Linear inequalities (2)

TARGET To understand how to solve linear inequalities.

STUDY GUIDE

Mathematical theorems and concepts are explained in detail. A scientific calculator is used to check and derive formulas according to the topic.

How to solve linear inequalities

An inequality expressing a range of possible values for x is called an **inequality** for x. The range of values of x that satisfy the inequality for x is called the **solution of the inequality**. Finding all the solutions to an inequality is called **solving the inequality**. When all the terms of an inequality are arranged on the left side, such as "(linear expression of x)≥ 0" or "(linear expression of x)<0", then the left side is an inequality expressed as a linear expression, which is called a **linear inequality**.

Ex. How to solve the linear inequality $3x-5<7$

From the properties of inequalities that if $a<b$ then $a+c<b+c$, we get $3x-5+5<7+5$ and $3x<12$ …(i)

From the properties of inequalities that when $m>0$, if $a<b$ then $\frac{a}{m}<\frac{b}{m}$, we get $\frac{3x}{3}<\frac{12}{3}$ and $x<4$ …(ii)

So, in the expression derived in (i) by the above transformation, we could consider moving -5, from $3x-5<7$, to the right side, which changed the sign to $+5$, so we could derive $3x<7+5$. In other words, the inequality can be solved by transposition, **the same as when solving equations**.

EXERCISE

Students learn basic examples based on the explanation in STUDY GUIDE.

1 Solve the following linear inequalities.

(1) $4x-3<2x+9$

Transpose to get $4x-2x<9+3$

Arrange to get $2x<12$

Divide both sides by 2 to get $x<6$

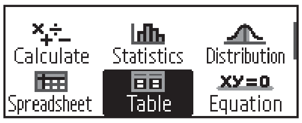
$x<6$

check

Explains how to use the scientific calculator to solve problems and check answers.

On the scientific calculator, use the Table function to compare both sides of the inequality.

Press ⌂, select [Table], press OK, then clear the previous data by pressing ⏻

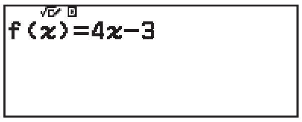


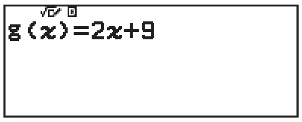
Press ⋯, select [Define f(x)/g(x)], press OK,

select [Define f(x)], press OK

After inputting f(x)$=4x-3$, press EXE

In the same way, input g(x)$=2x+9$.





Press ⋯, select [Table Range], press OK

After inputting [Start:4, End:7, Step:1], select [Execute], press EXE

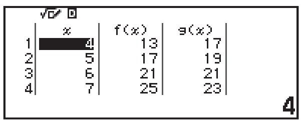
Check the start and end of the so

When $x<6$, then $4x-3<2x+9$,

when $x=6$, then $4x-3=2x+9$,

when $x>6$, then $4x-3>2x+9$, so each can be checked.

The content of this page is part of "1. Algebraic Expressions and Linear Functions".



(2) $5x+6 \geq 8x-9$

Transpose to get $5x-8x \geq -9-6$

Arrange to get $-3x \geq -15$

Divide both sides by -3 to get $x \leq 5$

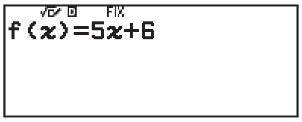
$x \leq 5$

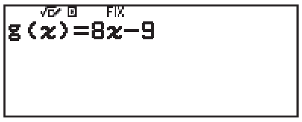
check

Press ⌂, select [Table], press OK, then clear the previous data by pressing ⏻

Press ⋯, select [Define $\mathrm{f}(x)/\mathrm{g}(x)$], press OK,

select [Define $\mathrm{f}(x)$], press OK



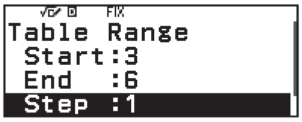


After inputting $\mathrm{f}(x)=5x+6$, press EXE

In the same way, input $\mathrm{g}(x)=8x-9$.

Press ⋯, select [Table Range], press OK

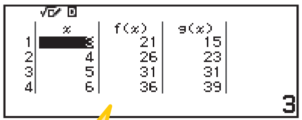
After inputting [Start:3, End:6, Step:1], select [Execute], press EXE



When $x<5$, then $5x+6>8x-9$,

when $x=5$, then $5x+6=8x-9$,

when $x>5$, then $5x+6<8x-9$, so each can be checked.



[Table] is one of the distinctive functions of the fx-991CW that is useful for summarizing a large number of values in a table for easy viewing, or for finding some kind of rule from the created table. The fx-991CW allows students to input two functions, $\mathrm{f}(x)$ and $\mathrm{g}(x)$. Here, they can use [Table] to examine how the values on both sides of a linear inequality change. This process is also useful for understanding the nature of linear inequalities.

(3) $\frac{1}{3}x-1 \leq \frac{1}{2}x+1$

Multiply both sides by 6 to get $\left(\frac{1}{3}x-1\right)$

Remove the brackets to get $2x-6 \leq 3x+6$

Transpose to get $2x-3x \leq 6+6$

Arrange to get $-x \leq 12$

Divide both sides by -1 to get $x \geq -12$

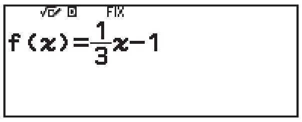
check

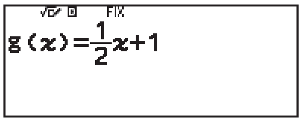
Press ⌂, select [Table], press OK, then clear the previous data by pressing ⏻

Press ⋯, select [Define $\mathrm{f}(x)/\mathrm{g}(x)$], press OK, select [Define $\mathrm{f}(x)$], press OK

After inputting $\mathrm{f}(x)=\frac{1}{3}x-1$, press EXE

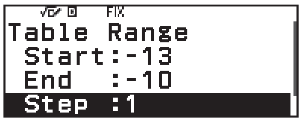
In the same way, input $\mathrm{g}(x)=\frac{1}{2}x+1$.





Press ⋯, select [Table Range], press OK

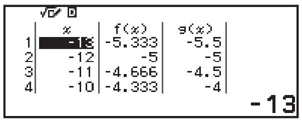
After inputting [Start:-13, End:-10, Step:1], select [Execute], press EXE



When $x<-12$, then $\frac{1}{3}x-1>\frac{1}{2}x+1$,

when $x=-12$, then $\frac{1}{3}x-1=\frac{1}{2}x+1$,

when $x>-12$, then $\frac{1}{3}x-1<\frac{1}{2}x+1$, so each can be checked.



Where to use the scientific calculator.

2 If 1 piece of item A costs $\$8$ and 1 piece of item B costs $\$5$, and you want to buy 10 pieces at a total price of less than or equal to $\$62$, then what is the maximum number of pieces of item A that you should buy?

Let the number of pieces of item A be x pieces.

