0) and (27.11, 0).



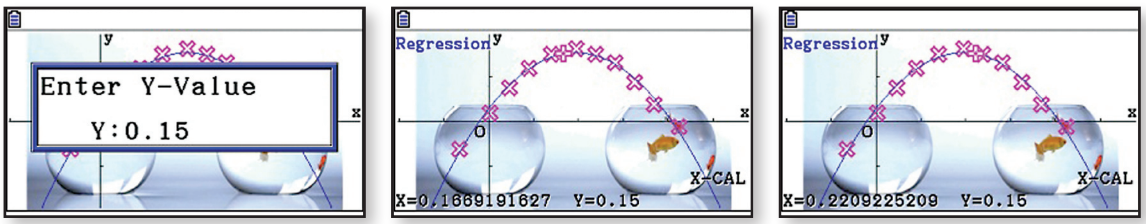


8. The first root should be (0, 0) the fish is jumping through the origin. The second root may be affected by the distortion of the fish landing in the water. The splash is behind the fish.

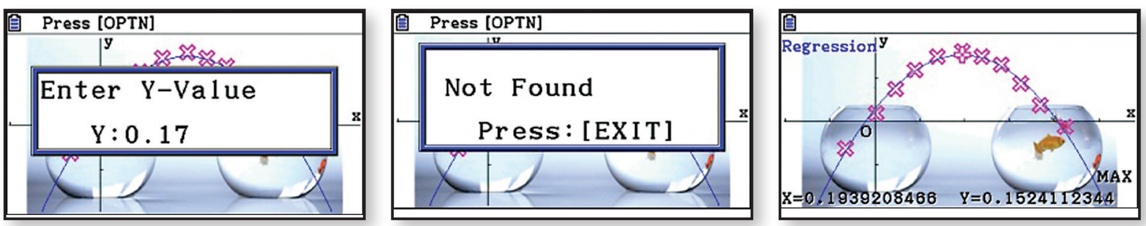
9. The two answers the calculator gives are (-10.05, -20) and (37.18, -20). The first answer does not make sense, so the second answer is the solution; 37.18 units over on the x-axis.



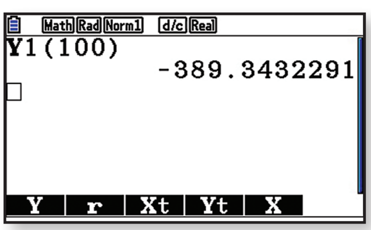
10. Placed the hoops at 9.70 and 17.43 units on the x-axis.



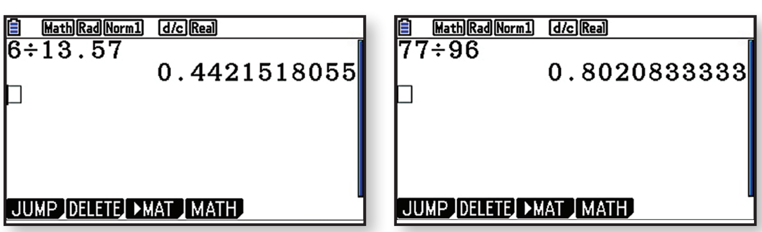
11. The fish will not make a jump of 15 units high. The calculator will give a "Not Found" message. The highest predicted jump is the maximum of the curve or 13.57.



12. Students will need to plug in a value of 100 by hand or change the maximum x value to be over 100 on the screen. From the RUN-MAT icon, enter [VARS] [F4] [F1] [1] [(] [1] [0] [0] [)] [EXE] to get a solution of -389.34 units.

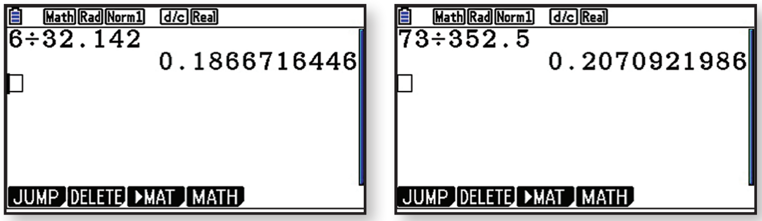


13. The height to jump ratio for the fish is 0.44 and the record height to jump ratio is 0.80. Javier, the record holder, is jumping higher.



14. Javier jumped 0.36 times as far as the fish.

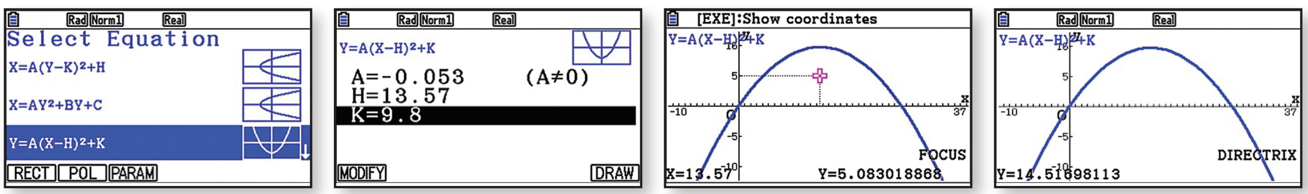
15. The fish jumped a distance of 32.142 units, so the height to jump ratio is 0.19. The world record ratio is 0.21; so Mike is a better long jumper.

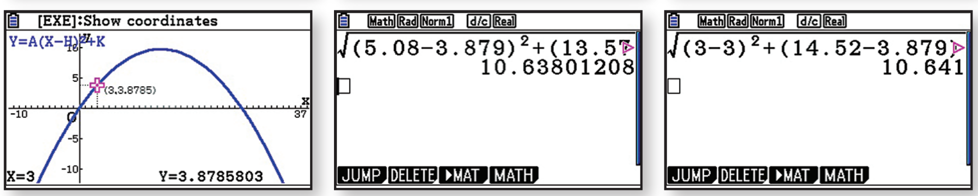


Extension Solutions

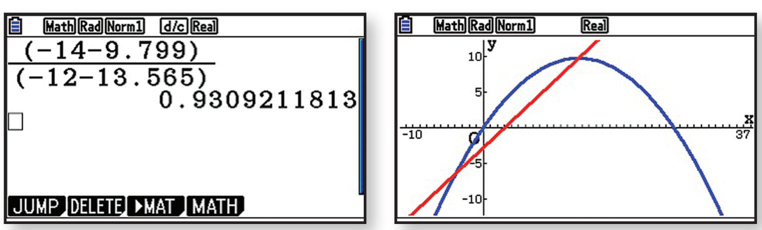
1. Answers will vary, depending on points plotted. If we use $y = -0.053x^2$ ◎ $1.45x$ ◎ 0.3, the answer should be $y = -0.053(x - 13.57)^2 + 9.80$.

2. The focus is at (13.57, 5.08), the directrix is at y = 14.52, a point on the curve is (3, 3.879). Using the distance formula, the distances to the focus is 10.638 and the distance to the directrix is 10.641.

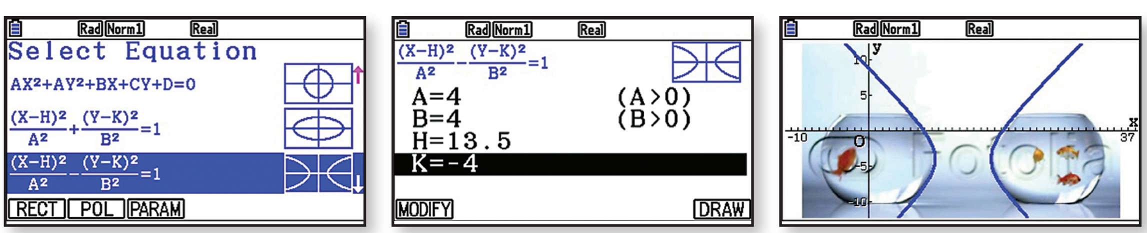




3. The maximum point is (13.565, 9.799). Using the slope formula, the slope of the line between the cat's starting point and the maximum point is 0.931. In point-slope form, the equation would be y + 14 = 0.931(x + 12). In slope-intercept form, the equation would be y = 0.931x – 2.828.



4. $\frac{(x - 14)^2}{16} - \frac{(x + 4)^2}{16} = 1$



DISCOVER THROUGH PRIZM™

Don't Block My View!

PRIZM WORKSHEET #104

CASIO EDUCATION

TOPIC AREAS: **Ratios and Conics**

NCTM STANDARDS:

- Apply appropriate techniques, tools and formulas to determine measurement.
- Use visualization, spatial reasoning, and geometric modeling to solve problems.

OBJECTIVE

Given a picture of a window and one measurement, the student will be able to use ratios to determine the dimensions of a new window design.

GETTING STARTED

Using the Casio Prizm, have students work in pairs or small groups to investigate graphically the distance between two points and to use ratios to determine the actual dimensions from the scale dimensions. Emphasis will also be on changing a semicircle to a semi-ellipse.

PRIOR TO USING THIS ACTIVITY:

- Students should be able to plot points and determine their coordinates.
- Students should have a basic understanding of ratios and proportions.
- Students should be able to determine distance between two points.
- Students should have a basic understanding of circles and ellipses.

WAYS STUDENTS CAN PROVIDE EVIDENCE OF LEARNING:

- If given a picture, students can plot points and find the distance between the two points
- If given a scale drawing, students can convert the scale dimensions of the drawing to actual dimensions.
- If given a semicircle and dimensions, students can determine a formula for an ellipse.

COMMON MISTAKES TO BE ON THE LOOKOUT FOR:

- Students may not have the correct points when finding distance.
- Students may not be using the correct formula for an ellipse.

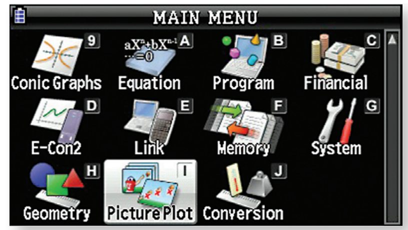
DEFINITIONS

- Center
- Circle
- Dimensions
- Distance
- Ellipse
- Major axis
- Minor axis
- Proportion
- Ratio
- Scale

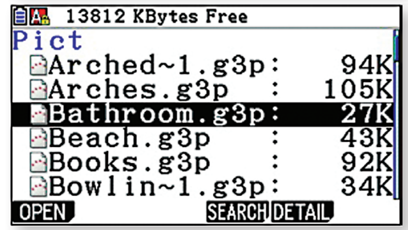
HOW TO **Don't Block My View!**

The following procedures will demonstrate how to retrieve an image, plot points on the picture, and use the equation solver to answer the desired questions.

TO OPEN A BACKGROUND IMAGE IN PICTURE PLOT:

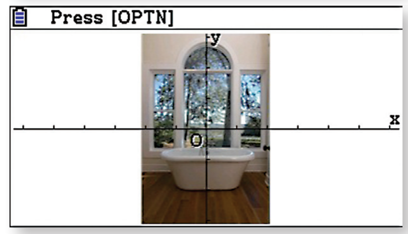


1. From the Main Menu, highlight the Picture Plot icon and press [EXE] or press [(].



2. Press [F1] (OPEN) to open the CASIO folder.

3. The g3p folder contains 47 background images. Press [▼] [F1] (OPEN) to open the folder. Scroll down the list of images and highlight the desired image. You will be using the "Bathroom" image in this activity. Press [F1] (OPEN).



TO PLOT POINTS ON THE IMAGE:

1. Press [OPTN] and choose [F2] (PLOT).

2. Move the Arrow to the desired position using [◀] [▶] [▲] [▼] and press [EXE]. In this activity, move the arrow to the upper left corner of the left window and press [EXE].

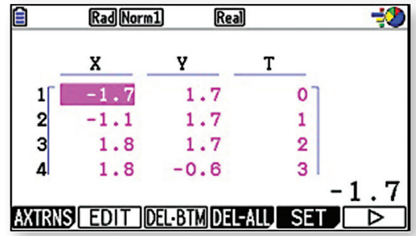
3. Continue to move the arrow and press [EXE] until you have all the points you want.

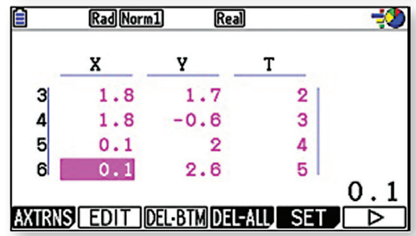
4. To stop plotting, press [EXIT].

TO FIND THE COORDINATES OF EACH POINT:

1. Press [F3] (List).

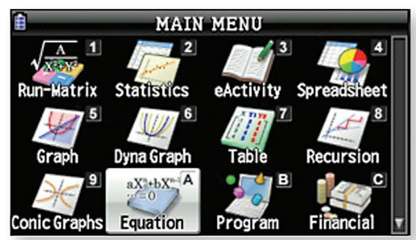
2. The X-column is the x-coordinate of the point, the Y-column is the y-coordinate of the point. The points in this list are in the order that they were plotted on the calculator. Scroll down to see the coordinates of all the points that were plotted.



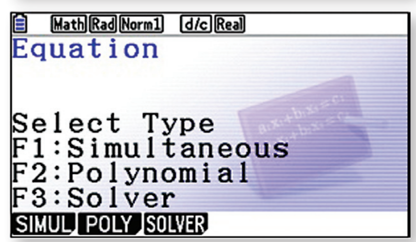


TO FIND THE DISTANCE BETWEEN TWO POINTS:

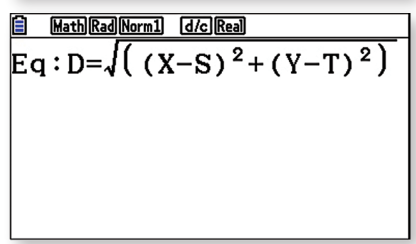
1. From the Main Menu, highlight the Equation icon and press [EXE] or press [X,θ,T].



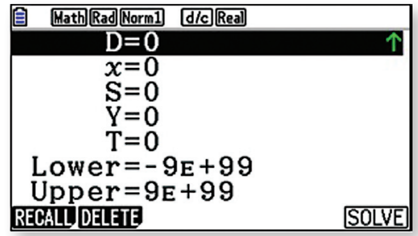
2. Choose [F3] (SOLVER).



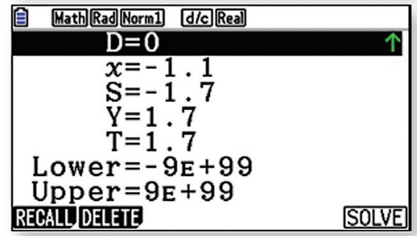
3. The distance formula between two points is $D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. However, you cannot enter subscripts into the calculator, therefore, enter the formula as $D = \sqrt{(X - S)^2 + (Y - T)^2}$; where D = distance between the two points; X = x-coordinate of the second point; S = x-coordinate of the first point; Y = y-coordinate of the second point; and T = y-coordinate of the first point.
   Press [ALPHA] [sin] [SHIFT] [·] [SHIFT] [x^2] [(] [ALPHA] [+] [ALPHA] [×] [)] [x^2] [+] [(] [ALPHA] [−] [−] [ALPHA] [÷] [)] [x^2] [EXE] [·]



4. Fill in D= 0 (even though you are looking for this number). Use point A as having coordinates (S, T) and point B as having coordinates (X, Y). Press [EXE] after each entry.

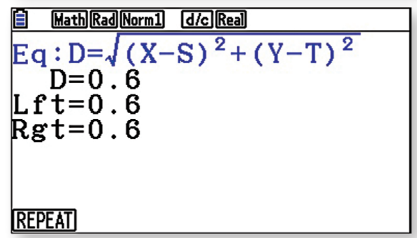


5. When finished entering the data, highlight the variable you are finding, in this case, highlight D = 0 and press [F6] (SOLVE).



6. The distance between point A and point B is 0.6 units.

7. To determine the distance between point A and point C, press [F1] (REPEAT). This allows you to just enter the new coordinates and use the formula you previously entered. Repeat this procedure for all points.

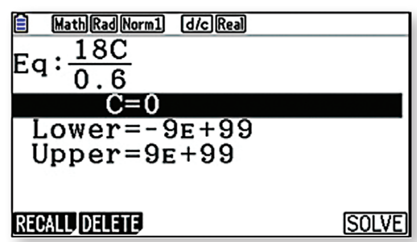


TO FIND THE ACTUAL DISTANCE BETWEEN TWO POINTS:

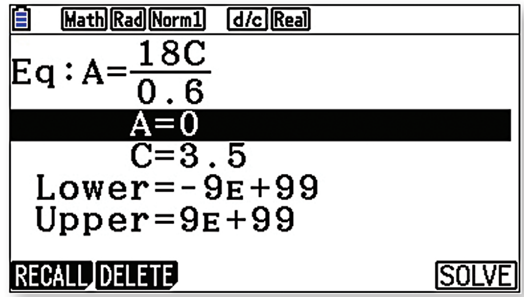
1. Use ratios and proportions to convert the calculator distance to actual distance. Since the actual distance from point A to point B is 18 inches and the calculator distance is 0.6 units, the proportion you will use is:

$$\frac{0.6}{18} = \frac{\text{calculator distance (C)}}{\text{actual distance (A)}}$$

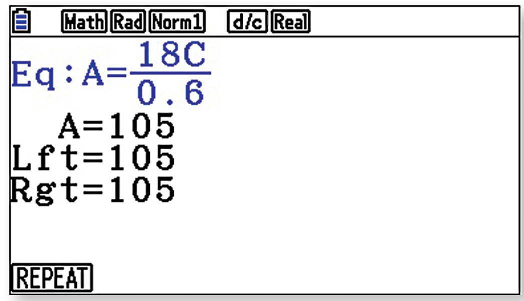
2. Use the Equation Solver as you did in the last Section, now solving for A in the proportion. Enter the new formula, $A = \frac{18C}{0.6}$, by pressing [ALPHA] [X,θ,T] [SHIFT] [•] [1] [8] [ALPHA] [ln] [a b/c] [0] [•] [6] [EXE].



3. Fill in the calculator distance for each C, highlight A and press F6 (SOLVE).

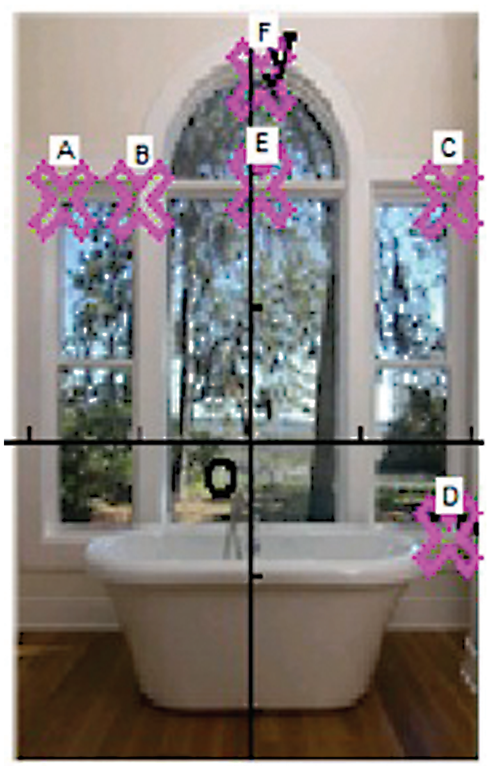


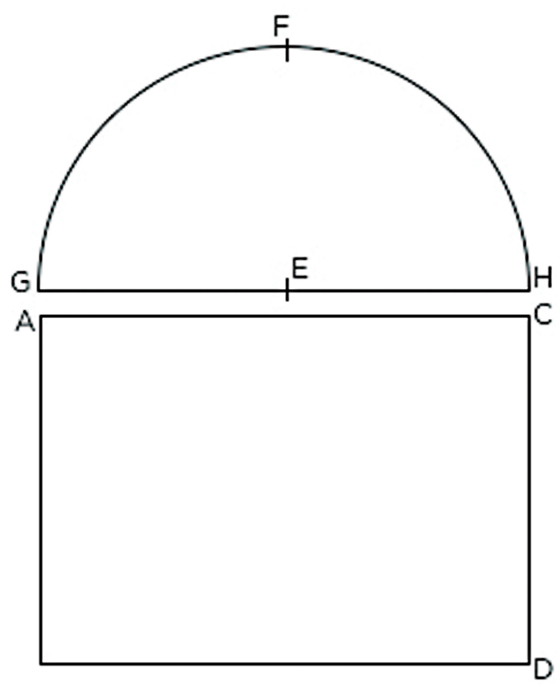
4. Press F1 (REPEAT) to solve additional problems.



The owner of this house found a major fault to this window design for the bathroom. When relaxing in the tub, the view is blocked by the inside border of the window. The owner wants to remove the window on the left and install a window with the design below right. The only actual measurement is the distance from point A to point B which is 18 inches. Point A and point B are the top corners of the glass of the present window. All points are to mark the edge of the glass, not the frame. The radius of the semicircle, from point E to point F, will be the same as half the minor axis for the semi-ellipse in the new design. Point E is not collinear with points A and C. The distance from A to C on the present design will be the same as the distance from A to C on the new design. The distance from point C to point D is also the same on both designs.

In this activity, you will need to find the actual dimensions of the rectangular section of the new design and the area. You will also need to find the coordinates of point G which is the left end of the major axis of the semi-ellipse and point H which is the right end of the major axis. You will need to determine the standard form of the equation of the ellipse, where point E is the center. Also, you will need to find the area of both the ellipse that is used and the semi-ellipse that is in the design of the new window. Finally, the total area of the glass needed for the new window.





Questions

1. Using the Casio Prizm, copy image "Bathroom" to the calculator screen. All the points that you will be graphing should be placed at the edge of the glass of the window, not in the frame, as shown in the previous drawing. Plot point A as the top left corner of the small window. Point B is the top right corner of the small window. Point C is the top right corner of the other small window. Point D is the bottom right corner of the right window. Point E is the center of the semicircle and point F is the highest point of that window. Plot all these point on the Prizm.

2. List the coordinates of points A, B, C and D.

POINT	COORDINATE
A	
B	
C	
D	

3. Find the distance between the following points:

POINTS	DISTANCE
A TO B	
A TO C	
C TO D	

4. Since the actual distance from points A to B is 18 inches, find the other actual dimensions of the rectangular section of the new window.

Length AC = ____________________

Height CD = ____________________

5. What is the area of the new rectangular window?

____

6. Determine the coordinates of the following points:

POINT	COORDINATE
E	
F	
G	
H	

7. Find the distance between the following points:

POINTS	DISTANCE
E TO F	
E TO G	
G TO H	

8. What is the length of the minor axis of the ellipse that is used?

____

9. What is the length of the major axis of the ellipse that is used?

____

10. Using the standard form for the equation of the ellipse, where E is the center of the ellipse at coordinates (h, k). The distance from E to G is a. The distance from E to F is b. Standard form is:

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

Find the standard form of this ellipse.

11. What is the actual length of a for the new window?

____

12. What is the actual length of b for the new window?

____

13. Using the formula for the area of an ellipse: $A = \pi ab$, determine the area of the ellipse.

____

14. What is the area of the semi-ecllipse?

____

15. What is the area of the total glass for both sections of the new window?

____

Extensions

1. Why will your answers not match those of your partner or other students in your class?

____

____

____

2. What would be the area of the new window if the upper window remains a semicircle, with the radius the length of EG?

____

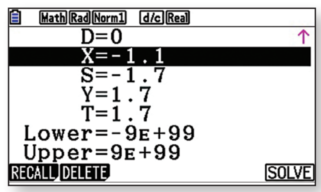
1.

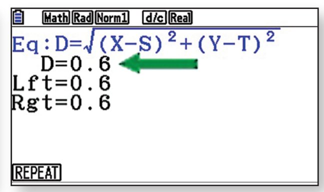
2.

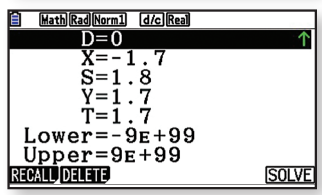
POINT	COORDINATE
A	(–1.7, 1.7)
B	(–1.1, 1.7)
C	(–1.8, 1.7)
D	(–1.8, –0.6)

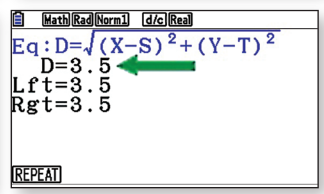
3.

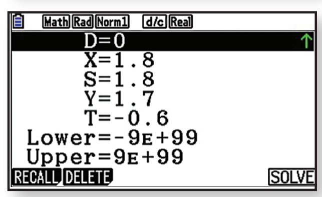
POINT	DISTANCE
A TO B	0.6
A TO C	3.5
C TO D	2.3

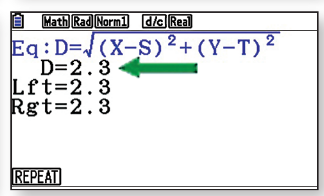




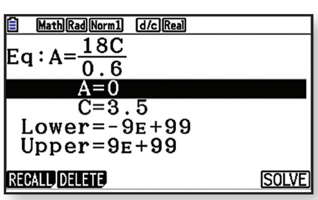


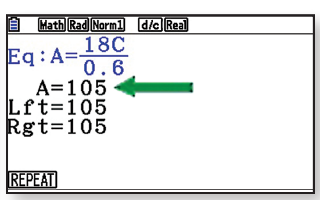


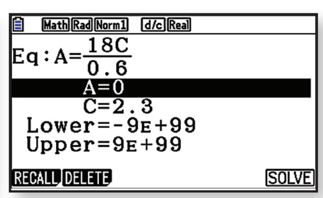


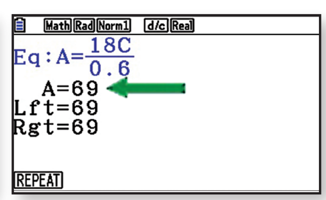


4. 105 inches; 69 inches

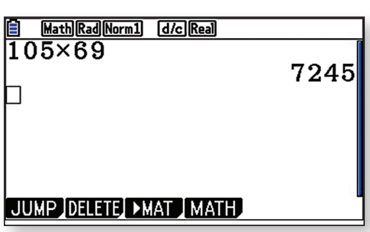








5. (105 in)(69 in) = 7,245 inches^2

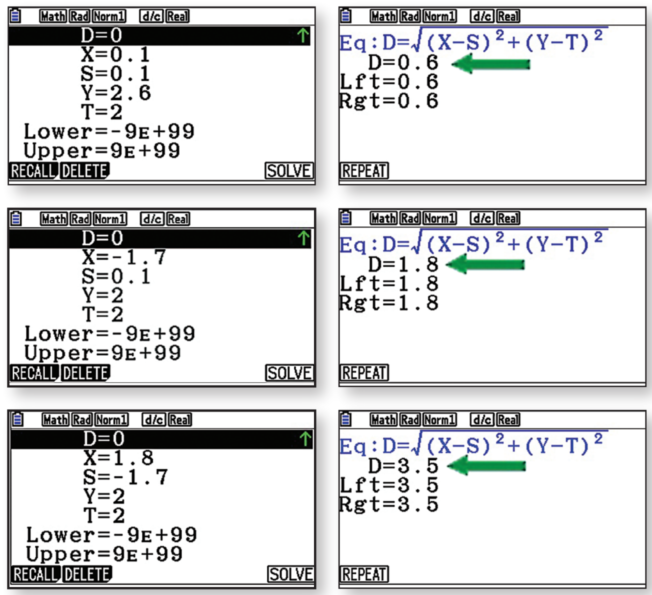


6.

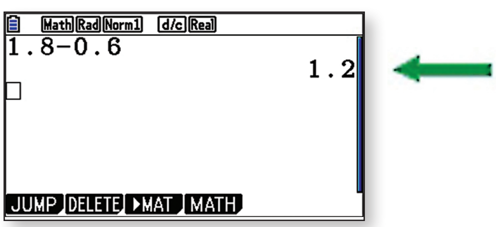
POINT	COORDINATE
A	(0.1, 2)
B	(0.1, 2.6)
C	(-1.7, 2)
D	(1.8, 2)

7.

POINTS	DISTANCE
E TO F	0.6
E TO G	1.8
G TO H	3.5



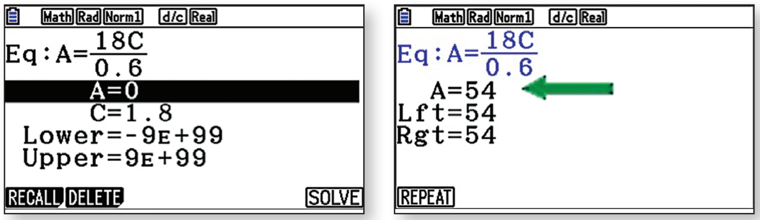
8. 1.8 – 0.6 = 1.2



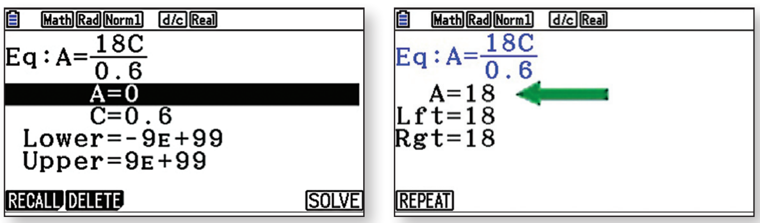
9. 3.5

10. $\frac{(x - 0.1)^2}{3.24} + \frac{(y - 2)^2}{0.36} = 1$

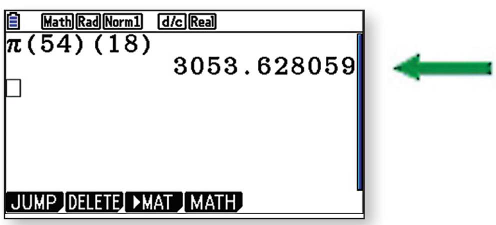
11. 54 inches



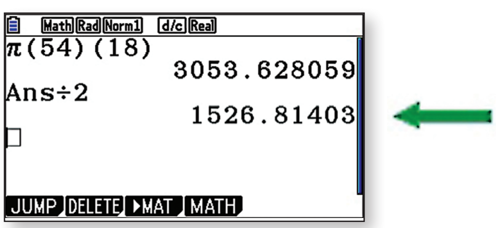
12. 18 inches



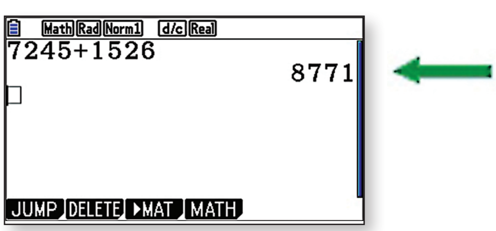
13. A = π(54)(18) = 3,054 inches^2



14. $\frac{3{,}054}{2}$ = 1526 inches^2



15. 7,245 + 1,526 = 8,771 inches^2



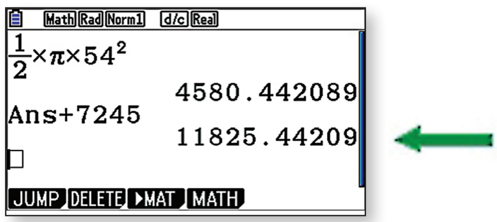
Extension Solutions

1. The calculator points are approximated.

2. 11,823 inches^{2w}

Area of semicircle = $\frac{1}{2}\pi r^2 = \frac{1}{2}\pi(54)^2$ = 4,580 inches^2

Total area = 7,245 + 4,580 = 11,825 inches^2



DISCOVER THROUGH PRIZM™

Area of Similar Figures

PRIZM WORKSHEET #105

CASIO EDUCATION

TOPIC AREA: Ratios of Similar Figures

NCTM STANDARDS:

- Use two-dimensional representations of three-dimensional objects to visualize and solve problems such as those involving surface area and volume.
- Use Cartesian coordinates and other coordinate systems, such as navigational, polar, or spherical systems, to analyze geometric situations.
- Investigate conjectures and solve problems involving two- and three-dimensional objects represented with Cartesian coordinates.

OBJECTIVES

Given a photo file, students will be able to find the coordinates for the endpoints of a radius for several circular objects. Using their knowledge of coordinate geometry and proportions, students will compare the ratio of the circumference and area to the radius for several circles.

GETTING STARTED

Have students work in pairs to determine the coordinates of the endpoints of the radius for several circles and use these coordinates to calculate the length of the radius. Using the radius, students will calculate the circumference and area of the circles and determine the relationship between the ratio of the radii and the measures of the circumference and area.

PRIOR TO USING THIS ACTIVITY:

- Students should be able to calculate the distance between two points.
- Students should know the formulas for finding the circumference and area of a circle.
- Students should be able to write a ratio and proportion.

WAYS STUDENTS CAN PROVIDE EVIDENCE OF LEARNING:

- Students will be able to calculate the length of the radius using the coordinates of the endpoints.
- Students will be able to compare the ratios between measures and write a conjecture.

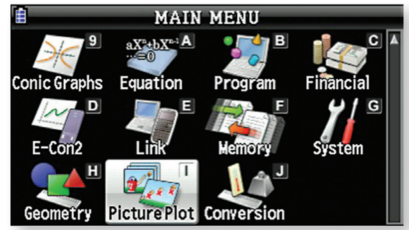
COMMON MISTAKES TO BE ON THE LOOKOUT FOR:

- Students may enter values into the formulas incorrectly.
- Students may be careless in choosing the endpoints of the radius.
- Students may write the proportions incorrectly in order to comparing the relationships.

DEFINITIONS:

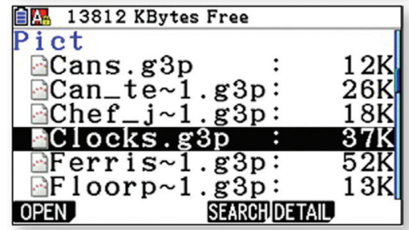
- Radius
- Circumference
- Area

The following will walk you through the keystrokes and menus required to successfully complete the area of similar figures activity.



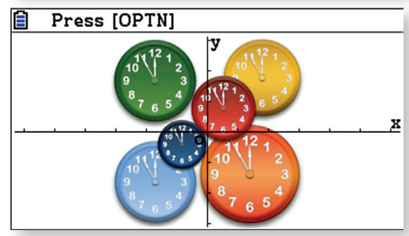
TO OPEN A BACKGROUND IMAGE IN PICTURE PLOT:

1. From the Main Menu, highlight the Picture Plot icon and press EXE or press (.

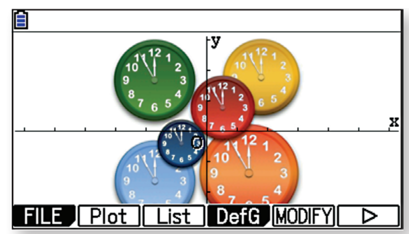


2. Press F1 (OPEN) to open the CASIO folder.

3. The g3p folder contains 47 background images. Press ▽ F1 (OPEN) to open the folder. Scroll down the list of images and highlight the desired image. You will be using the "Clocks" image in this activity. Press F1 (OPEN).

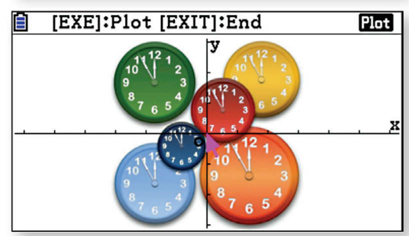


© FOTODDELLI - FOTOLIA.COM

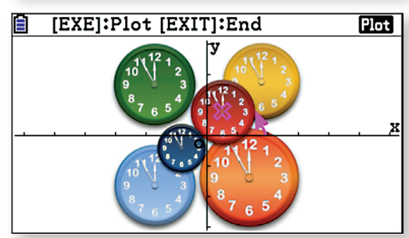


TO FIND THE COORDINATES OF THE ENDPOINTS FOR THE RADIUS:

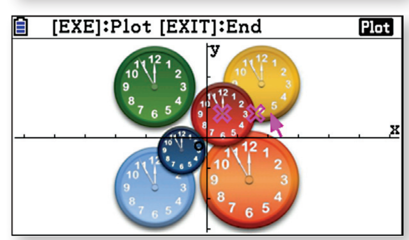
1. The status bar at the top of the screen prompts what buttons you have to choose from. For this image, you need to press OPTN .



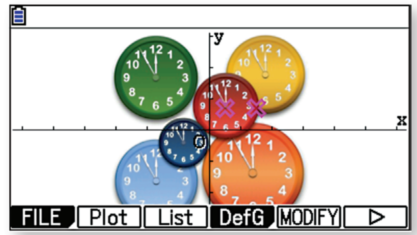
2. To plot points on the picture, press F2 (Plot). A pink arrow will appear. Use ◁ △ ▷ ▽ to move the arrow to the center of the red clock. (Any of the number keys can also be used to jump to different areas on the screen). Press EXE to plot the point on the image.



3. Use ◁ △ ▷ ▽ to move the arrow to the edge of the red clock and press EXE to plot the point on the image. You now have the endpoints of the radius for the red clock.