The number of pieces of item B is $(10-x)$ pieces, so the total cost is $\$\{8x+5(10-x)\}$.

Since it is less than or equal to $\$62$, we get $8x+5(10-x) \leq 62$.

So, the answer is $8x+50-5x \leq 62$, $8x-5x \leq 62-50$, $3x \leq 12$, so $x \leq 4$

Therefore, you should buy 4 or fewer pieces of item A.

4 or fewer pieces

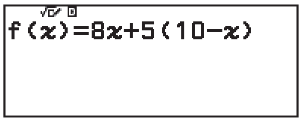
check

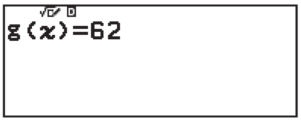
Press ⌂, select [Table], press OK, then clear the previous data by pressing ⏻

Press ⋯, select [Define f(x)/g(x)], press OK,

select [Define f(x)], press OK

After inputting $f(x)=8x+5(10-x)$, press EXE



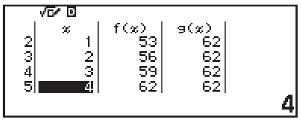


In the same way, input $g(x)=62$.

Press ⋯, select [Table Range], press OK

After inputting [Start:0, End:4, Step:1], select [Execute], press EXE

When $x \leq 4$, then we can confirm that $8x+5(10-x) \leq 62$.



Students can do practice problems similar to those in EXERCISE. They can also practice using the scientific calculator as they learned to in check.
In PRACTICE and ADVANCED the answers are printed in red. (Separate data is also available without the red parts, so it can be used for exercises.)

PRACTICE

◆ Solve the following linear inequ

(1) $7x-4<4x+8$

Transpose to get $7x-4x<8+4$

Arrange to get $3x<12$

Divide both sides by 3 **to get** $x<4$

$x<4$

check

Press ⌂**, select** [Table]**, press** OK**, then clear the previous data by pressing** ⏻

Press ⋯**, select** [Define $\mathrm{f}(x)/\mathrm{g}(x)$]**, press** OK**, select** [Define $\mathrm{f}(x)$]**, press** OK

After inputting $\mathrm{f}(x)=7x-4$**, press** EXE

In the same way, input $\mathrm{g}(x)=4x+8$.

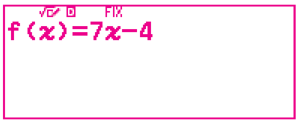
Press ⋯**, select** [Table Range]**, press** OK

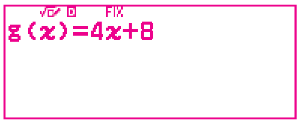
After inputting [Start:2, End:5, Step:1]**, select** [Execute]**, press** EXE

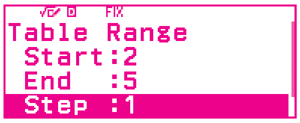
When $x<4$**, then** $7x-4<4x+8$,

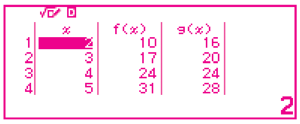
when $x=4$**, then** $7x-4=4x+8$,

when $x>4$**, then** $7x-4>4x+8$**, so each can be checked.**









(2) $2x+5\geq 3x-2$

Transpose to get $2x-3x\geq -2-5$

Arrange to get $-x\geq -7$

Divide both sides by -1 **to get** $x\leq 7$

$x\leq 7$

check

Press ⌂**, select** [Table]**, press** OK**, then clear the previous data by pressing** ⏻

Press ⋯**, select** [Define $\mathrm{f}(x)/\mathrm{g}(x)$]**, press** OK**, select** [Define $\mathrm{f}(x)$]**, press** OK

After inputting $\mathrm{f}(x)=2x+5$**, press** EXE

In the same way, input $\mathrm{g}(x)=3x-2$.

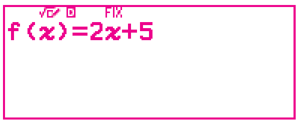
Press ⋯**, select** [Table Range]**, press** OK

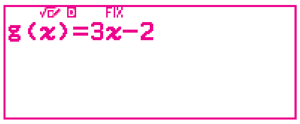
After inputting [Start:6, End:9, Step:1]**, select** [Execute]**, press** EXE

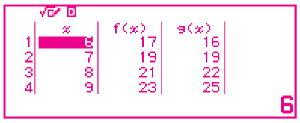
When $x<7$**, then** $2x+5>3x-2$,

when $x=7$**, then** $2x+5=3x-2$,

when $x>7$**, then** $2x+5<3x-2$**, so each can be checked.**







(3) $7x-2>3(3x+4)+6$

Remove the brackets to get $7x-2>9x+12+6$

Transpose to get $7x-9x>12+6+2$

Arrange to get $-2x>20$

Divide both sides by -2 to get $x<-10$

$x<-10$

check

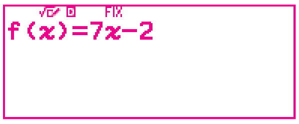
Press ⌂, select [Table], press OK, then clear the previous data by pressing ⏻

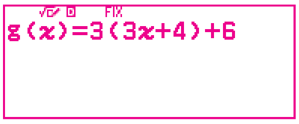
Press ⋯, select [Define $f(x)/g(x)$],

press OK, select [Define $f(x)$], press OK

After inputting $f(x)=7x-2$, press EXE

In the same way, input $g(x)=3(3x+4)+6$.

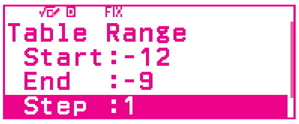




Press ⋯, select [Table Range], press OK

After inputting [Start:-12, End:-9, Step:1],

select [Execute], press EXE

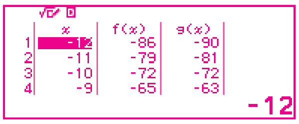


When $x<-10$, then $7x-2>3(3x+4)+6$,

when $x=-10$, then $7x-2=3(3x+4)+6$,

when $x>-10$, then $7x-2<3(3x+4)+6$,

so each can be checked.



PRACTICE

1 Determine the axes and vertexes of the graph of the quadratic function $y = x^2 - 8x + 10$.

$$y = x^2 - 8x + 10 = (x - 4)^2 - 4^2 + 10 = (x - 4)^2 - 6$$

Therefore, we get an axis of $x=4$ and a vertex of $(4, -6)$.

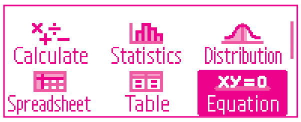
$x=4, (4, -6)$

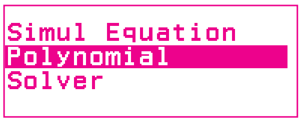
check

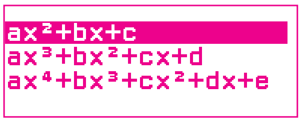
Press ⌂**, select [Equation], press** OK

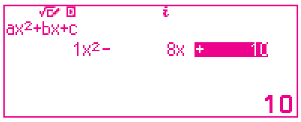
Select [Polynomial], press OK**, select [$ax^2 + bx + c$], press** OK

1 EXE − 8 EXE 1 0 EXE EXE

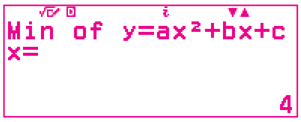


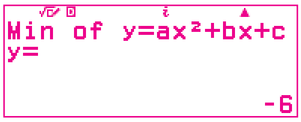


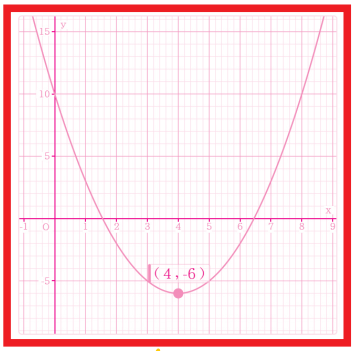




EXE EXE EXE **Press** ⇧ x**, scan the QR code to display a graph.**







Students can generate a QR code based on the calculation results and data displayed on the scientific calculator, and then scan it with a smartphone or tablet to display a graph. This is one of the distinctive features of the fx-991CW. Here, a graph of $y=x^2-8x+10$ is drawn to help students understand visually that the vertex of the graph is $(4,-6)$.

The content of this page is part of "2. Quadratic Functions".

2 Draw the graphs of the following functions.

(1) $y = 2x^2 - 8x + 7$

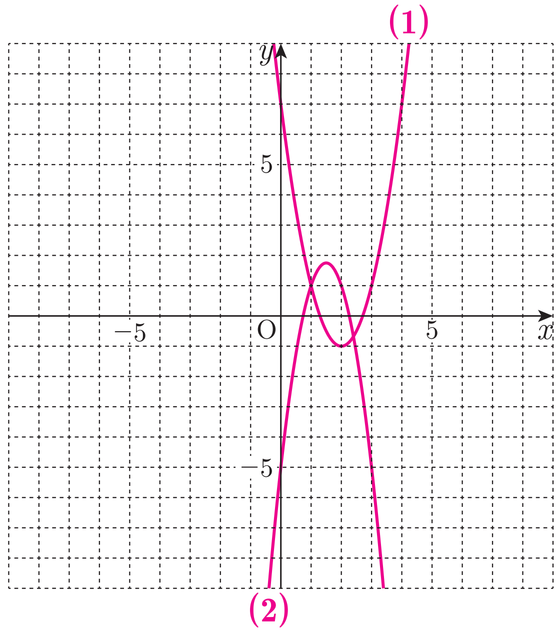
$$y = 2x^2 - 8x + 7 = 2(x^2 - 4x) + 7$$
$$= 2\{(x-2)^2 - 2^2\} + 7 = 2(x-2)^2 - 1$$

Therefore, we get an axis of $x=2$ and a vertex of $(2, -1)$.

(2) $y = -3x^2 + 9x - 5$

$$y = -3x^2 + 9x - 5 = -3(x^2 - 3x) - 5$$
$$= -3\left\{\left(x - \frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2\right\} - 5 = -3\left(x - \frac{3}{2}\right)^2 + \frac{7}{4}$$

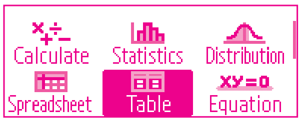
Therefore, we get an axis of $x = \frac{3}{2}$ and a vertex of $\left(\frac{3}{2}, \frac{7}{4}\right)$.

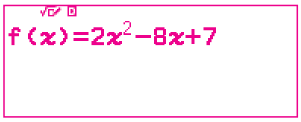


check

Press ⌂, select [Table], press OK, then clear the previous data by pressing ⏻

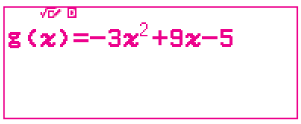
Press ⋯, select [Define f(x)/g(x)], press OK, select [Define f(x)], press OK, after inputting $f(x) = 2x^2 - 8x + 7$, press EXE



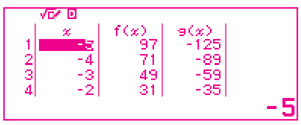


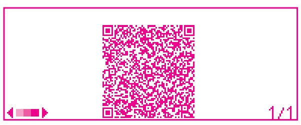
In the same way, input $g(x) = -3x^2 + 9x - 5$.

Press ⋯, select [Table Range], press OK, after inputting [Start:−5, End:5, Step:1], select [Execute], press EXE

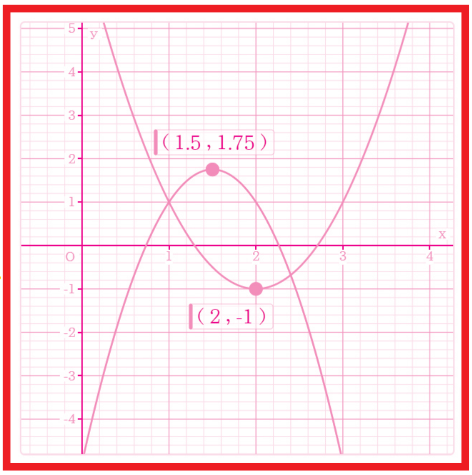


Press ⇧ ⓧ, scan the QR code to display a graph.





By using [Table], students can show the points that a graph passes through on a table. By plotting these points, they can even easily draw the graph freehand.
In addition, students can use a QR code to confirm whether the graph they have drawn is correct.



ADVANCED

Practical problems have been included in several topics. Solutions using a scientific calculator are also presented as necessary.

3 We know that when an object is thrown vertically upward from a height of 0 m at a velocity of vm/s, the relation between the height y m of the object and the time x seconds since it was thrown is $y=-4.9x^2+vx$. So, when a ball is thrown vertically from a height of 0 m at a velocity of 9.8 m/s, find the time it takes to reach the highest point and the height of that highest point. Note that the air resistance can be ignored.

$$y=-4.9x^2+9.8x=-4.9(x^2-2x)=-4.9\{(x-1)^2-1^2\}=-4.9(x-1)^2+4.9$$

Therefore, the graph of this function is convex upward, the axis is $x=1$, and the vertex is $(1, 4.9)$.

Thus, the ball takes 1 second to reach its highest point, and the height of that highest point is 4.9 m.