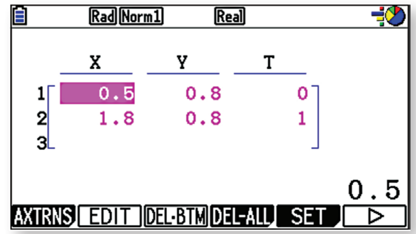


4. To stop plotting, press [EXIT].

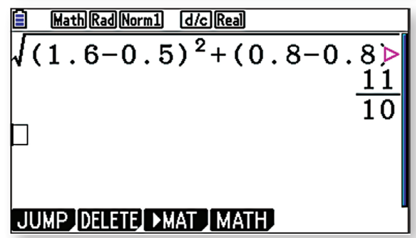


TO FIND THE LENGTH OF THE RADIUS:

1. To see the coordinates of the endpoints, press [F3] (List). Enter these coordinates into the table.

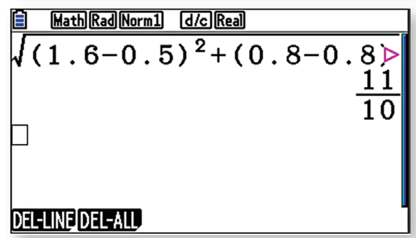


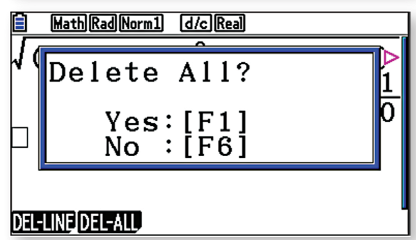
2. Press [MENU] and use ◀ ▲ ▶ ▼ to highlight the RUN-MAT icon or press [1]. Use the distance formula, $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, and the coordinates to calculate the length of the radius for the red clock.



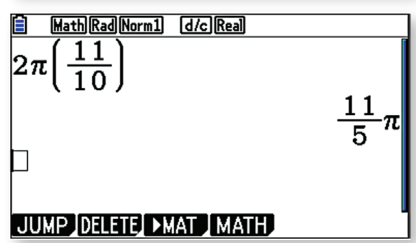
TO FIND THE CIRCUMFERENCE AND AREA OF THE RED CLOCK:

1. Press [F2] (DELETE) [F2] (DEL-ALL) [F1] (Yes) to clear the screen.

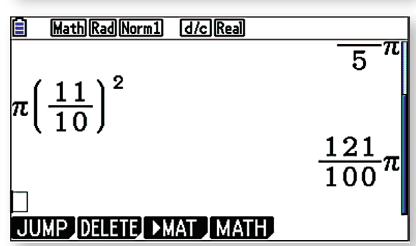




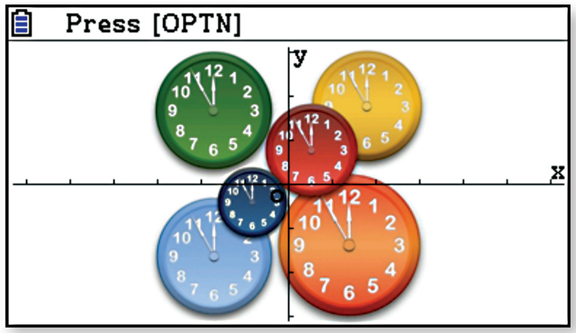
2. Using the formula C = 2πr, press [2] [SHIFT] [EXP] [COS] [a b/c] [1] [1] ▼ [1] [0] ▶ [)] [EXE] to calculate the circumference of the red clock. Enter the circumference into the table.



3. Using the formula A = 2πr, press [SHIFT] [EXP] [(] [a b/c] [1] [1] ▼ [1] [0] ▶ [)] [x²] [EXE] to calculate the area of the red clock. Enter the area into the table.



It seems logical that if one doubles the dimensions of an object then the area the object covers would also double; this is not the case. In this activity we will compare the perimeter and area of two similar objects when one dimension is altered.



Questions

1. Find the coordinates of the endpoints of the radius for each of the clock faces and enter this into the table. Use the distance formula to find the length of each radius.

COLOR	P_0	P_1	RADIUS
DARK BLUE			
RED			
GREEN			
GOLD			
ORANGE			

2. Find the circumference for each clock, using $C = 2\pi r$, and enter this information into the table.

COLOR	RADIUS	CIRCUMFERANCE
DARK BLUE		
RED		
GREEN		
GOLD		
ORANGE		

3. Divide the radius of the Red Clock by the radius of the Dark Blue Clock.
Divide the circumference of the Red Clock by the circumference of the Dark Blue Clock.
What do you notice?

____

____

4. Divide the radius of the Gold Clock by the radius of the Orange Clock.
Divide the circumference of the Gold Clock by the circumference of the Orange Clock.
What do you notice?

____

____

5. Does this hold true for another pair of clocks?

____

6. Write a conjecture about the ratio of the radii of circles and the ratio of the corresponding circumferences.

____

____

7. Find the area for each clock using $A = 2\pi r2$ and enter this information into the table.

COLOR	RADIUS	AREA
DARK BLUE		
RED		
GREEN		
GOLD		
ORANGE		

8. Divide the radius of the Green Clock by the radius of the Red Clock. Divide the area of the Green Clock by the area of the Red Clock. What do you notice?

____

____

9. Divide the radius of the Green Clock by the radius of the Gold Clock.
Divide the area of the Green Clock by the area of the Gold Clock. What do you notice?

____

____

10. Does this hold true for another pair of clocks?

____

11. Write a conjecture about the ratio of the radii of circles and the ratio of the corresponding areas.

____

____

Extension

1. Make a table for several squares using the side and perimeter. Does your conjecture for the circumference hold true for the perimeter of squares as well?

____

2. Make a table for several equilateral triangles using the side and area. Does your conjecture hold true for the area of equilateral triangles as well?

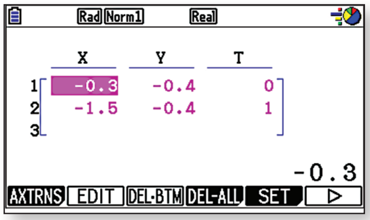
____

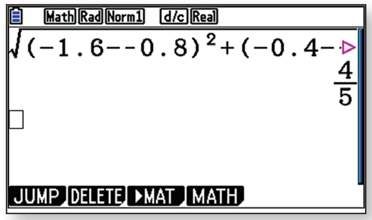
3. Since the ratio of length is equal to each other and the ratio of area is the square of the lengths, what do you think would happen with length and volume of regular figures?

____

____

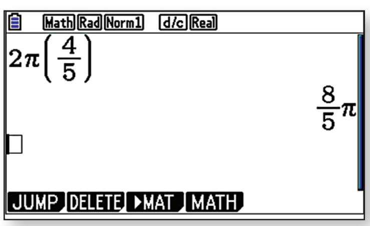
1. Answers may vary according to the plotted points.
[Screen Shots for Dark Blue Clock]





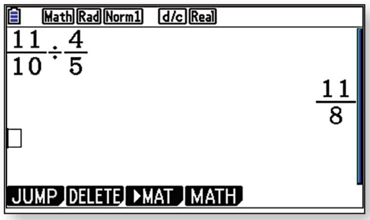
COLOR	P_0	P_1	RADIUS
DARK BLUE	(−0.8, −0.4)	(−1.6, −0.4)	$\frac{4}{5}$
RED	(0.5, 0.8)	(1.6, 0.8)	$\frac{11}{10}$
GREEN	(−1.7, 1.7)	(−1.7, 0.4)	$\frac{13}{10}$
GOLD	(1.8, 1.8)	(1.8, 3)	$\frac{6}{5}$
ORANGE	(1.4, −1.4)	(3, −1.4)	$\frac{8}{5}$

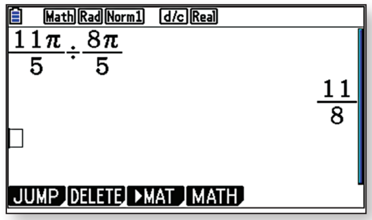
2. Screen Shot for Dark Blue Clock



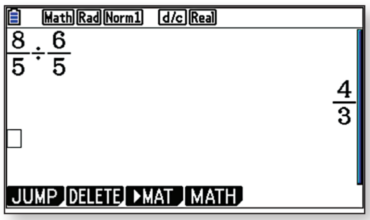
COLOR	RADIUS	CIRCUMFERENCE
DARK BLUE	$\frac{4}{5}$	$\frac{8}{5}\pi$
RED	$\frac{11}{10}$	$\frac{11}{5}\pi$
GREEN	$\frac{13}{10}$	$\frac{13}{5}\pi$
GOLD	$\frac{6}{5}$	$\frac{12}{5}\pi$
ORANGE	$\frac{8}{5}$	$\frac{16}{5}\pi$

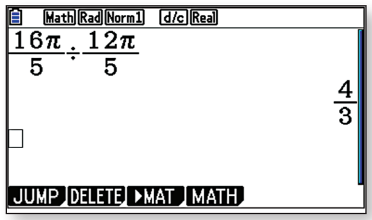
3. The ratios are equal.





4. The ratios are equal.

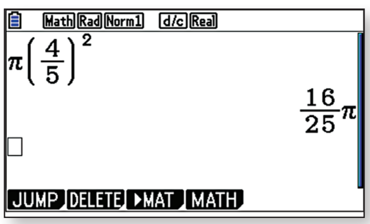




5. Yes

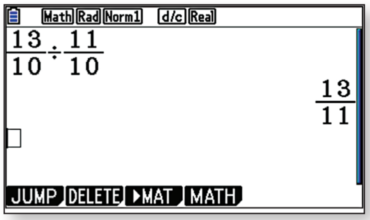
6. The ratio of the circumferences of two similar circles is equal to the ratio of their radii.

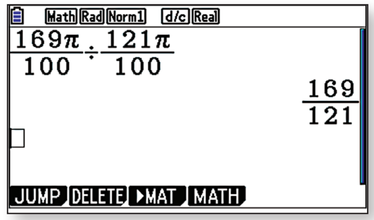
7. Screen Shot for Dark Blue Clock



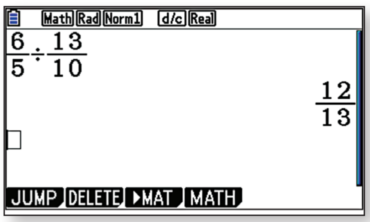
COLOR	RADIUS	CIRCUMFERENCE
DARK BLUE	$\frac{4}{5}$	$\frac{16}{25}\pi$
RED	$\frac{11}{10}$	$\frac{121}{100}\pi$
GREEN	$\frac{13}{10}$	$\frac{169}{100}\pi$
GOLD	$\frac{6}{5}$	$\frac{36}{25}\pi$
ORANGE	$\frac{8}{5}$	$\frac{64}{25}\pi$

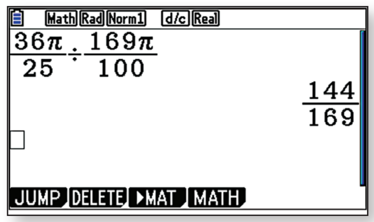
8. The ratio of the areas is the square of the ratio of the radii.





9. The ratio of the areas is the square of the ratio of the radii.





10. Yes

11. The ratio of the areas of two similar circles is equal to the square of the ratio of their radii.

Extension Solutions

1. The tables will vary. The conjecture holds true

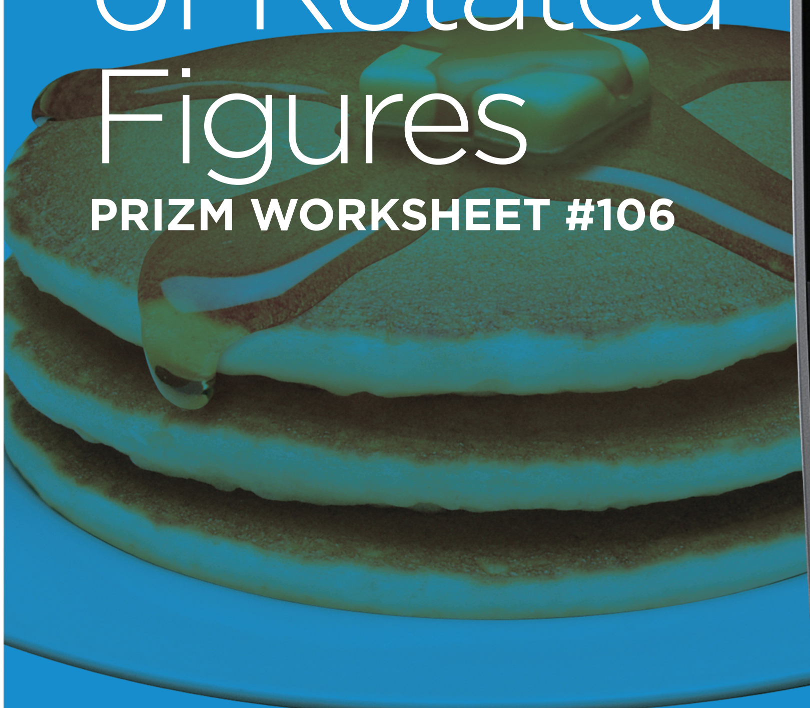
2. The tables will vary. The conjecture holds true.

3. Since the first is equal and the second is the square, the third would be the cube of the ratio of the lengths.

DISCOVER THROUGH PRIZM™

Volume of Rotated Figures

PRIZM WORKSHEET #106



CASIO® EDUCATION

TOPIC AREA:

Volume of Solids by Shell Method

NCTM STANDARDS:

- Use two-dimensional representations of three-dimensional objects to visualize and solve problems such as those involving surface area and volume;
- 100Use Cartesian coordinates and other coordinate systems, such as navigational, polar, or spherical systems, to analyze geometric situations;

OBJECTIVES

Given a photo file, students will be able to fit an equation onto the picture and find the equation for the line of best fit. Using their knowledge of polynomial functions and integral calculus, the students will find the volume of a figure formed by rotating a polynomial around a specified axis.

GETTING STARTED

Have students work in pairs to determine the coordinates of points used to find the line of best fit for a polynomial and use this equation to solve problems involving rotation of solids.

PRIOR TO USING THIS ACTIVITY:

- Students should have an understanding of how to find a regression line.
- Students should have an understanding of polynomial functions.
- Students should be able to write and solve an integral given the function with upper and lower bounds.

WAYS STUDENTS CAN PROVIDE EVIDENCE OF LEARNING:

- Students will be able to use their knowledge of geometry to determine upper and lower bounds for an integral.
- Students will be able to find the volume of a figure using the shell method.

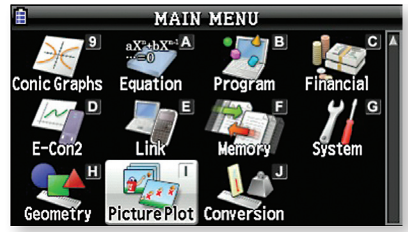
COMMON MISTAKES TO BE ON THE LOOKOUT FOR:

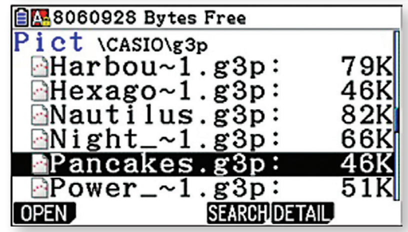
- Students may have difficulty determining upper and lower bounds.
- Students may have difficulty with the integration of a polynomial.
- Students may use the wrong polynomial for the integration.

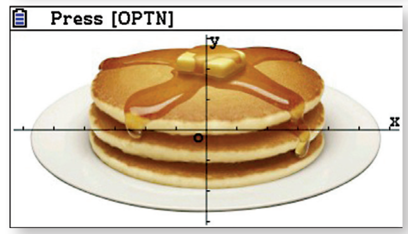
DEFINITIONS:

- Regression
- Upper and Lower Bounds
- Integral

The following will walk you through the keystrokes and menus required to successfully complete the volume of rotated figures activity.







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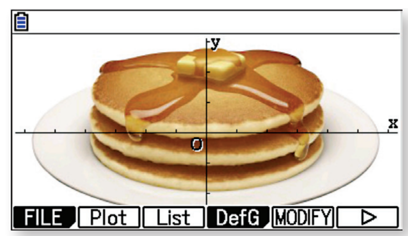
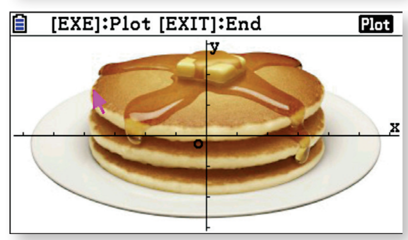
TO OPEN A BACKGROUND IMAGE IN PICTURE PLOT:

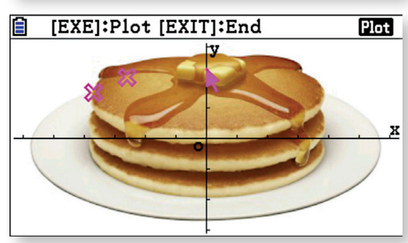
1. From the Main Menu, highlight the Picture Plot icon and press [EXE] or press [(].
2. Press [F1] (OPEN) to open the CASIO folder.
3. The g3p folder contains 47 background images. Press [▼] [F1] (OPEN) to open the folder. Scroll down the list of images and highlightthe desired image. You will be using the "Pancakes" in this activity. Press [F1] (OPEN).

TO PLOTS POINTS ON THE IMAGE AND CREATE A LIST OF POINTS:

1. The status bar at the top of the screen displays what operations you have to choose from; for this image, you will need to press [OPTN].
2. To plot points on the image, press [F2] (Plot). A pink arrow will appear. Use [◀] [▲] [▶] [▼] to move the arrow where you would like to plot a point. (Any number of keys can also be used to jump to different areas on the screen). Press [EXE] to plot a point on the picture.

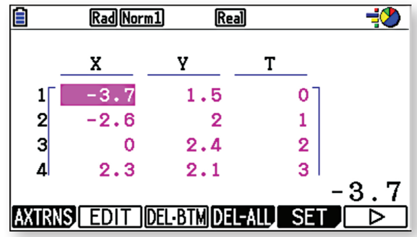


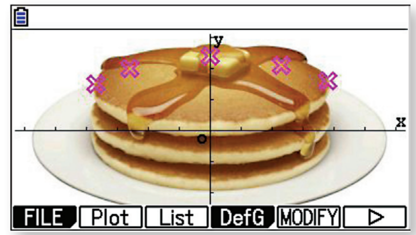




TO VIEW THE LIST OF DATA POINTS YOU SELECTED:

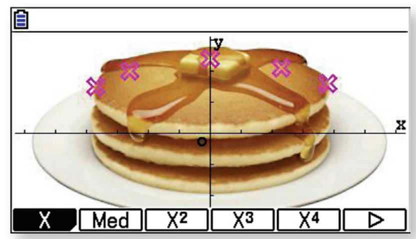
1. Press [OPTN] [F3] to view the list of points plotted. Press [EXIT] to go back to the image and points.



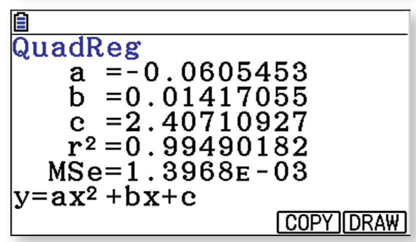


TO CREATE A LINE OF BEST FIT:

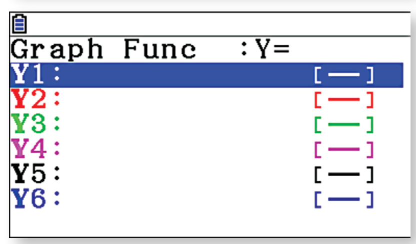
1. Press [F6] (▷) and [F2] (REG).



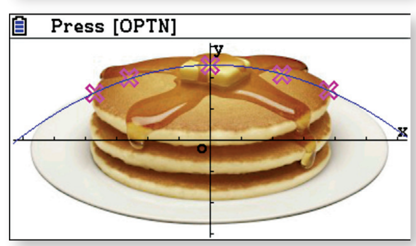
2. In this case the regression line is a quadratic, therefore press [F3] (X^2).



3. Press [F5] (Copy) and [EXE] to copy the equation into the Graph Menu, then [EXIT].



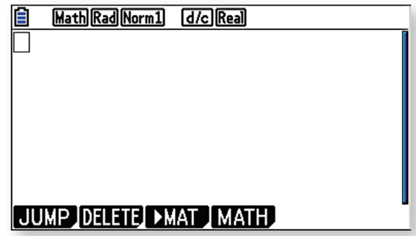
4. Press [F6] (DRAW) to see the regression line generated by your selected points.



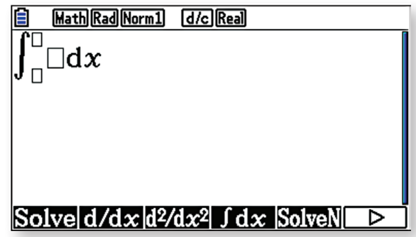
HOW TO **Volume of Rotated Figures**

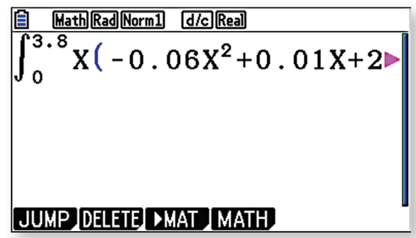
TO FIND THE VOLUME OF THE PANCAKES BY THE SHELL METHOD:

1. Press MENU, highlight the RUN•MATRIX icon and press EXE.

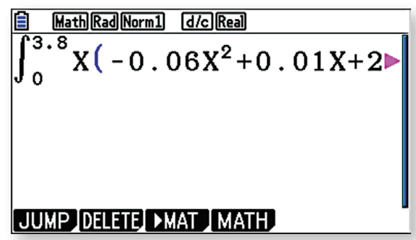


2. Press OPTN F4 (CALC) F4 (∫dx). Using the formula for finding the volume of an area rotated about the y-axis, enter this into the calculator.

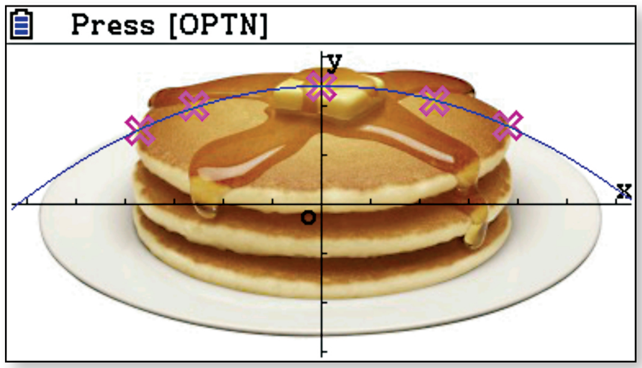




3. Press EXE to see the volume.



People are starting to think more about their diet and what they eat. To some, a pancake breakfast is a great way to start a Saturday or Sunday morning. However, these delicious breakfast icons are made from flour which is considered high in calories, especially if made with processed or white flour. In the picture below you will see a stack of three pancakes. You will be estimating the number of calories based on the volume of a stack of pancakes.



1. List the coordinates for the curve of the top of the pancake.

X-COORDINATE	Y-COORDINATE
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.

2. What is the regression line for the curve across the top of the pancakes?

____

3. In terms of the radius, what is the range for the radius of the pancake (assuming each unit on the graph is 1 inch)?

____

4. The volume of a solid formed by rotating an area about the y-axis is $v=2\pi\int_a^b ph\,dx$, where p is the radius and h is the height.
Write the integral to find the volume of the stack of pancakes using the regression line as the height and the range for the radius as the lower and upper bounds.

5. Find the volume of the stack of pancakes.

____

6. For these pancakes, 1 oz. ≈ .55 $in.^3$ and 8 oz. = 453 calories. Find the number of calories (without the butter and syrup) for the stack of pancakes.

____

Extension

1. If the pancakes had a radius of 2 in., what would be the volume of the stack of pancakes?

____

2. What was the change in volume of the pancakes?

____

3. What would be the volume and calorie count for a single pancake?

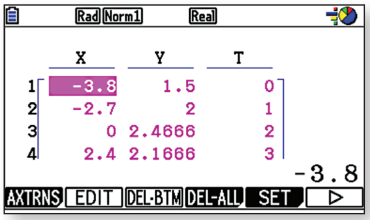
a. Volume:

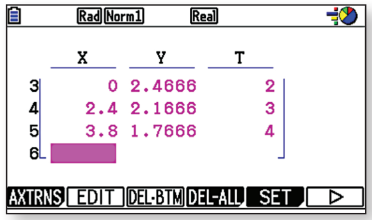
____

b. Calorie Count:

____

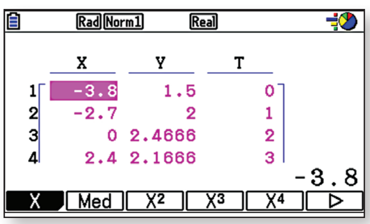
1.

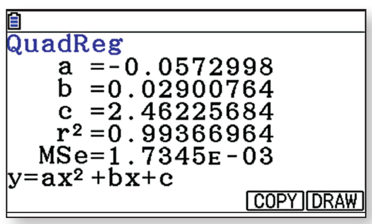


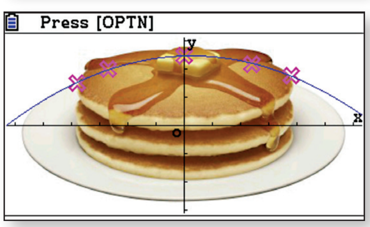


X-COORDINATE	Y-COORDINATE
1. **-3.8**	1. **1.5**
2. **-2.7**	2. **2.0**
3. **0**	3. **2.6**
4. **2.4**	4. **2.2**
5. **3.8**	5. **1.8**

2.





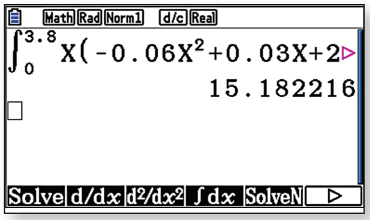


$y = -0.06x^2 + 0.03x + 2.46$

3. $0 < x < 3.8$ in.

4. $v = 2\pi\int_0^{3.8} x(-0.06x^2 + 0.03x + 2.46)$

5.

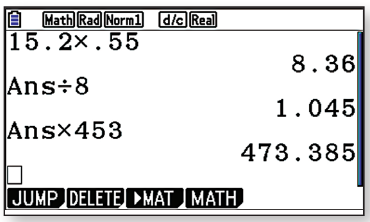


V = 15.2 in.^{3}

6. 15.2 in.3 = 8.36 oz.

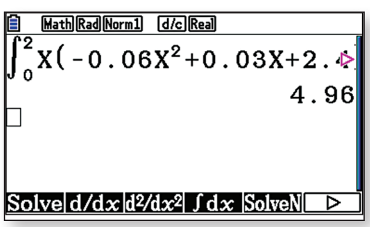
8.36 ÷ 8 = 1.045 (No. of 8 oz.)

1.045 x 453 Cal. per 8 oz. = 473 Cal.



Extensions

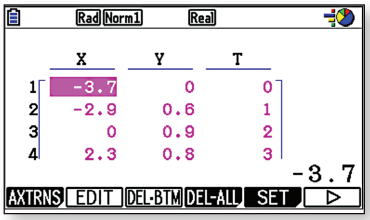
1.

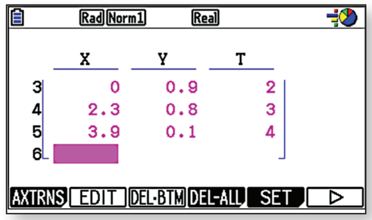


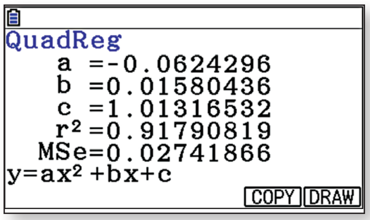
V = 4.96 in.^{3}

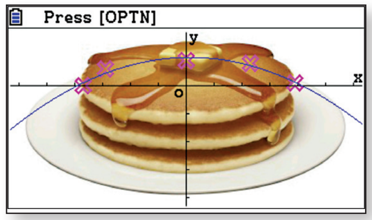
2. 8.36 – 4.96 = 3.4 in.^{3}

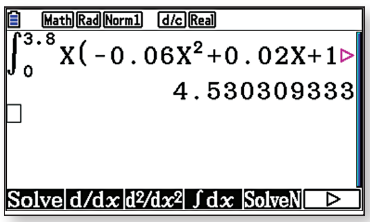
3.

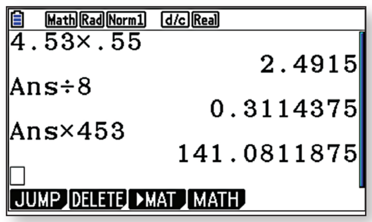












V = 4.53 in.^{3}

Calorie Count = 141 Cal.

DISCOVER THROUGH PRIZM™

A Bridge Over Icy Waters

PRIZM WORKSHEET #107

CASIO EDUCATION

TOPIC AREA:

Graphs for sinusoidal and polynomial functions; statistical regression

NCTM STANDARDS:

All students should be able to:

- Display a scatterplot and describe its shape.
- Determine regression coefficients and equations.

OBJECTIVE

Each student will be able to determine sinusoidal models for a given set of points.

Each student will suggest and determine an alternate model for the points.

GETTING STARTED

Consider letting the students work in pairs or in small groups to determine the points and the models. If each student or group selects points, the results will be different. An option for the first few questions is to demonstrate the point and have all students or groups use the same values.

PRIOR TO USING THIS ACTIVITY:

- Students should have a basic understanding of sinusoidal graphs.
- Students should be familiar with transformations of functions.
- Students should know amplitude and period.
- Students should be familiar with basic calculator methods for regression.

WAYS STUDENTS CAN PROVIDE EVIDENCE OF LEARNING:

- Students should write equations for the functions.
- Students should display graphs that match the photograph.

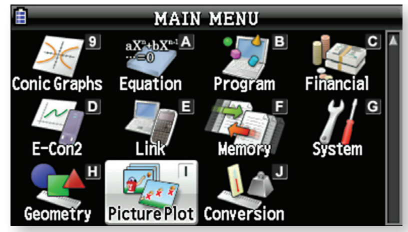
COMMON MISTAKES TO BE ON THE LOOKOUT FOR:

- Students may be unclear on left and right horizontal shifts.
- Students may not realize that the vertical distance from a "middle" point to a "high" point is the amplitude (and not half the amplitude).
- Students may not realize that the horizontal distance from a "middle" point to a "high" point is one fourth of the period.
- Students may not recall how to compute the coefficient b from the period.
- Students may not be aware that all sinusoidal graphs can be written as sine and cosine functions.
- Students may not realize that only one coefficient changes when a sine function is rewritten as a cosine function.

DEFINITIONS

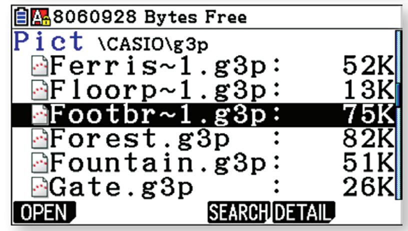
- Sinusoid
- Amplitude
- Period
- Horizontal shift
- Vertical shift
- Regression

The following will walk you through the keystrokes and menus required to successfully complete this activity.

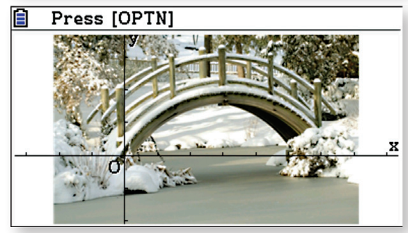


TO OPEN A BACKGROUND IMAGE IN PICTURE PLOT:

1. From the Main Menu, highlight the Picture Plot icon and press [EXE] or press [(].



2. Press [F1] (OPEN) to open the CASIO folder.



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3. The g3p folder contains 47 background images. Press [▼] [F1] (OPEN) to open the folder. Scroll down the list of images and highlight the desired picture. You will be using the "Footbr~1" image in this activity. Press [F1] (OPEN).

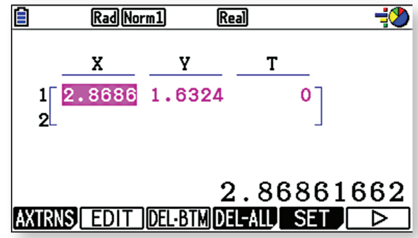
TO PLOT POINTS ON THE IMAGE:

1. The status bar at the top of the screen prompts what buttons you have to choose from. For this picture, you will need to press [OPTN].

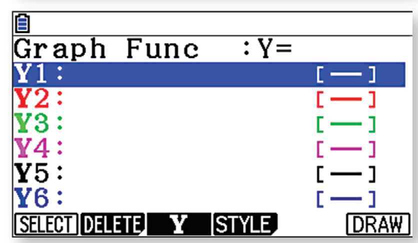
2. To plot a point, press [F2] (Plot). Use [◄] [►] [▲] [▼] to move to the desired location and press l to plot a point there. Press [EXIT] when all points are plotted.

TO UTILIZE IMAGE BACKGROUNDS:

1. To see the coordinates of plotted points, press F3 (List).



2. To edit (define) a function, press F4 (DefG).

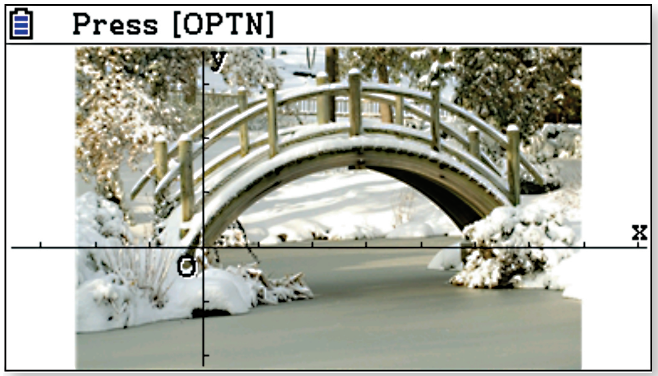


3. For regression, press F6 (▷) and F2 (REG). For sinusoidal, press F6 (▷) and F4 (Sin).

Structures, such as the St. Louis Arch, Chicago's Millennium Bean, and bridges often have a shape that can be described by a mathematical model.

In this activity, you will determine models for two portions of a bridge, and review some useful concepts about functions.

Open the footbridge photo titled "Footbr~1.g3p". Notice that a window for the axes has already been created, so we will use that setting.



Questions

1. Describe the shape of the bridge. What parts of the bridge did you focus on to describe the shape? Name several mathematical functions that could be used to model parts of the bridge.

____

____

2. Let's use the bottom of the bridge and model it as a sine function. For now, we'll assume that the bottom passes through the origin and that the function has no vertical or horizontal shift. Plot a point at the highest point of the bottom of the bridge and write its coordinates.

____

____

3. If the sine model has no vertical shift, what is the amplitude? Explain.

____

____

4. If the sine model has no vertical or horizontal shifts, what is the period? Explain. If the model is a function, with equation $y = a \cdot \sin(b \cdot x)$, compute the coefficient b.

5. Graph the function.

6. It is also possible to model the bottom of the bridge as a cosine function. In this case, there will be a horizontal shift. Describe the shift in magnitude and direction. Write the model as a function, with equation $y = a \cdot \cos(b(x - c))$.

7. Graph the function. How does this graph compare to the first model?

8. It is also possible to model the bottom of the bridge as a cosine function, shifted in the opposite direction. Describe the shift in magnitude and direction. Write the model as a function, with equation $y = a \cdot \cos(b(x + c))$. Check your result by graphing.

9. You have created this model using only two points. It is possible to use many additional points. Plot four to six additional points. Then use sinusoidal regression to write a model as a function with equation $y = d + a \cdot \sin(b(x - c))$. Check your result by graphing. Describe how this model is similar to the first model and how it is different than the first model.

____

____

____

10. Now, consider the upper portion of the bridge, that is, the curved railing. Plot five to seven points and use sinusoidal regression to write a model as a function with equation $y = d + a \cdot \sin(b(x - c))$. Check your result by graphing. Describe how this model is similar to the model for the lower portion and how it is different than that model.

____

____

____

Extensions

1. Use the graphing menu and shade the region between the models for the upper and lower parts of the bridge.

2. Use a different type of function to model the bridge. Determine the equation and make a graph.

3. A quadratic regression will display a value for R^2. For a certain set of points selected for the bridge, $R^2 = 0.9968$. Interpret the meaning of this value.

____

1. The bridge is shaped like an arch, based on the main part where you walk, as well as the railings. The model could be a parabola, a sine curve, or a higher degree polynomial.

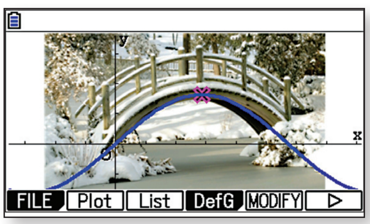
2. Answers will vary; for example, (2.8686, 1.6324).