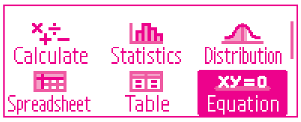
1 second, 4.9 m

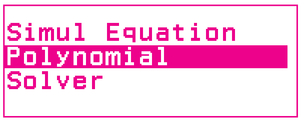
check

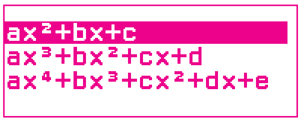
Press ⌂, select [Equation], press OK

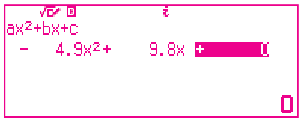
Select [Polynomial], press OK, select [ax^2+bx+c], press OK

⊖ 4 . 9 EXE 9 . 8 EXE 0 EXE EXE

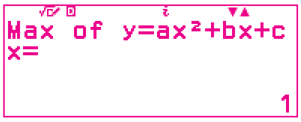


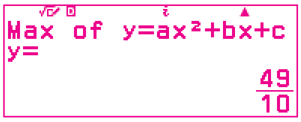


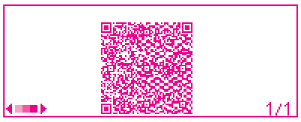




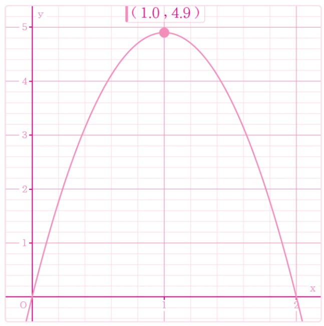
EXE EXE EXE **Press ⇧ x, scan the QR code to display a graph.**







Real-life problems are also described. In this question, students can use what they learned in PRACTICE two pages earlier.



At the vertex of the graph, y (height) starts to decrease, so we know the instant that the ball begins to fall. Note that the graph is not the trajectory of the ball.

Finding an angle

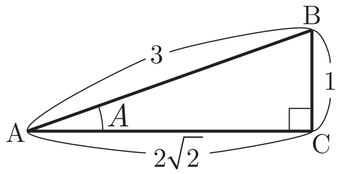
TARGET To understand how to use 2 sides of a right-angled triangle to find the angles of the right-angled triangle.

Students can identify the objective to learn in each section.

STUDY GUIDE

Inverse functions of trigonometric ratio (inverse trigonometric functions)

Angles can be found by using the values of the trigonometric ratio.



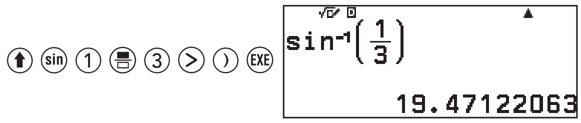
Ex. When $\sin A = \frac{1}{3}$, angle A is expressed as follows.

Simple examples on how to apply equations and theorems.

$A = \mathrm{si}$

$$\sin A = \frac{1}{3} \quad \Rightarrow \quad A = \sin^{-1}\frac{1}{3} = 19.47^\circ$$

(*Namely $\sin 19.47^\circ = \frac{1}{3}$)



In the same way, from $\cos A = \frac{2\sqrt{2}}{3}, \tan A = \frac{1}{2\sqrt{2}}$

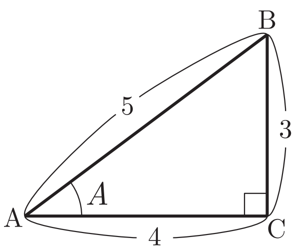
we can show $A = \cos^{-1}\frac{2\sqrt{2}}{3}, A = \tan^{-1}\frac{1}{2\sqrt{2}}$.

19.47°

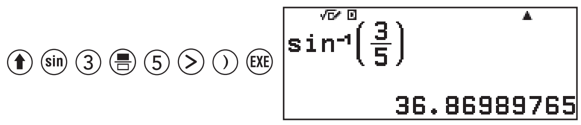
EXERCISE

◆ Find the measure of A to the 2nd decimal place for the $\triangle$ABC below.

(1)

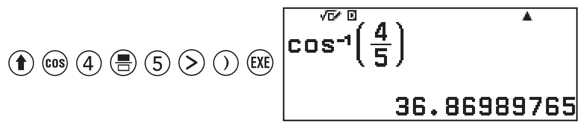


When $\sin A = \frac{3}{5}$, we get $A = \sin^{-1}\frac{3}{5} = 36.87^\circ$.



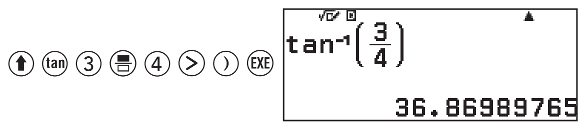
36.87°

OTHER METHODS From $\cos A = \frac{4}{5}$, we get $A = \cos^{-1}\frac{4}{5} = 36.87^\circ$.



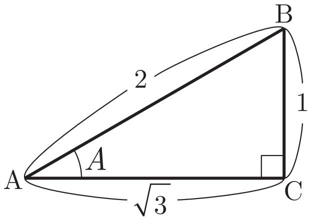
OTHER METHODS From $\tan A = \frac{3}{4}$, we get $A = \tan^{-1}\frac{3}{4} = 36.87^\circ$.

Alternative solutions and different verification methods for previously presented problems.



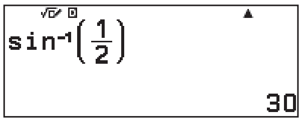
The content of this page is part of "3. Trigonometry".

(2)



From $\sin A = \dfrac{1}{2}$, we get $A = \sin^{-1}\dfrac{1}{2} = 30^\circ$.

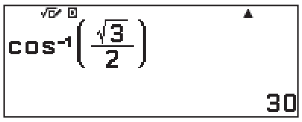
SHIFT sin 1 ⊟ 2 ▷) EXE



30°

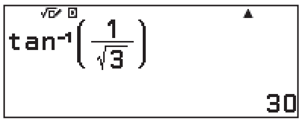
OTHER METHODS From $\cos A = \dfrac{\sqrt{3}}{2}$, we get $A = \cos^{-1}\dfrac{\sqrt{3}}{2} = 30^\circ$.

SHIFT cos √■ 3 ▷ ⊟ 2 ▷) EXE



OTHER METHODS From $\tan A = \dfrac{1}{\sqrt{3}}$, we get $A = \tan^{-1}\dfrac{1}{\sqrt{3}} = 30^\circ$.

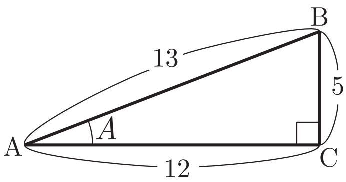
SHIFT tan 1 ⊟ √■ 3 ▷ ▷) EXE



PRACTICE

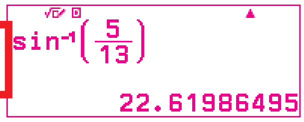
◆ Find the measure of A to the 2nd decimal place for the $\triangle$ABC below.

(1)



From $\sin A = \dfrac{5}{13}$, we get $A = \sin^{-1}\dfrac{5}{13} = 22.62^\circ$.