3. If there is no vertical shift, the amplitude is the same as the y-coordinate of the highest point, about 1.6324.

4. The horizontal distance from (0, 0) to (2.8686, 1.6324) is one fourth of the period, so the period is about 11.4744. The coefficient $b = \frac{2\pi}{114744} \approx 0.5476$.

$y = 1.6324 \cdot \sin(0.5476x)$

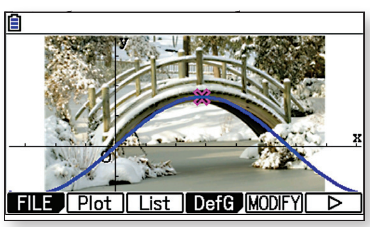
5.



6. The shift is 2.8686 units to the right.

$y = 1.6324 \cdot \cos(0.5476(x - 2.8686))$

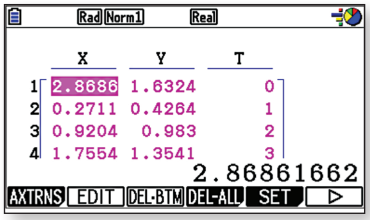
7.

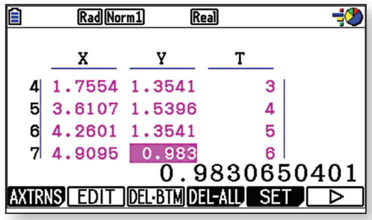


8. If the period is 11.4744, then a horizontal shift left would be 11.4744 – 2.8686 = 8.6058 units.

$y = 1.6324 \cdot \cos(0.5476(x + 8.6058))$

9. Points chosen will vary. For example,

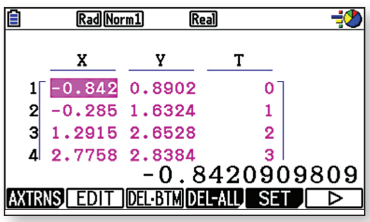


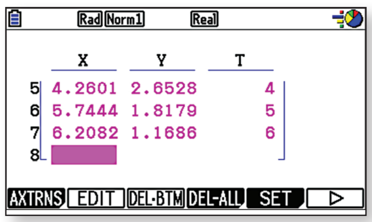


y = 11.6565•sin(0.1693x + 1.0711) – 10.0326

This model is fairly close to our original model.

10. Points chosen will vary. For example,



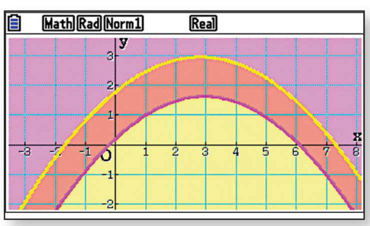


y = 24.0066•sin(0.1114x + 1.2565) – 21.0560

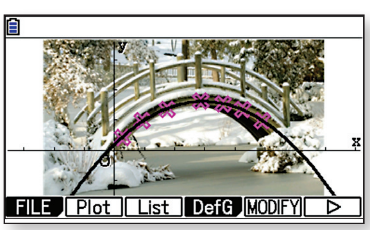
This model has a much larger vertical shift and amplitude, a larger period and horizontal shift.

Extensions

1.



2. Answers will vary. A quadratic function is a likely choice.



3. A value for R^2 = 0.9968, which is very close to 1, means there is a strong quadratic relationship for these points.

DISCOVER THROUGH PRIZM™

Rack 'Em Up

PRIZM WORKSHEET #108

CASIO EDUCATION

TOPIC AREAS:
Linear Inequalities, Linear Programming, Area

NCTM STANDARDS:

- Identify functions as linear or nonlinear and contract their properties from tables, graphs, or equations;
- Use coordinate geometry to represent and examine the properties of geometric shapes;

OBJECTIVE:

Given a photo file, students will be able to construct linear inequalities that create a polygon with specific dimensions and calculate the area of a polygon formed by linear programming.

GETTING STARTED:

Prior to beginning the activity, determine the students' readiness to graph a system of linear inequalities design to form a particular shape.

PRIOR TO USING THIS ACTIVITY:

- Students should be able to graph a linear function.
- Students should be able to graph a linear inequality.
- Students should understand the differences between greater than, less than, greater than or equal to, and less than or equal to.
- Students should be able to calculate the area of a triangle.

WAYS STUDENTS CAN PROVIDE EVIDENCE OF LEARNING:

- Given a linear inequality, students should be able to graph it by using pencil and paper, as well as with a graphing calculator.
- Given a triangle which is drawn on/off a coordinate plane, students should be able to calculate its area.

COMMON MISTAKES TO BE ON THE LOOKING OUT FOR:

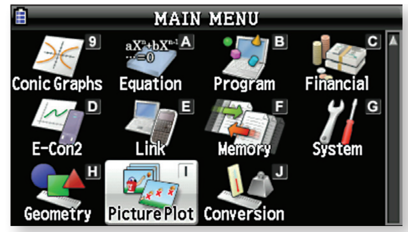
- Students may use the incorrect inequality symbol when graphing a linear inequality on the coordinate plane.
- Students may misidentify the slope and y-intercept of a linear inequality.
- Students may incorrectly calculate the area of a triangle.

DEFINITIONS:

- Linear Inequalities
- Slope
- y-intercept
- Triangle

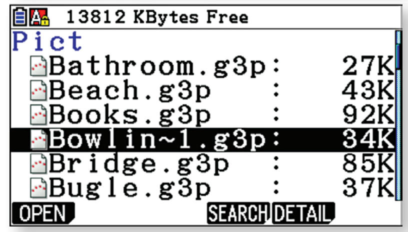
HOW TO **Rack 'Em Up**

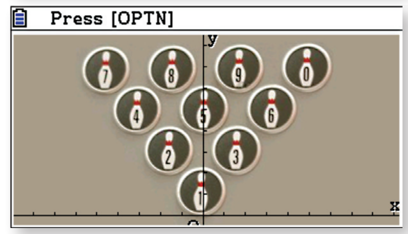
The following will walk you through the keystrokes and menus required to successfully complete the Rack 'Em Up activity.



TO OPEN A BACKGROUND IMAGE IN PICTURE PLOT:

1. From the Main Menu, highlight the Picture Plot icon and press [EXE] or press [(] .
2. Press [F1] (OPEN) to open the CASIO folder.
3. The g3p folder contains 47 background images. Press [▼] [F1] (OPEN) to open the folder. Scroll down the list of images and highlight the desired image. You will be using the "Bowlin~1" image in this activity. Press [F1] (OPEN).





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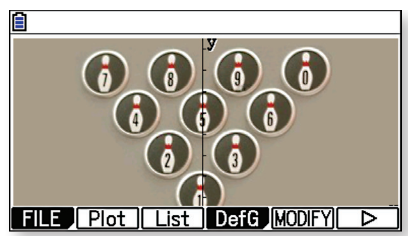
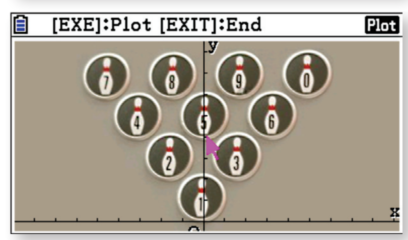
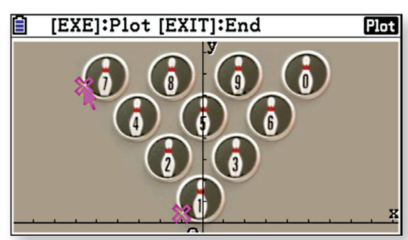
TO PLOT POINTS ON THE IMAGE AND CREATE A LIST OF POINTS:

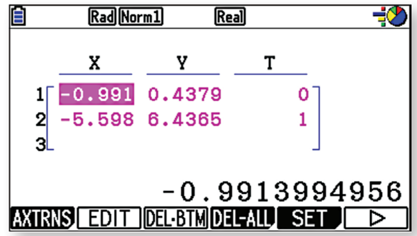
1. The status bar at the top of the screen prompts what buttons you have to choose from. For this image, you will need to press [OPTN] .
2. To plot points on the image, press [F2] (Plot).A pink arrow will appear; use [◀] [▶] [▲] [▼] to move the arrow to where you would like for itto plot a point. (Any of the number keys can also be used to jump to different areas on the screen). Press [EXE] to plot the point on the picture.
3. Continue moving the arrow and pressing [EXE] until you have all the points you want. To stop plotting, press [EXIT] .







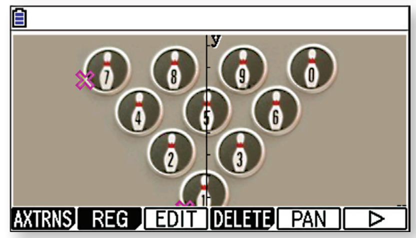
TO VIEW THE LIST OF DATA POINTS:



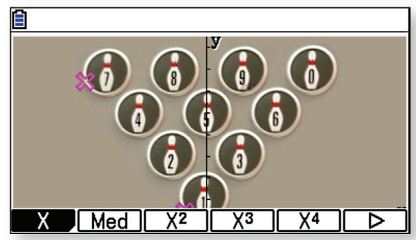
1. Press F3 (LIST) to view the list of points plotted.

   Press EXIT to go back to the image and points.

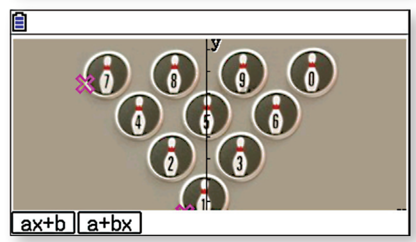
TO CREATE A BEST FIT LINE OR CURVE OF BEST FIT:

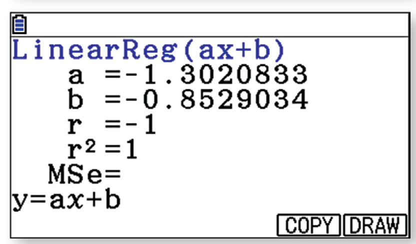


1. Press F6 and F2 (REG).

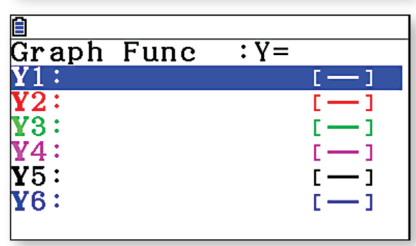


2. Choose the appropriate regression model. In this case, it will be X, so press F1 and F1 (ax+b).

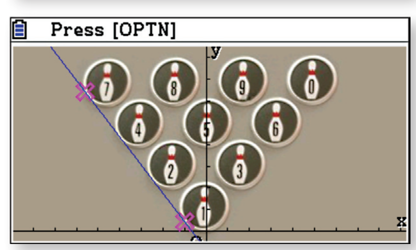




3. Press F5 (Copy) and EXE to copy the equation to the Graph menu.

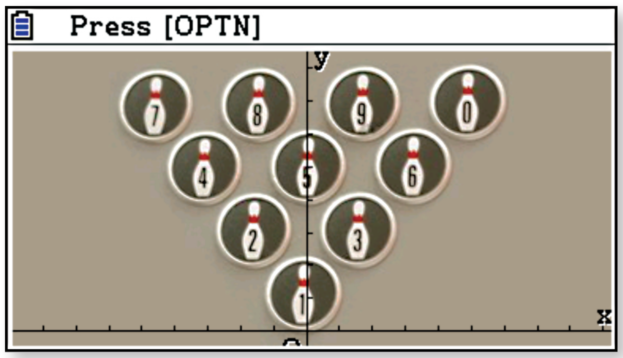


4. Press F6 (DRAW) to see the regression curve and the points.



Have you ever been bowling? If you have, you know the bowling pins are arranged at the end of the lane in the shape of a triangle. Each bowling pin is evenly spaced 12" apart from the center of one pin to the next. So, how much space is actually taken up by the bowling pins at the end of the lane? Perhaps knowing a little more about this will help you the next time you go bowling.

In this activity, you will determine the lines formed by the ten pins on a bowling lane. You will also determine how much space the bowling pins use and how to graphically represent that on your calculator. Finally, you will determine the amount of area the pins use at the end of a bowling lane.



Questions

1. Plot points for bowling pin #1 and bowling pin #7? What are the coordinates?

____

2. What is the equation of the line containing bowling pin #1 and bowling pin #7?

____

3. Plot points for bowling pin #1 and bowling pin #10. What are the coordinates?

____

4. What is the equation of the line containing bowling pin #1 and bowling pin #10?

____

5. Plot points for bowling pin #7 and bowling pin #10. What are the coordinates?

____

6. What is the equation of the line containing bowling pin #7 and bowling pin #10?

____

7. Write each equation as a linear inequality, which will shade the area of the triangle formed by the bowling pins?

____

Extension

1. Is the line containing bowling pins #1, #2, #4 and #7 parallel with the line containing bowling pins #3, #5, and #8, as well as the line containing bowling pins #6 and #9? Explain your answer and prove your answer by accompanying screen shots.

____

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____

____

2. Is the line containing pins #1 and #7 perpendicular to the line containing pins #7 and #10? Explain your answer and prove your answer by accompanying screen shots.

____

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____

____

3. What is the area of the triangle formed when the bowling pin deck is shaded?

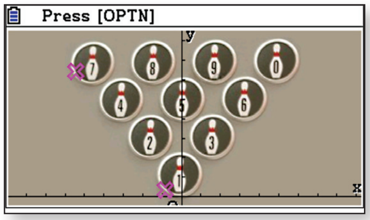
____

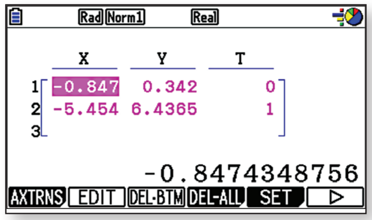
____

ANSWERS WILL VARY, DEPENDING ON POINTS PLOTTED.

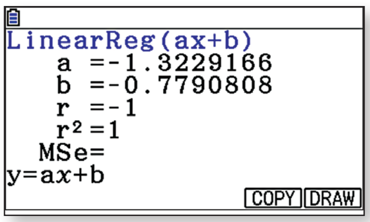
1. Bowling Pin #1: (–0.847, 0.342)

Bowling Pin #7: (–5.454, 6.437)



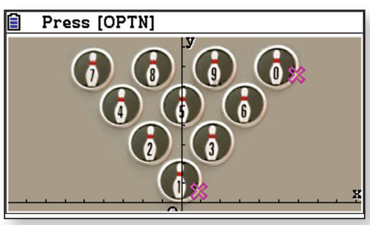


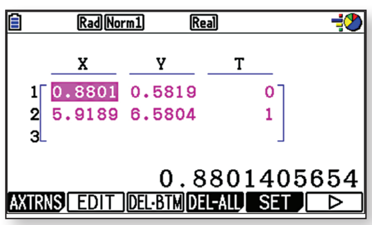
2. y = –1.323x – 0.779



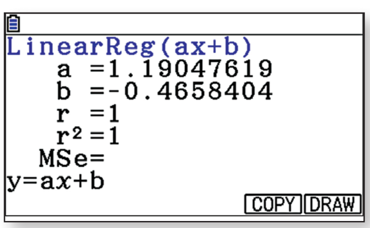
3. Bowling Pin #1: (0.880, 0.582)

Bowling Pin #10: (5.919, 6.580)



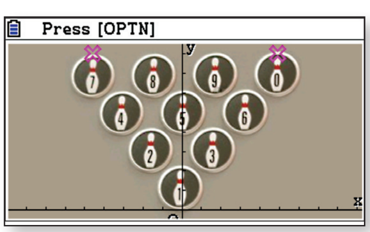


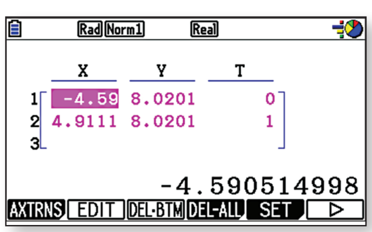
4. y = 1.190x – 0.466



5. Bowling Pin #7: (–4.591, 8.020)

Bowling Pin #10: (4.911, 8.020)



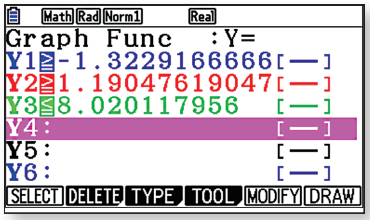


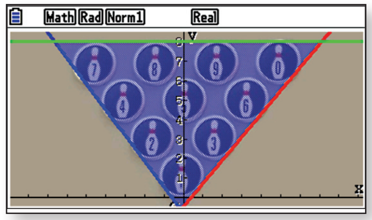
6. y = 8.020

7. **Y1** ≥ −1.323x − 0.779

Y2 ≥ 1.190x − 0.466

Y3 ≤ 8.020





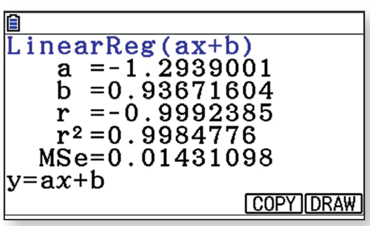
Extension Solutions:

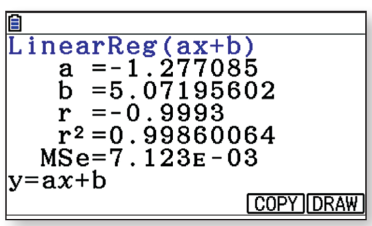
1. Yes, the three lines are parallel. After plotting points and performing linear regressions, we got the following equations:
Linear Equation for pins #1, 2, 4, 7: y =−1.294x + 0.937

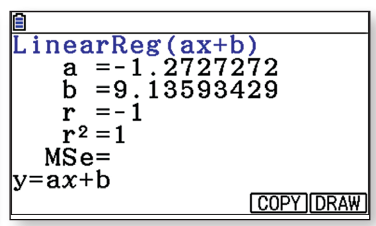
Linear Equation for pins #3, 5, 8: y = −1.277x + 5.072

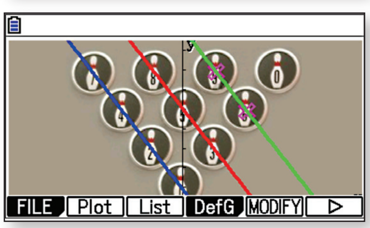
Linear Equation for pins #6, 9: y = −1.273x + 9.136

In order for lines to be parallel, the slopes must be the same. Our three slopes are roughly the same. They may not be the exact same because of the points we plotted. We can not guarantee that we plotted each point in the middle of each pin, each time.









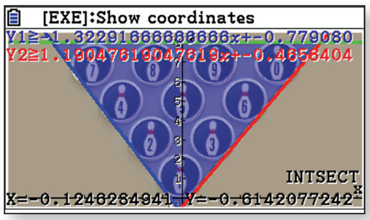
2. No, the two lines are not perpendicular. In order for two lines to be perpendicular, the slopes need to be the opposite reciprocals of each other.
Essentially, if the slope of one line is $\frac{a}{b}$, then the slope of the line perpendicular to it needs to be $-\frac{b}{a}$. This is not the case with our two lines. The slope of the line containing pins #7 and #10 is 0. The equation of a line perpendicular to it would be x =.

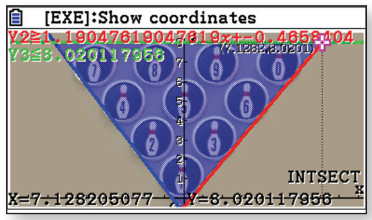
3. To find the area of the triangle formed by the lines, we first need to find the points that represent each corner. To do so, while viewing the graph of the three inequalities, press F5 (G-Solv) F5 (INTSECT).

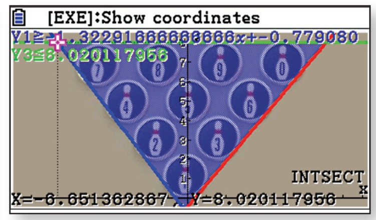
Point 1 (Intersection of **Y1** and **Y2**): (−0.125, −0.614)

Point 2 (Intersection of **Y2** and **Y3**): (7.128, 8.020)

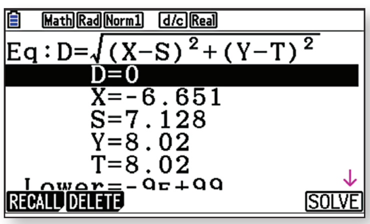
Point 3 (Intersection of **Y1** and **Y3**): (−6.651, 8.002)

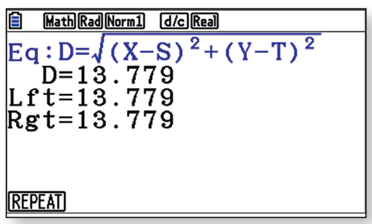




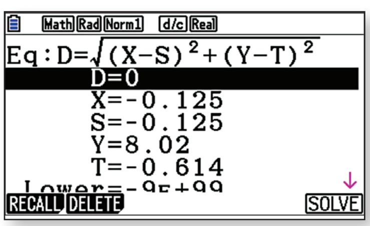


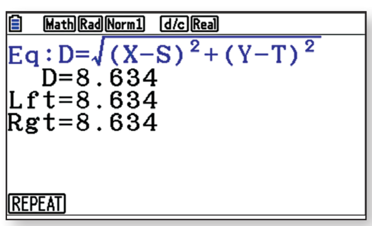
Now that we have the three corners, we can find the length of the base and the length of the height of the triangle. To find the length of the base, we need to find the distance from Point 2 to Point 3. (b = 13.779)





To find the height of the triangle, we will find the distance from Point 1 to point (-0.125, 8.020). We can't use Point 2 or Point 3, because the height has to be perpendicular to the base. (h = 8.634)





The area of a triangle is $A = \frac{1}{2}bh$. So, the area of our triangle,

is $A = \frac{1}{2}(13.779)(8.634) = 59.484$ units^2

DISCOVER THROUGH PRIZM™

Double Helix

PRIZM WORKSHEET #109

CASIO. EDUCATION

TOPIC AREA: Curve fitting; periodic functions; parametric equations

NCTM STANDARDS:

- Recognize and apply mathematics in contexts outside of mathematics.
- Identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships.
- Use a variety of symbolic representations, including recursive and parametric equations, for functions and relations.

OBJECTIVE

Given a photo of a screw, students will be able to fit a periodic function onto a section of the screw. Using their knowledge of trigonometric functions and how to restrict the domain of a function, students will create a regression based on plotting and analyzing a series of points. As an extension, students will define a model of the thread of a screw using parametric equations.

GETTING STARTED

Have students work in small groups to help determine how to identify the amplitude, period, and phase shift of a sinusoidal equation.

PRIOR TO USING THIS ACTIVITY:

- Students should understand how to graph parametric equations.
- Students should be able to determine an appropriate regression given a scatter plot.

WAYS STUDENTS CAN PROVIDE EVIDENCE OF LEARNING:

- Students should display graphs that match the photograph.
- Students should select appropriate regressions, given a scatter plot.

COMMON MISTAKES TO BE ON THE LOOKOUT FOR:

- Students may be careless in the placement of points.
- Students may have difficulty setting parameter values.

DEFINITIONS

- Three-dimensional helix

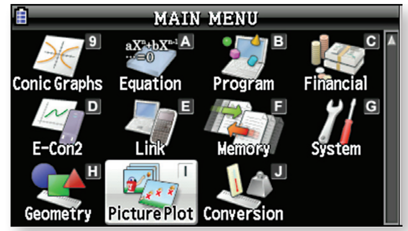
FORMULAS

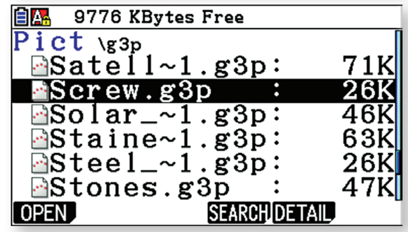
- Parametric Helix Equations: $x_t = a \cdot \cos t$, $y_t = a \cdot \sin t$, $z_t = bt$

The following will walk you through the keystrokes and menus required to successfully complete the Double Helix activity.

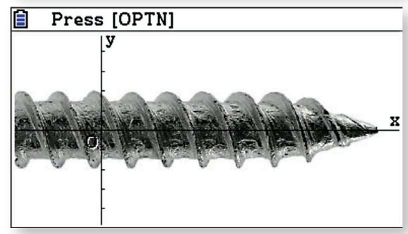
TO OPEN A BACKGROUND IMAGE IN PICTURE PLOT:

1. From the Main Menu, highlight the Picture Plot icon and press [EXE] or press [( ].
2. Press [F1] (OPEN) to open the CASIO folder.





3. The g3p folder contains 47 background images. Press [▼] [F1] (OPEN) to open the folder. Scroll down the list of images and highlightthe desired picture. You will be using the "Screw" image in this activity. Press [F1] (OPEN).



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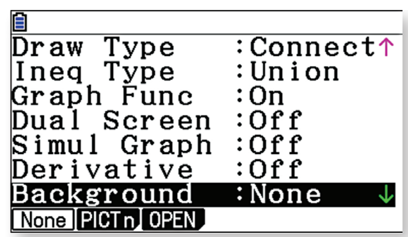
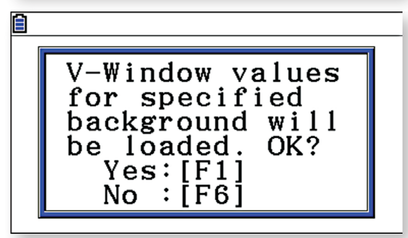
TO ADD A BACKGROUND IMAGE TO THE GRAPH MENU:

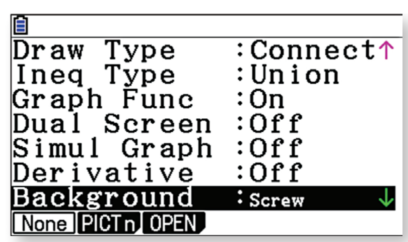
1. From within the GRAPH menu, press [SHIFT] [MENU] (SET UP) and [▼] until BACKGROUND is highlighted.



2. Press [F3] (OPEN), arrow down to the desired file and press [F1] (OPEN).

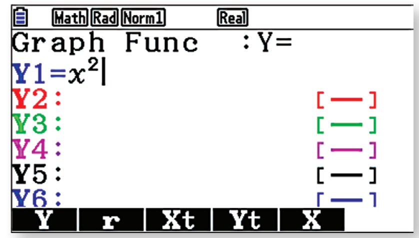


3. Press [F1] (YES) to accept the specified View Window.

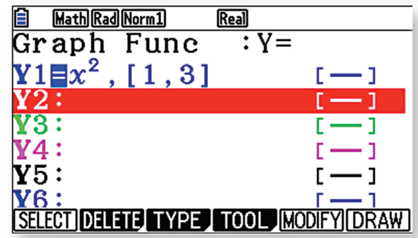


TO RESTRICT THE DOMAIN OF A FUNCTION:

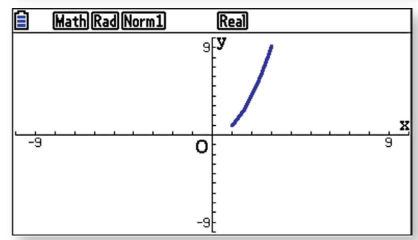
1. From within the GRAPH menu, enter the desired function, by pressing [X,θ,T] [x^2].



2. Domain restrictions are placed inside brackets. The restricted domain is [1, 3]. Immediately following the function, enter the following: [,] [SHIFT] [+] [1] [,] [3] [SHIFT] [−] [EXE].

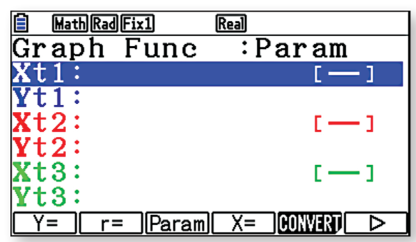


3. To see the restricted graph, press [F6] (DRAW).



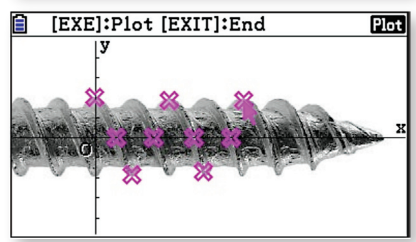
TO SET THE EQUATION LINE TO RECEIVE PARAMETRIC EQUATIONS:

1. From within the GRAPH menu, go to an empty line and press [F3] (TYPE) [F3] (PARAM).

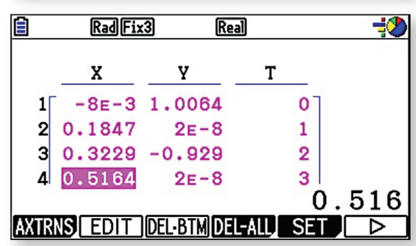


TO PLOT POINTS ON THE IMAGE AND CREATE A LIST OF POINTS:

1. To plot points, press [OPTN] [F2] (PLOT). A pink arrow will appear; use [◀] [▶] [▲] [▼] to move the arrow to where you would like for it to plot a point. (Any of the number keys can also be used to jump to different areas on the screen). Press [EXE] to plot the point.



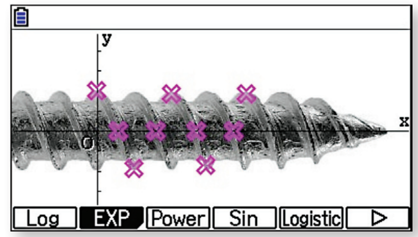
2. Continue moving the arrow and pressing [EXE] until you have all the desired points. Press [EXIT] to stop plotting.



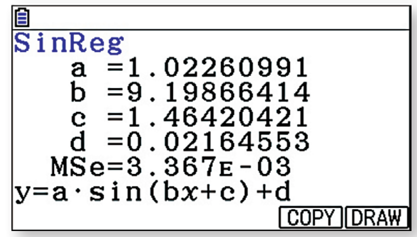
3. Press [OPTN] [F3] (LIST) to view the list of points plotted. Press [EXIT] to go back to the image and points or press [F4] (DEL-ALL) to delete all points.

TO PERFORM A SINUSOIDAL REGRESSION:

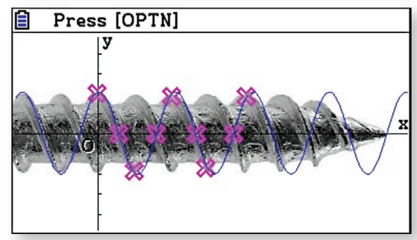
1. From within the Picture Plot icon, press OPTN F6 (▷) F2 (REG) F6 (▷) F4 (Sin).



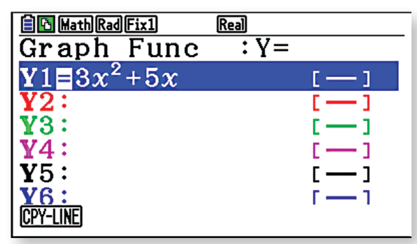
2. From the regression screen, press F5 (COPY) to record the regression information into a line in the Graph icon.



3. Press F6 (DRAW) to sketch the regression through the plotted points.



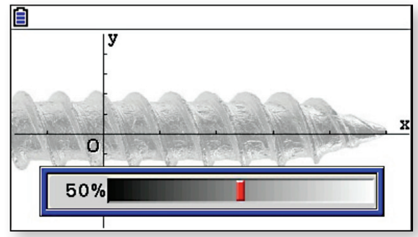
TO COPY AND PASTE A FUNCTION TO ANOTHER LOCATION:



1. From within the Graph icon, highlight the desired function and press SHIFT 8 **(CLIP)** F1 (CPY-LINE) to copy.

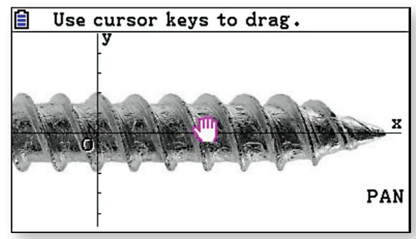
2. Highlight the desired destination and press SHIFT 9 **(PASTE)** to paste.

TO ADJUST THE BRIGHTNESS OF AN IMAGE:



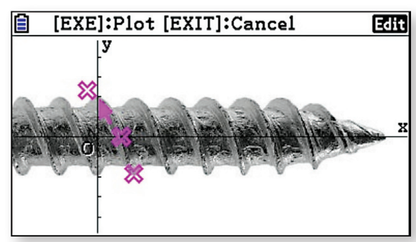
1. From within the Picture Plot icon, press OPTN F6 (▷) F6 (▷) F3 (FadeI/O).
2. Use the replay buttons ◀ ▶ ▲ ▼ to lighten the image with each successive press.
3. Press EXE to accept the desired level of brightness.

TO PAN THE AXES OVER AN IMAGE:

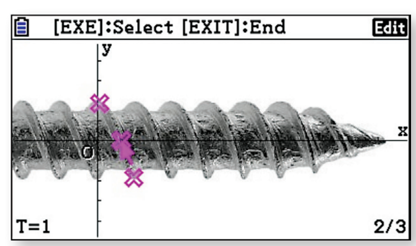


1. From within the Picture Plot icon, press OPTN F6 (▷) F5 (PAN) EXE to grab the axes.
2. Use the replay buttons ◀ ▶ ▲ ▼ to move the axes to the desired location.
3. Press EXE to accept the desired movement of the axes.

TO MOVE A PREVIOUSLY PLOTTED POINT:



1. While the Picture Plot screen is displayed, press OPTN F6 (▷) F3 (EDIT).
2. Use replay buttons ◀ ▶ ▲ ▼ to move the pointer to the desired plot, then press EXE.



3. Use the cursor to move the pointer to the location to which you want to move the point, then press EXE.
4. After you have finished moving all the points you want, press EXIT.

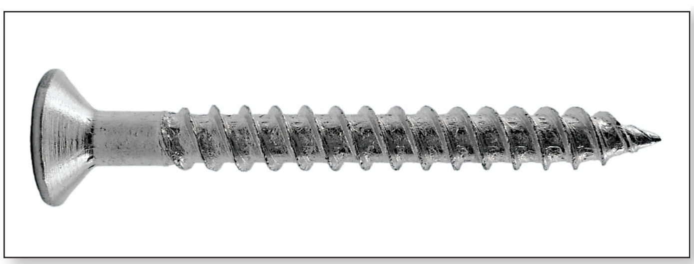
On April 25, 1953, the molecular biologists James D. Watson and Francis H. C. Crick published a pivotal paper in which they described the geometric shape of DNA, the molecule of life. The two biologists described the molecule as being in the form of a double helix — two helices that spiral around each other, connected by molecular bonds, to resemble nothing more than a rope ladder that has been repeatedly twisted along its length.

Given the neat way the two intertwined helices in DNA function in terms of genetic reproduction, you might think that the helix had important mathematical properties. Unfortunately, the equation of the helix is quite unremarkable and there seems to be relatively little to catch the mathematician's attention.

On a more productive note, helices are common in the world around us. Various sea creatures have helical shells and climbing vines wind around supports to trace out a helix. In the technological world of our own making, spiral staircases, corkscrews, drills, bedsprings, and telephone handset chords are helix-shaped. And what kind of a world would we have without the binding capacity the helix provides in the form of various kinds of bolts and screws.

In this activity, you will need to find a function to model the thread of a screw. You will accomplish this by plotting points along a screw's thread and performing a regression on the plotted points. You will need to be able to adjust the plotted points to obtain a more accurate regression. A second regression will be drawn so that it is the reflection of the first regression. You will also need to determine an appropriate domain for each equation.

In the extension, you will need to plot appropriate points along features of a screw to determine various lengths. These lengths will be used as parameters for the equation for a helix projected in two-dimensions and will be drawn using parametric equations.



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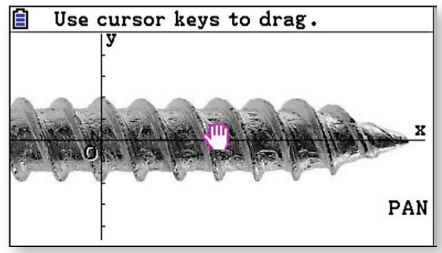
Questions

1. Pan the axes until the x-axis passes through the center of the screw and the y-axis passes through the apex of the third thread from the left. Record the resulting viewing window settings in the spaces provided and round each number to the nearest tenth.

X MIN	
X MAX	
Y MIN	
Y MAX	



2. Which regression on the Casio Prizm should be used to model the thread of the screw?

____

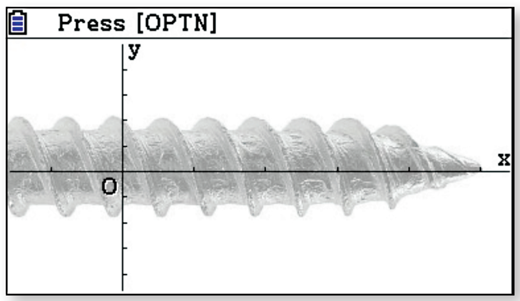
3. Notice that there are two separate spirals that form the thread of a screw.
Plot at least 7 points along one spiral of the thread that follows the regression selected in Question 2. Round each coordinate to the nearest tenth and record the coordinates in the space provided

POINT 1	
POINT 2	
POINT 3	
POINT 4	
POINT 5	
POINT 6	
POINT 7	
POINT 8	
POINT 9	
POINT 10	

4. Perform the appropriate regression and write the regression equation below. Round each parameter to the nearest hundredth.