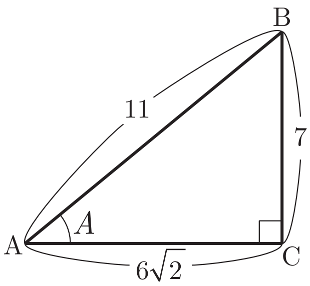
SHIFT sin 5 ⊟ 1 3 ▷) EXE



22.62°

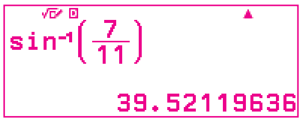
Key operations are carefully explained to students who are using a scientific calculator for the first time.

(2)



From $\sin A = \dfrac{7}{11}$, we get $A = \sin^{-1}\dfrac{7}{11} = 39.52^\circ$.

SHIFT sin 7 ⊟ 1 1 ▷) EXE



39.52°

Tutorial videos are also available for students who are new to the fx-991CW. Be sure to view these!

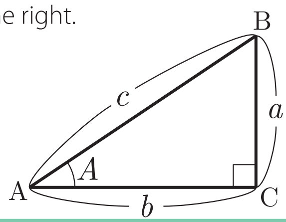
Trigonometric ratio formula including tangent

TARGET To understand a trigonometric ratio formula including tangent.

STUDY GUIDE

Correlation of trigonometric ratios (2)

The following relations are derived from the right-angled triangle shown in the diagram on the right.



$$(1)\ \frac{\sin A}{\cos A} = \tan A \qquad (2)\ 1 + \tan^2 A = \frac{1}{\cos^2 A}$$

explanation

(1) From the right-angled triangle shown in the diagram on the right

$$\frac{\sin A}{\cos A} = \frac{\frac{a}{c}}{\frac{b}{c}} = \frac{a}{b} = \tan A$$

$$\Rightarrow \frac{\sin A}{\cos A} = \tan A$$

EXTRA Info.

Knowledge and information on formulas and other supplementary information in other units.

Calculating fractions

$$\frac{\frac{A}{B}}{\frac{C}{D}} = \frac{A \times D}{B \times C}$$

Proof $\frac{\frac{A}{B}}{\frac{C}{D}} = \frac{A}{B} \div \frac{C}{D} = \frac{A}{B} \times \frac{D}{C} = \frac{A \times D}{B \times C}$

Ex. $\frac{\frac{3}{5}}{\frac{7}{4}} = \frac{3 \times 4}{5 \times 7} = \frac{12}{35}$

Press ⌂, select [Calculate], press OK

3 ⊟ 5 ▷ ⊟ 7 ⊟ 4 EXE

(2) Divide both sides of $\sin^2 A + \cos^2 A = 1$ by $\cos^2 A$

such that $\frac{\sin^2 A}{\cos^2 A} + \frac{\cos^2 A}{\cos^2 A} = \frac{1}{\cos^2 A}$

$$\left(\frac{\sin A}{\cos A}\right)^2 + 1 = \frac{1}{\cos^2 A}$$

$$\Rightarrow \tan^2 A + 1 = \frac{1}{\cos^2 A}$$

The content of this page is part of "3. Trigonometry".

Proofs and checks of mathematical formulas.

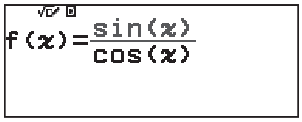
proof

(1) Calculate the value of the function to confirm the formula (1).

Press ⌂, select [Table], press OK, then clear the previous data by pressing ⏻

Press ⋯, select [Define $f(x)/g(x)$], press OK, select [Define $f(x)$], press OK

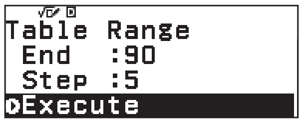
Input $f(x)=\dfrac{\sin x}{\cos x}$. sin x) ÷ cos x) EXE



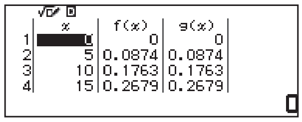
Press ⋯, select [Define $f(x)/g(x)$], press OK, select [Define $g(x)$], press OK

Input $g(x)=\tan x$. tan x) EXE

Press ⋯, select [Table Range], press OK, after inputting [Start:0, End:90, Step:5], select [Execute], press EXE



We can confirm that $\dfrac{\sin A}{\cos A}=\tan A$ holds regardless of the value of A.



When $x=0$, then $f(x)=g(x)=0$

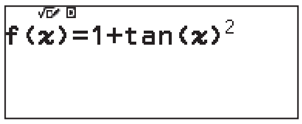
When $x=5$, then $f(x)=g(x)=0.0874$

As shown below, for any x, we get $f(x)=g(x)$.

(2) Calculate the value of the function to confirm the formula (2).

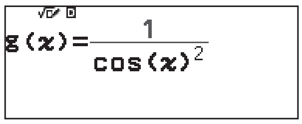
Press ⏻ to clear the previous data, press ⋯, select [Define $f(x)/g(x)$], press OK, select [Define $f(x)$], press OK

Input $f(x)=1+\tan^2 x$. 1 + tan x) x² EXE

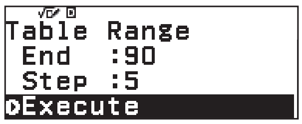


Press ⋯, select [Define $f(x)/g(x)$], press OK, select [Define $g(x)$], press OK

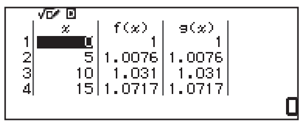
Input $g(x)=\dfrac{1}{\cos^2 x}$. 1 ÷ cos) x² EXE



Press ⋯, select [Table Range], press OK, after inputting [Start:0, End:90 p:5], select [Execute], press EXE



We can confirm that $1+\tan^2 A=\dfrac{1}{\cos^2 A}$ holds regardless of the va A.



On this page, students are using [Table] to check what the formula is asserting. Specifically, for each of the formulas (1) and (2) on the previous page, the students are checking that $f(x)=g(x)$ holds for any value of x, with $f(x)$ on the left side and $g(x)$ on the right side. This is an effective way to correctly understand the meaning of a new formula.

Uses of exponential functions

TARGET To understand how to solve problems using graphs of exponential functions.

EXERCISE Real-life problems are also described.

◆ Suppose we examined the transmission of an infectious disease every day, and our results showed that the number of newly infected people increased by r times after 1 day. For example, on a given day 100 people are infected, and if r=1.5, then after 1 day the number of newly infected people would be 100×1.5=150 (people), and after 2 days it would be 150×1.5=225 (people). Calculating this from 2 days ago, gives us 100×1.5^2 (people). From this, we can realize the following. That after x days, the number of newly infected people will be y times the initial number of people, which we can express as $y=r^x$. Now, solve the following problems.

(1) Given that r=1.5 does not change. Find how many times the number of people infected on the first day would be newly infected after 10 days. Also, given that 100 people were infected on the first day, find how many people would be newly infected after 10 days.

Substitute x=10 into $y=1.5^x$, such that $y=1.5^{10}=57.66\cdots$ (times).