5. Draw the regression. Adjust plotted points as necessary to provide a tighter regression. If adjustments were made, rewrite the regression equation.

6. Show the results to your classmates and your teacher. Sketch the regression over the image below.



7. Does the regression model the entire screw thread? If not, why do you think it does not? Explain.

8. Restrict the domain of the regression equation so that no more than two periods are displayed. What is the domain?

9. How could the second spiral be modeled without having to plot points, or performing another regression? Explain.

10. Write both equations for each spiral of the screw's thread.

Extension

The parametric model for a screw's thread is similar to the three dimensional helix. The two dimensional projection can be given by the equations:

$Xt = T$

$Yt = R \bullet \sin \frac{2\pi}{P} (T = w)$

,where R = radius of the screw
P = distance between successive turns
w = the phase shift

1. What is the radius of the screw?

____

2. What is the period (distance between successive turns) of the screw?

____

3. What is the phase shift (horizontal distance to correct for a standard sine function) of the screw?

____

4. The Picture Plot icon can only graph Y = f(x), therefore we must switch to the Graph icon, when graphing parametric equations. Write the parametric equation to model one spiral of the screw's thread, rounding all coefficients and constants to the nearest hundredth.

____

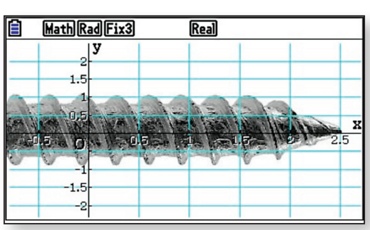
____

5. Graph the parametric function on the Casio Prizm in the Graph icon, with the appropriate values for T θ, to create a model displaying at most two periods of the spiral. Show the results to your classmates and teacher. What values did you select for T θmin and T θmax?

____

____

6. Sketch the parametric function over the image below.

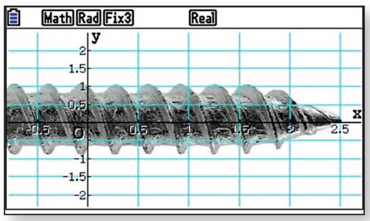


7. What is the **reflection** of the 1st parametric equation?

____

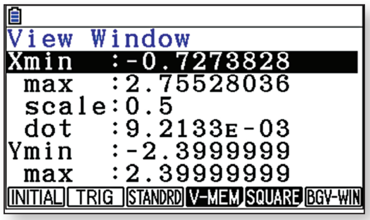
____

8. Sketch both parametric functions over the image below using appropriate values for T θ min and T θ max.



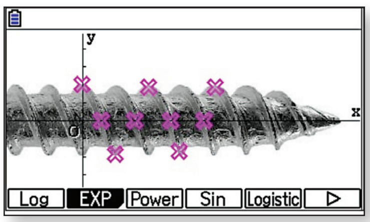
Answers will vary, depending on the placement of the axes and plotted points.

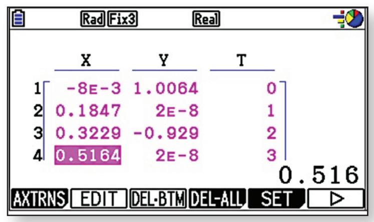
1. Xmin: –0.7 Xmax: 2.8 Ymin: –2.4 Ymax: 2.4



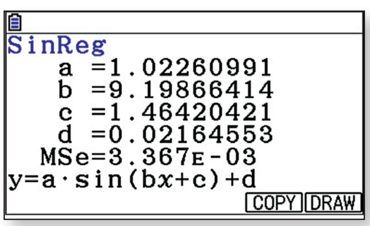
2. Sinusoidal

3.



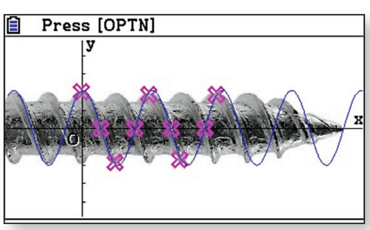


4. Regression equation: y = 1.02•sin(9.2x + 1.46) + 0.02



5. The adjustment is not necessary for this example.

6.



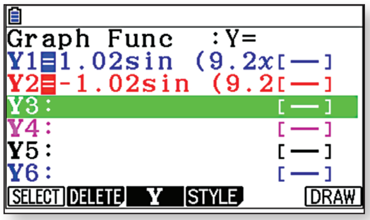
7. No. Possible reasons include, but are not limited to: (a) any regression is only valid over a very specific domain, (b) the radius of the screw is not constant, or (c) the entire screw's thread is actually two spirals, not one spiral.

8. Answers may vary; one possible domain of regression equation is: [0, 1]

9. The second spiral equation is the reflection (over the x-axis) of the first spiral equation.

10. Y1 = 1.02 • sin(9.2x + 1.46) + 0.02

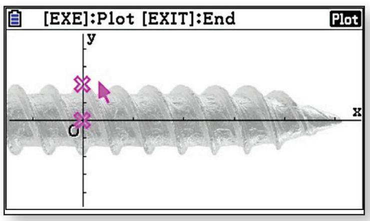
Y2 = –1.02 • sin(9.2x + 1.46) + 0.02

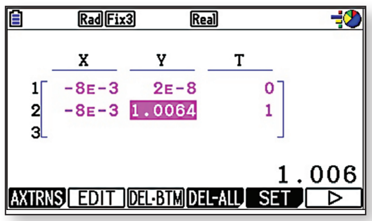


Extension Solutions

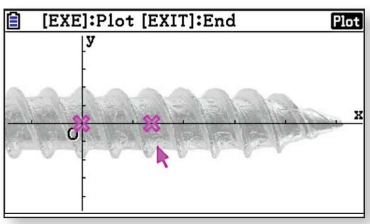
Answers will vary, depending on plotted points.

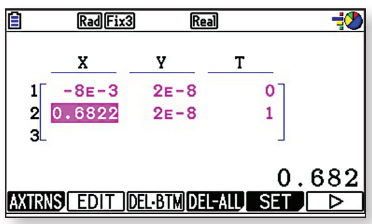
1. Radius = 1.01



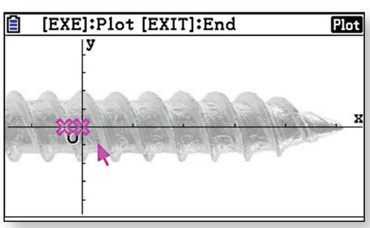


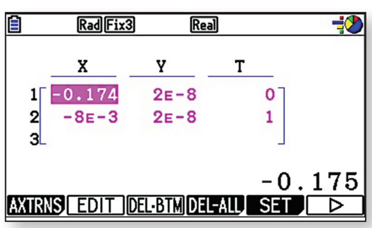
2. Period = 0.68





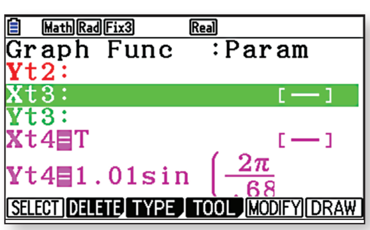
3. Phase Shift = 0.18





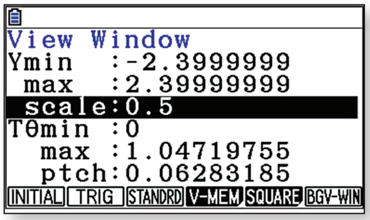
4. Xt = T

Yt = 1.01 • sin $\frac{2\pi}{0.68}$ (T + 0.18)

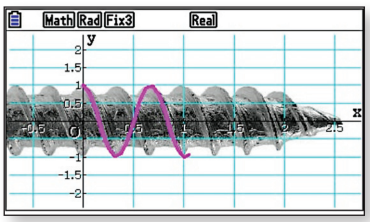


5. T θ min = 0

T θ max = $\frac{\pi}{3} \approx 1.0472$

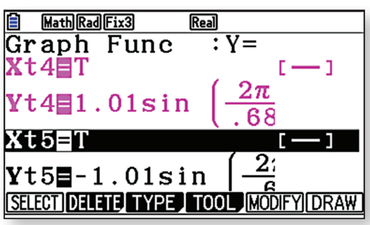


6.

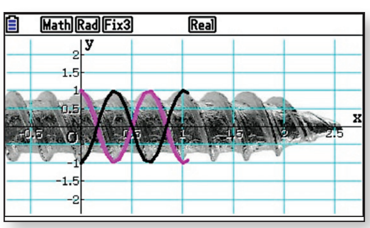


7. Xt = T

Yt = 1.01 • sin $\frac{2\pi}{0.68}$ (T + 0.18)



8.



DISCOVER THROUGH PRIZM™

Loop the Loop

PRIZM WORKSHEET #110

CASIO. EDUCATION

TOPIC AREA:

Curve fitting; piecewise-defined functions; parametric regressions

NCTM STANDARDS:

- Use Cartesian coordinates to analyze geometric situations.
- Identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships.
- Use a variety of symbolic representations, including recursive and parametric equations, for functions and relations
- Apply and adapt a variety of appropriate strategies to solve problems

OBJECTIVE

Given a photo of a rollercoaster, students will be able to fit a piecewise-defined function onto a section of the coaster. Using their knowledge of the standard equation of a circle and how to restrict the domain of a function, students will create multiple equations based on plotting and analyzing a series of points. As an extension, students will learn how to perform a parametric regression.

GETTING STARTED

Have students work in small groups to help determine how to separate the standard equation for a circle into two explicit functions.

PRIOR TO USING THIS ACTIVITY:

- Students should understand how to solve literal equations.
- Students should be able to create the standard equation for a circle, given the center and radius.
- Students should be able to determine an appropriate regression given a scatter plot.

WAYS STUDENTS CAN PROVIDE EVIDENCE OF LEARNING:

- Students should display graphs that match the photograph.
- Students should select appropriate regressions given a scatter plot.

COMMON MISTAKES TO BE ON THE LOOKOUT FOR:

- Students may be careless in the placement of points.
- Students may have difficulty setting parameter values.

DEFINITIONS

Piecewise-defined function

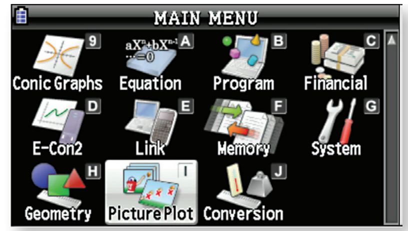
FORMULAS

Standard Equation of a Circle $(x - h)^2 + (y - k)^2 = r^2$

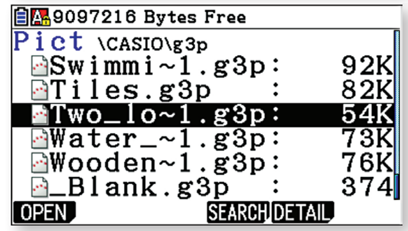
Explicit Equations for a Circle: $y \pm \sqrt{r^2 - (x - h)^2 + k}$

The following will walk you through the keystrokes and menus required to successfully complete the Loop the Loop activity.

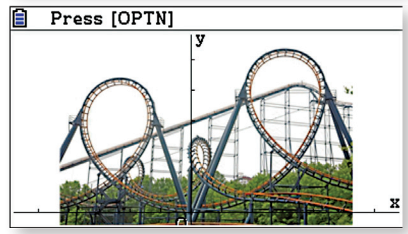
TO OPEN A BACKGROUND IMAGE IN PICTURE PLOT:



1. From the Main Menu, highlight the Picture Plot icon and press [EXE] or press [(].



2. Press [F1] (OPEN) to open the CASIO folder.



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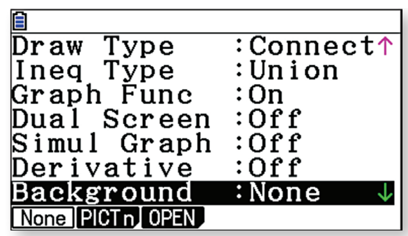
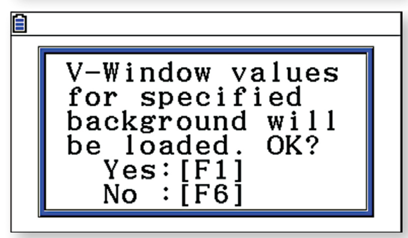
3. The g3p folder contains 47 background images. Press [▼] [F1] (OPEN) to open the folder. Scroll down the list of images and highlight the desired image. You will be using the "Two_lo~1" image in this activity. Press [F1] (OPEN).

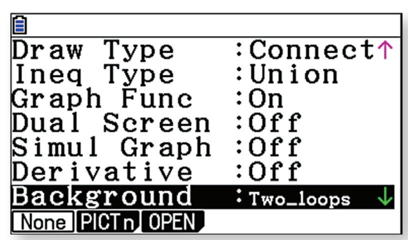
TO ADD A BACKGROUND IMAGE TO THE GRAPH MENU:



1. From within the GRAPH menu, press [SHIFT] [MENU] (SET UP) and [▼] until BACKGROUND is highlighted.

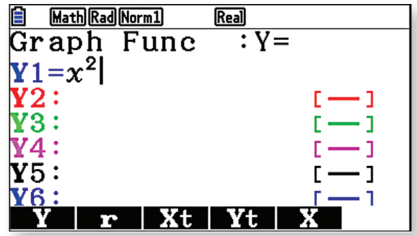


2. Press [F3] (OPEN), arrow down to the desired file and press [F1] (OPEN).

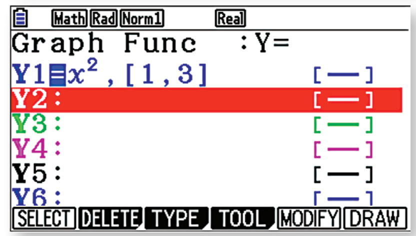


3. Press [F1] (YES) to accept the specified View Window.

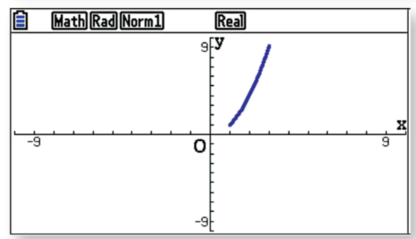
TO RESTRICT THE DOMAIN OF A FUNCTION:



1. From within the GRAPH menu, enter the desired function, by pressing [X,θ,T] [x^2].

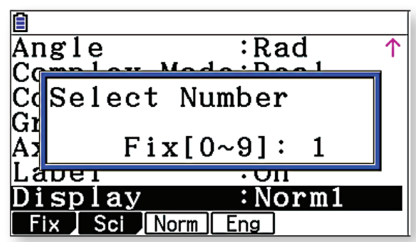


2. Domain restrictions are placed inside brackets. The restricted domain is [1, 3]. Immediately following the function, enter the following: [,] [SHIFT] [+] [1] [,] [3] [SHIFT] [−] [EXE].



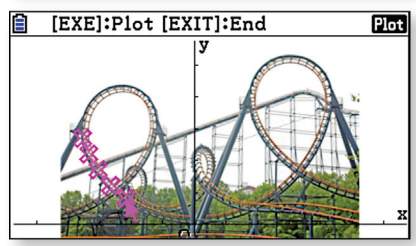
3. To see the restricted graph, press [F6] (DRAW).

TO SET THE NUMBER OF DECIMAL PLACES FROM 0 TO 9:

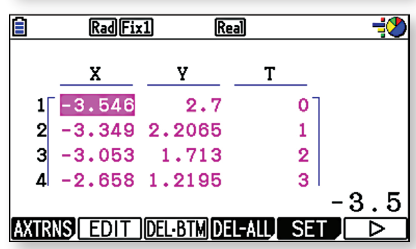


1. To set decimal places to [1], from within the GRAPH menu press [SHIFT] [MENU] **(SET UP)** [▲] [F1] (FIX) [1] [EXE].

TO PLOT POINTS ON THE PICTURE AND CREATE A LIST OF POINTS:



1. To plot points, press [OPTN] [F2] (PLOT). A pink arrow will appear; use [◀] [▶] [▲] [▼] to move the arrow to where you would like for it to plot a point. (Any of the number keys can also be used to jump to different areas on the screen). Press [EXE] to plot the point.



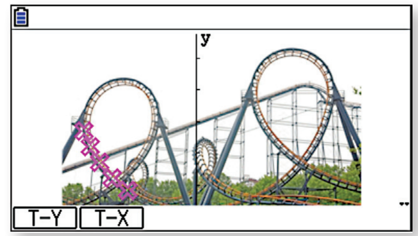
2. Continue moving the arrow and pressing [EXE] until you have all the desired points. Press [EXIT] to stop plotting.

3. Press [OPTN] [F3] (LIST) to view the list of points plotted. Press [EXIT] to go back to the picture and points or press [F4] (DEL-ALL) to delete all points.

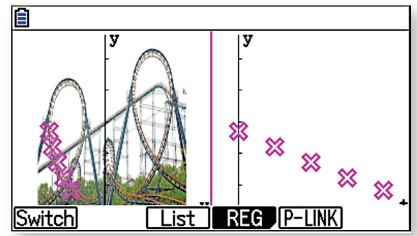
HOW TO **Loop the Loop**

TO PERFORM A PARAMETRIC REGRESSION OF Y ON T:

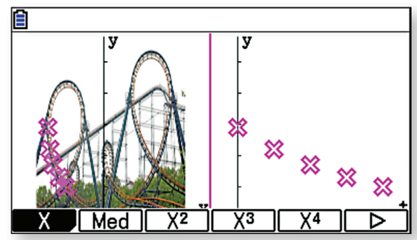
1. From within the PICTURE PLOT menu, press [OPTN] [F6] (▷) [F1] (AXTRNS) [F1] (T-Y).

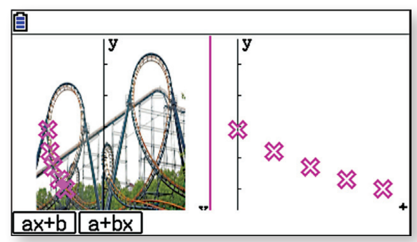


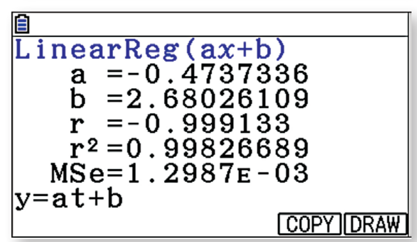
2. From the dual screen, press [OPTN] [F4] (REG).