The number of people newly infected after 10 days would be

100×57.66⋯=5766.⋯ (people).

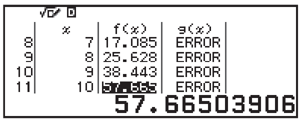
Press ⌂, select [Table], press OK, then clear the previous data by pressing ⏻

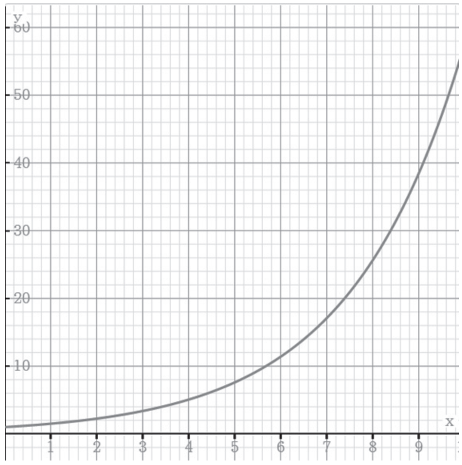
Press ⋯, select [Define f(x)/g(x)], press OK, select [Define f(x)], press OK

After inputting $f(x)=1.5^x$, press EXE

Press ⋯, select [Table Range], press OK

After inputting [Start:0, End:10, Step:1], select [Execute], press EXE





Approximately 58 times, approximately 5800 people

(2) Find a value for r that would reduce the number of newly infected people.

For example, examine what happens when r=0.8 and 0.4.

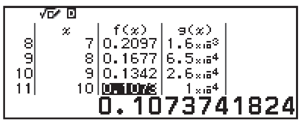
Press ⌂, select [Table], press OK, then clear the previous data by pressing ⏻

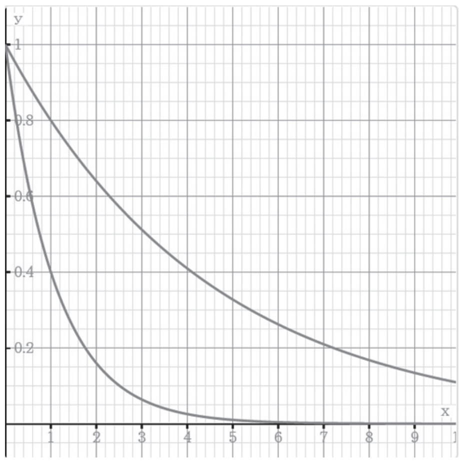
Press ⋯, select [Define f(x)/g(x)], press OK, select [Define f(x)], press OK, after inputting $f(x)=0.8^x$, press EXE

In the same way, input $g(x)=0.4^x$.

Press ⋯, select [Table Range], press OK

After inputting [Start:0, End:10, Step:1], select [Execute], press EXE





When $0<r<1$, the curve falls to the right, so we know it is decreasing.

$0<r<1$

The content of this page is part of "5. Exponential and Logarithmic Functions".

(3) Find a range of values for r such that the number of people infected on the 20th day would be fewer than 100 people, given that 10000 people are initially infected.

For there to be 100 people after 20 days, the ratio to the initial number of people would be $\frac{100}{10000}=\frac{1}{100}$ (times).

Substitute x=20 and $y=\frac{1}{100}$ into $y=r^x$ to find r.

From $\frac{1}{100}=r^{20}$, we get $r=\left(\frac{1}{100}\right)^{\frac{1}{20}}=0.794\cdots$.

Given that after x days, the number of newly infected people will be z people, we can express this as $z=10000r^x$.

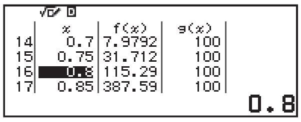
Press ⌂, select [Table], press OK, then clear the previous data by pressing ⏻

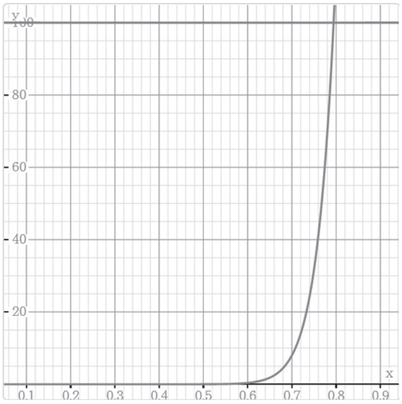
Press ⋯, select [Define f(x)/g(x)], press OK, select [Define f(x)], press OK, after inputting $f(x)=10000x^{20}$, press EXE

In the same way, input $g(x)=100$.

Press ⋯, select [Table Range], press OK

After inputting [Start:0.05, End:0.95, Step:0.05], select [Execute], press EXE

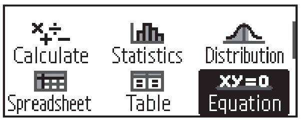


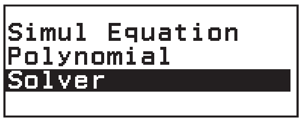


From the graph, we can find it is somewhat below 0.8.

Furthermore, by using the Solver function to solve $10000x^{20}=100$, we get x=0.794…≃0.79.

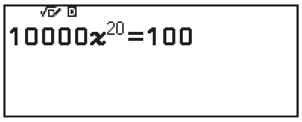
Press ⌂, select [Equation], press OK, select [Solver], press OK





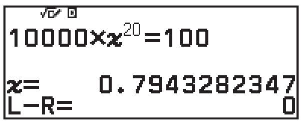
Input 10000x^{20} = 100.

1 0 0 0 0 × x ■ 2 0 ▷ ⇧ (1 0 0 EXE



Input 0.8.

0 . 8 EXE EXE



0<r≤0.79

When tackling real-life problems, we may need to use mathematical formulas that we cannot process by hand calculations, but we can do them smoothly by using a scientific calculator.
In this question, the student needs to solve the equation $10000x^{20}=100$, so they can use the scientific calculator's [Table] and graphs to guess a solution then use [Solver] to improve the accuracy of their solution.

Distance between points and lines

TARGET To understand about distances between points and lines.

STUDY GUIDE

Distance between points and lines

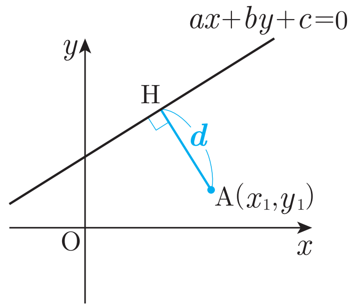
When we let H be the intersection of a perpendicular line drawn from point A to the line l, then the length d of AH is called the **distance** between point A and the line l, which we can find as follows.

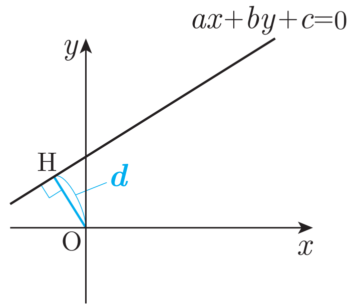
(1) d is the distance between $A(x_1, y_1)$ and line $ax+by+c=0$

$$d=\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$$

(2) d is the distance between the origin O and line $ax+by+c=0$

$$d=\frac{|c|}{\sqrt{a^2+b^2}}$$





Formulas and their supplementary explanations.

explanation

Let the points be $A(x_1, y_1)$ and $H(x_2, y_2)$, such that $d=AH=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $\ldots$(i)

Since the line AH is perpendicular to the line $ax+by+c=0$, we get $bx-ay+c'=0$ $\ldots$(ii).

It passes through point $A(x_1, y_1)$, so we get $bx_1-ay_1+c'=0$, $c'=-bx_1+ay_1$

By substituting this into (ii), we get $bx-ay-bx_1+ay_1=0$, so $b(x-x_1)-a(y-y_1)=0$

The line AH passes through $H(x_2, y_2)$, so we get $b(x_2-x_1)-a(y_2-y_1)=0$ $\ldots$(iii)

Since point $H(x_2, y_2)$ is a point on the line $ax+by+c=0$, we get $ax_2+by_2+c=0$

By transforming this, we get $a(x_2-x_1)+b(y_2-y_1)+ax_1+by_1+c=0$ $\ldots$(iv)

If we solve (iii) and (iv) for x_2-x_1 and y_2-y_1, then we get

$$x_2-x_1=-\frac{a(ax_1+by_1+c)}{a^2+b^2}, y_2-y_1=-\frac{b(ax_1+by_1+c)}{a^2+b^2}$$

These are substituted into (i), for

$$d=\sqrt{\left\{-\frac{a(ax_1+by_1+c)}{a^2+b^2}\right\}^2+\left\{-\frac{b(ax_1+by_1+c)}{a^2+b^2}\right\}^2}=\sqrt{\frac{(a^2+b^2)(ax_1+by_1+c)^2}{(a^2+b^2)^2}}=\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$$

If the point A is the origin, then $x_1=0$ and $y_1=0$, so $d=\frac{|c|}{\sqrt{a^2+b^2}}$

The content of this page is part of "6. Equations of Lines and Circles".

EXERCISE

Find the distances between the following points and lines.

(1) Point $(2, 3)$ and line $5x-4y+8=0$

$$d=\frac{|5\cdot 2-4\cdot 3+8|}{\sqrt{5^2+(-4)^2}}=\frac{6}{\sqrt{41}}=\frac{6\sqrt{41}}{41}$$

$$\frac{6\sqrt{41}}{41}$$

(2) The origin and line $2x+3y-9=0$

$$d=\frac{|-9|}{\sqrt{2^2+3^2}}=\frac{9}{\sqrt{13}}=\frac{9\sqrt{13}}{13}$$

$$\frac{9\sqrt{13}}{13}$$

(3) Point $(1, -5)$ and line $y=2x+3$

By transforming $y=2x+3$, we get $2x-y+3=0$

$$d=\frac{|2\cdot 1-1\cdot(-5)+3|}{\sqrt{2^2+(-1)^2}}=\frac{10}{\sqrt{5}}=2\sqrt{5}$$

$$2\sqrt{5}$$

Repeating similar calculations is one of the things that a scientific calculator is good at. By using [VARIABLE], students can see how values change when they change multiple variables (in this question, when the values of **A**, **B**, **C**, x, and y are changed). By trying out various values, the students are likely to notice some unexpected regularities!

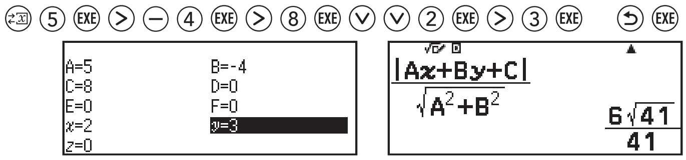
check

On the scientific calculator, use the VARIABLE function to calculate the distance between a point and a line.

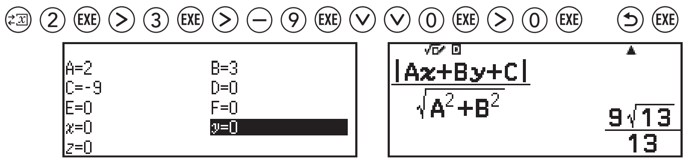
Press ⓐ, select [Calculate], press OK

Input the formula for the distance between a point and a line.

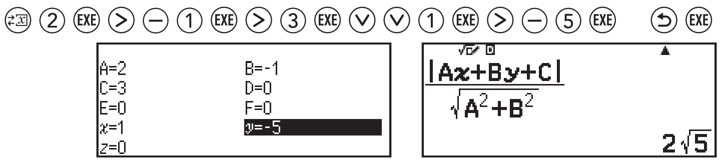
(1) In the VARIABLE screen, input [A=5, B=−4, C=8, x=2, and y=3], and then calculate.



(2) In the same way, input [A=2, B=3, C=−9, x=0, and y=0], and then calculate.



(3) In the same way, input [A=2, B=−1, C=3, x=1, and y=−5], and then calculate.



EXTRA Info.

Use the scientific calculator to confirm that equations and inequalities hold.

EXERCISE

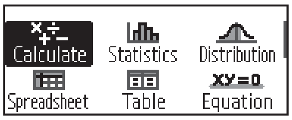
◆ Use the scientific calculator to confirm that the following equations and inequalities hold.

(1) $a^3+b^3=(a+b)^3-3ab(a+b)$

$a^3+b^3=(a+b)^3-3ab(a+b) \Leftrightarrow (a+b)^3-3ab(a+b)-(a^3+b^3)=0$

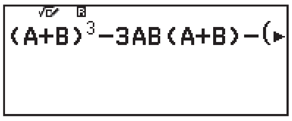
Therefore, we simply need to confirm that the right side − the left side =0 (or, the left side − the right side =0).

Press ⌂, select [Calculate], press OK



Input $(\mathrm{A}+\mathrm{B})^3-3\mathrm{AB}(\mathrm{A}+\mathrm{B})-(\mathrm{A}^3+\mathrm{B}^3)$.

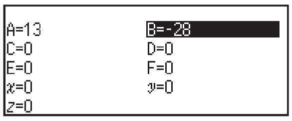
(⇧ 4 + ⇧ 5) ■□ 3 > − 3 ⇧ 4 ⇧ 5 (⇧ 4 + ⇧ 5)
− (⇧ 4 ■□ 3 > + ⇧ 5 ■□ 3 >)

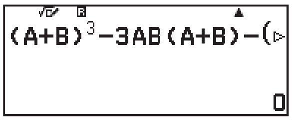


Use the VARIABLE function to assign any values to A and B.

(Example) To input A=13 and B=−28

VARIABLE 1 3 EXE > − 2 8 EXE ↺ EXE





We can confirm $(\mathrm{A}+\mathrm{B})^3-3\mathrm{AB}(\mathrm{A}+\mathrm{B})-(\mathrm{A}^3+\mathrm{B}^3)=0$ using any values, so we can verify the validity of $a^3+b^3=(a+b)^3-3ab(a+b)$ holds. You may want to verify other examples.