3. Select the appropriate regression. In this case, it is linear, so press [F1] (X) [F1] (ax+b).

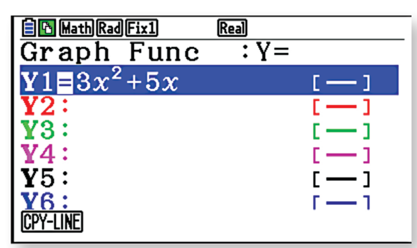






TO COPY AND PASTE A FUNCTION TO ANOTHER LOCATION:

1. From within the GRAPH menu, press the arrow to the desired function. Press [SHIFT] [8] **(CLIP)** [F1] (CPY-LINE) to copy.
2. Arrow to desired destination and press [SHIFT] [9] **(PASTE)** to paste.



When most people first view a looping roller coaster (Figure 1), they think that the loop is a circle; this is a common misconception. If you were to look more closely, you might notice that the top of the loop may look like a half-circle, whereas the bottoms look different, having an increasing radius of curvature closer to the ground (Figure 2). These type of vertical loops have a tear drop shape and actually follow a special shape called a clothoidloop, which involves equations and mathematics that are beyond the scope of this activity.

In this activity, you will need to find a piecewise-defined function consisting of three different equations for a circle. You will accomplish this by determining the coordinates of the center of each circle and at least two points of the circle, that lie on the axes that has each respective center as the origin. You will need to be able to write an explicit function for either the top or bottom half of each circle. You will also need to determine an appropriate domain for each equation.

In the extension, you plot appropriate points along one loop of the rollercoaster and perform a parametric regression to model the loop. You will also become familiar with changing various settings and using features of the Casio Prizm.

Figure 1

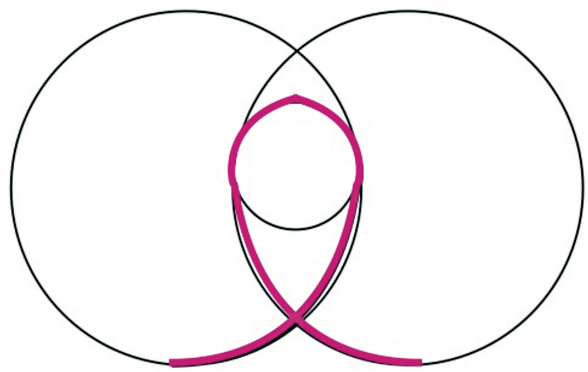


Figure 2

Questions

1. Using the leftmost loop of the rollercoaster, plot one point that best represents the center of the small circle at the top of the loop and two points on the small circle that lie directly horizontal and directly vertical from the center. Write the coordinates in the spaces provided. Round each coordinate to the nearest tenth.

Center ________________

Point 1 ________________

Point 2 ________________

2. Using the coordinates in Question 1, determine the radius of the small circle by averaging the distances between both points and the center of the small circle.

3. Using the center and the radius of the small circle, write the standard equation of the circle.

4. Rewrite the standard equation for the small circle as an explicit function.

5. What is the domain of the explicit function for the small circle?

6. Using the leftmost loop of the rollercoaster, plot one point that best represents the center of the larger circle entering the loop and two points on the circle that lie directly horizontal and directly vertical from the center. Write the coordinates in the spaces provided. Round each coordinate to the nearest tenth.

Center ________________

Point 1 ________________

Point 2 ________________

7. Using the coordinates in Question 6, determine the radius of the entering circle by averaging the distances between both points and the center of the entering circle.

8. Using the center and the radius of the entering circle, write the standard equation of the circle.

____

9. Rewrite the standard equation for the entering circle as an explicit function.

____

10. What is the domain of the explicit function for the entering circle?

____

11. Using the leftmost loop of the rollercoaster, plot one point that best represents the center of the larger circle exiting the loop and two points on the circle that lie directly horizontal and directly vertical from the center. Write the coordinates in the spaces provided. Round each coordinate to the nearest tenth.

Center ______________

Point 1 ______________

Point 2 ______________

12. Using the coordinates in Question 11, determine the radius of the exiting circle by averaging the distances between both points and the center of the exiting circle.

____

13. Using the center and the radius of the exiting circle, write the standard equation of the circle.

____

14. Rewrite the standard equation for the exiting circle as an explicit function.

____

15. What is the domain of the explicit function for the exiting circle?

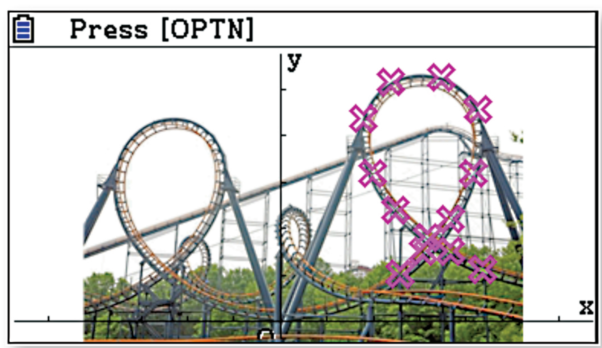
____

16. Graph each explicit function in the PICTURE PLOT menu on the Casio Prizm, with appropriate domains to create a piecewise-defined model of the leftmost rollercoaster loop. Show the results to your classmates and teacher. Sketch the piecewise-defined function over the image below.

Extension

Plot at least 10 points along the rightmost loop of the rollercoaster. Round each coordinate to the nearest tenth. Record the coordinates in the space provided.

POINT 1	
POINT 2	
POINT 3	
POINT 4	
POINT 5	
POINT 6	
POINT 7	
POINT 8	
POINT 9	
POINT 10	



2. Use the Casio Prizm to plot the Y (vertical) coordinates with respect to T (time) and perform the appropriate Y-T regression. Copy the full regression equation into the GRAPH menu of the Casio Prizm. Round all coefficients and constants to the nearest hundredth and record the regression equation below.

____

3. Use the Casio Prizm to plot the X (horizontal) coordinates with respect to T (time) and perform the appropriate X-T regression. Copy the full regression equation into the GRAPH menu of the Casio Prizm. Round all coefficients and constants to the nearest hundredth and record the regression equation below.

____

4. Open the GRAPH menu and copy both regression equations into the appropriate line of a parameterized equation. [Note: The PICTURE PLOT menu can only graph Y = f(x), therefore we must alter each regression equation to include the parameter T when graphing parametric equations in the GRAPH menu.] Write the parametric equation below. Round each coefficient and constants to the nearest hundredth.

____

____

5. Graph the parametric function on the Prizm in the GRAPH mode, with the appropriate values for the parameter, to create a model of the rightmost rollercoaster loop. Show the results to your classmates and teacher. Write the values selected for Tmin/Tmax below.

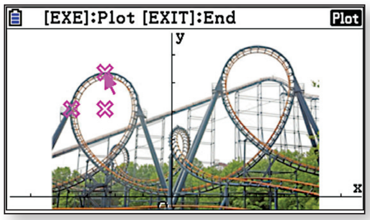
____

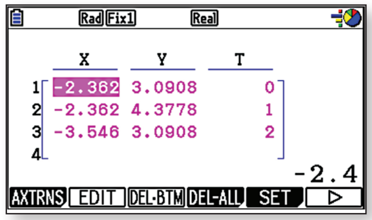
6. Sketch the parametric function over the image below.

SOLUTIONS **Loop the Loop**

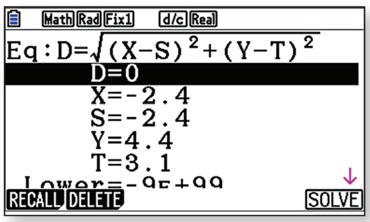
ANSWERS WILL VARY, DEPENDING ON THE PLOTTED POINTS.

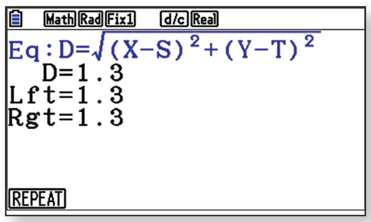
1. Center (-2.4, 3.1)   Point 1 (-2.4, 4.4)   Point 2 (-3.5, 3.1)

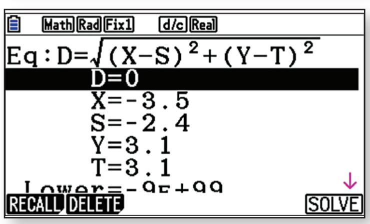


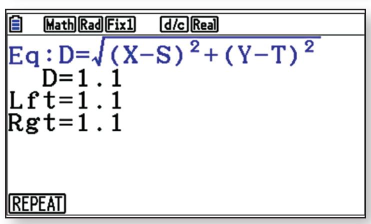


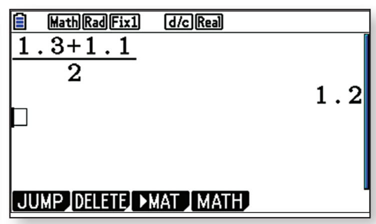
2. Radius = 1.2











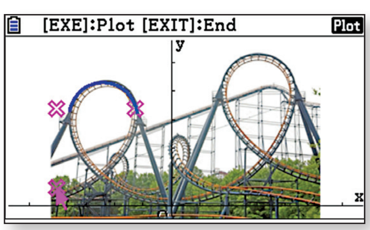
3. Standard equation: $(x + 2.4)^2 + (y - 3.1)^2 = 1.2^2$

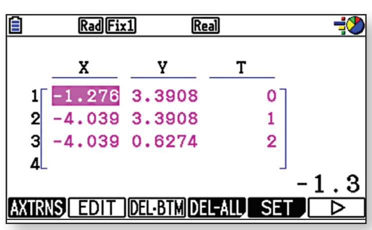
4. Explicit functions: $y = \pm\sqrt{1.2^2 - (x + 2.4)^2} + 3.1$

5. The domain is not necessary, however it is [-3.5, -1.2].

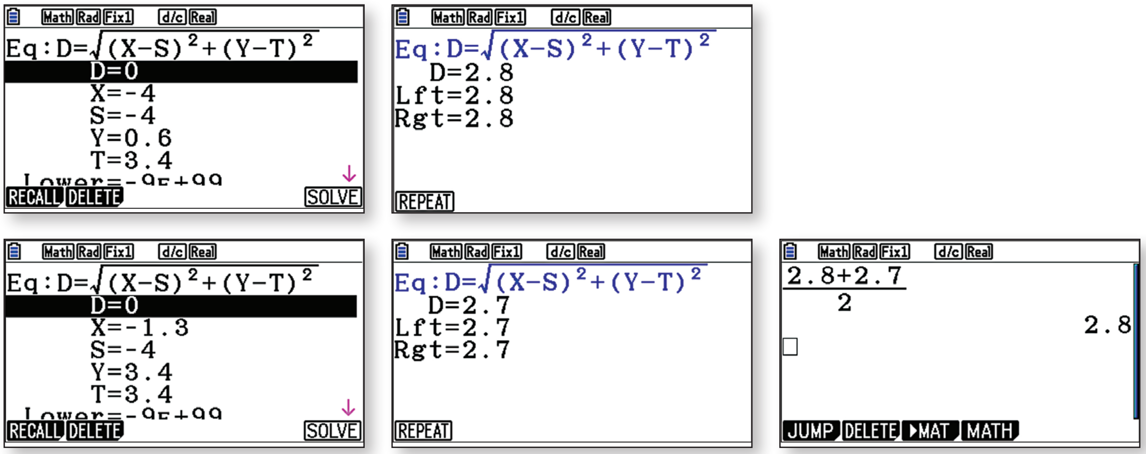
6. Center (-4.0, 3.4)   Point 1 (-4.0, 0.6)   Point 2 (-1.3, 3.4)

[Hint: To more accurately place the entering circle, draw the top circle for reference points]





7. Radius = 2.8

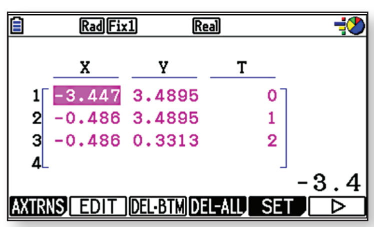


8. Standard equation: $(x + 40)^2 + (y - 3.4)^2 = 2.8^2$

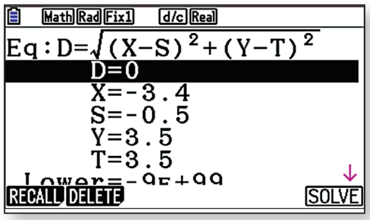
9. Explicit functions: $y \pm \sqrt{2.8^2 - (x + 4.0)^2 + 3.4}$

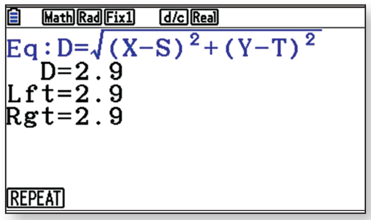
10. The domain used to get a better fit was [-3.2, -1.2].

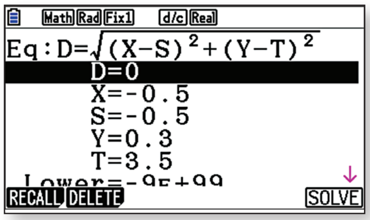
11. Center (-0.5, 3.5) Point 1 (-3.4, 3.5) Point 2 (-0.5, 0.3)
[Hint: To more accurately place the exiting circle, draw the top circle for reference points.]

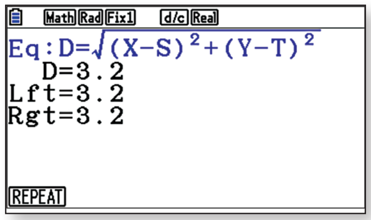


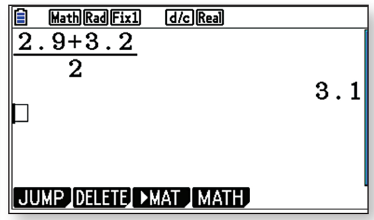
12. Radius = 3.1









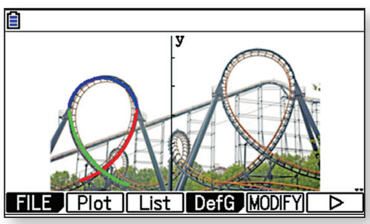


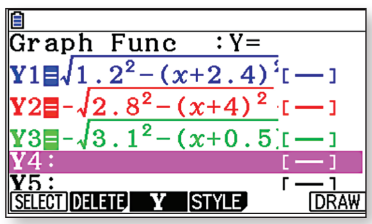
13. Standard equation: $(x + 0.5)^2 + (y - 3.5)^2 = 3.1^2$

14. Explicit functions: $y \pm \sqrt{3.1^2 - (x + 0.5)^2 + 3.5}$

15. The domain used to get a better fit was [-3.9, -1.5]

16.





$Y1 = \pm\sqrt{1.2^2 - (x + 2.4)^2 + 3.1}$

$Y2 = \pm\sqrt{2.8^2 - (x + 4.0)^2} + 3.4$, [-3.2,-1.2]

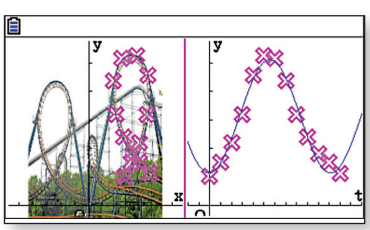
$Y3 = \pm\sqrt{3.1^2 - (x + 0.5)^2} + 3.5$, [-3.9,-1.5]

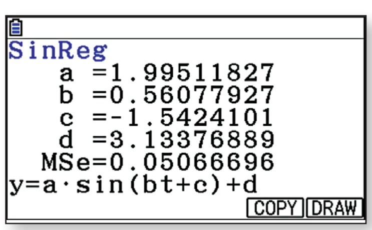
Extension Solutions

ANSWERS WILL VARY, DEPENDING ON THE PLOTTED POINTS.

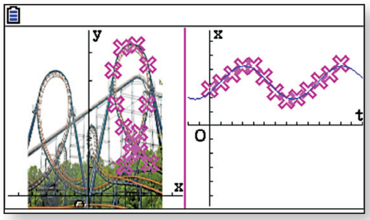
1. Answers may vary, depending on the plotted points.

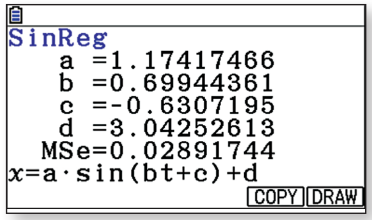
2. One possible solution: $Y - T = 2\sin(0.56T - 1.54) + 3.13$



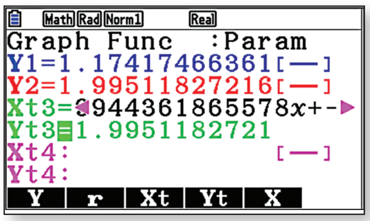


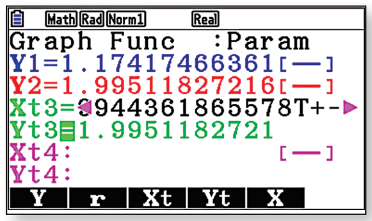
3. One possible solution: X –T = 1.17sin (07T –0.63) + 3.04





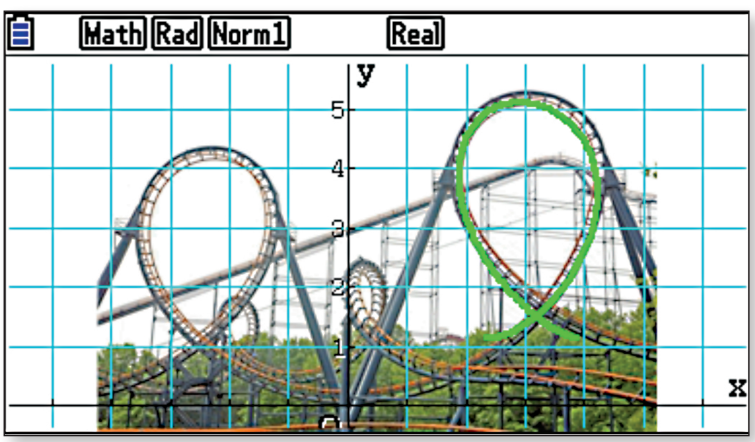
4. One possible solution: Xt = 1.17sin (0.7T –0.63) + 3.04
Yt = 2sin (0.56T –1.54) + 3.13





5. One possible response: T θmin = 0
T θmax = 1

6.



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