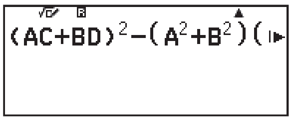
(2) $(ac+bd)^2 \leq (a^2+b^2)(c^2+d^2)$

$(ac+bd)^2 \leq (a^2+b^2)(c^2+d^2) \Leftrightarrow (ac+bd)^2-(a^2+b^2)(c^2+d^2) \leq 0$

Therefore, we simply need to confirm that the left side − the right side ≤0 (or, the right side − the left side ≥ 0).

Input $(\mathrm{AC}+\mathrm{BD})^2-(\mathrm{A}^2+\mathrm{B}^2)(\mathrm{C}^2+\mathrm{D}^2)$.

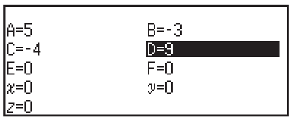
(⇧ 4 ⇧ 6 + ⇧ 5 ⇧ 1) ■²
− (⇧ 4 ■² + ⇧ 5 ■²) (⇧ 6 ■² + ⇧ 1 ■²)

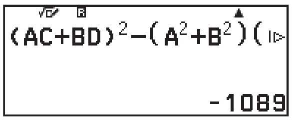


Use the VARIABLE function to assign any values to A, B, C, D.

(Example) To input A=5, B=−3, C=−4, and D=9

VARIABLE 5 EXE > − 3 EXE > − 4 EXE > 9 EXE ↺ EXE





We can confirm $(\mathrm{AC}+\mathrm{BD})^2-(\mathrm{A}^2+\mathrm{B}^2)(\mathrm{C}^2+\mathrm{D}^2)<0$ using any values, so we can verify the validity of $(ac+bd)^2 \leq (a^2+b^2)(c^2+d^2)$ holds. You may want to verify other examples.

On this page, students use [VARIABLE] to verify that the equality and inequality of $a^3+b^3=(a+b)^3-3ab(a+b)$ and $(ac+bd)^2 \leq (a^2+b^2)(c^2+d^2)$ hold.

The content of this page is part of "7. Formulas and Proofs".

7. Formulas and Proofs 28

Sequences

TARGET To understand how to express a sequence and its general term.

STUDY GUIDE

General terms of sequences

Numbers arranged in a row according to a rule are called a **sequence**, and each individual number is called a **term**.

A sequence with a finite number of terms is called a **finite sequence**, and a sequence with an infinite number of terms is called an **infinite sequence**.

We say that a finite sequence has a **number of terms** and there is a **first term** and a **last term**.

Sequences are generally expressed as follows using letters with indices.

$$\boxed{a_1, a_2, a_3, \cdots\cdots, a_n, \cdots\cdots}$$

The sequence above can also be expressed as $\{a_n\}$.

When the nth term in the sequence $\{a_n\}$ is expressed in an expression as n, we say it is the **general term** of the sequence $\{a_n\}$.

Ex. Given the sequence $\{a_n\}$ of 2, 4, 6, 8, and 10 has 5 terms: $a_1=2$(first term), $a_2=4$, $a_3=6$, $a_4=8$, $a_5=10$(last term)

From $a_1=2\cdot1$, $a_2=2\cdot2$, $a_3=2\cdot3$, $a_4=2\cdot4$, and $a_5=2\cdot5$, we can estimate that the general term is $a_n=2n$.

EXERCISE

1 Find the first to 4th terms of the sequence $\{a_n\}$ whose general term is expressed by the following expression.

(1) $a_n=3n+2$

When the first term is $n=1$, we get $a_1=3\cdot1+2=5$

When the 2nd term is $n=2$, we get $a_2=3\cdot2+2=8$

When the 3rd term $n=3$, we get $a_3=3\cdot3+2=11$

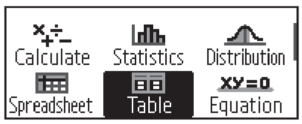
When the 4th term is $n=4$, we get $a_4=3\cdot4+2=14$

$$a_1=5, a_2=8, a_3=11, a_4=14$$

check

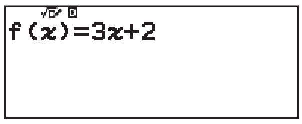
On the scientific calculator, use the Table function to confirm each term of the sequence.

Press ⌂, select [Table], press OK, then clear the previous data by pressing ⏻



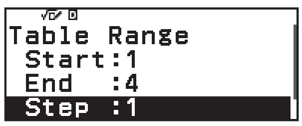
Press ⋯, select [Define $f(x)/g(x)$], press OK, select [Define $f(x)$], press OK

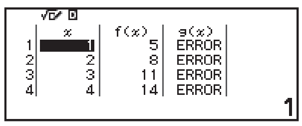
After inputting $f(x)=3x+2$, press EXE



Press ⋯, select [Table Range], press OK

After inputting [Start:1, End:4, Step:1], select [Execute], press EXE





The content of this page is part of "10. Sequences".

(2) $a_n = n^2 - 4n$

When the first term is $n=1$, we get $a_1=1^2-4\cdot1=-3$

When the 2nd term is $n=2$, we get $a_2=2^2-4\cdot2=-4$

When the 3rd term is $n=3$, we get $a_3=3^2-4\cdot3=-3$

When the 4th term is $n=4$, we get $a_4=4^2-4\cdot4=0$

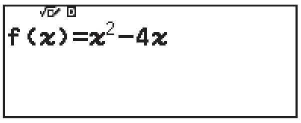
$$\boldsymbol{a_1=-3, a_2=-4, a_3=-3, a_4=0}$$

check

Press (HOME), select [Table], press (OK), then clear the previous data by pressing (⏻)

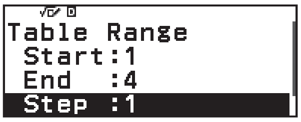
Press (⋯), select [Define f(x)/g(x)], press (OK), select [Define f(x)], press (OK)

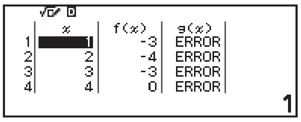
After inputting f$(x)=x^2-4x$, press (EXE)



Press (⋯), select [Table Range], press (OK)

After inputting [Start:1, End:4, Step:1], select [Execute], press (EXE)





[2] Estimate the general terms of the sequences $\{a_n\}$ below.

(1) $-5, -10, -15, -20, \cdots\cdots$

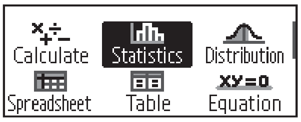
$a_1=-5\cdot1,\ a_2=-5\cdot2,\ a_3=-5\cdot3,\ a_4=-5\cdot4,\ \cdots\cdots$

Therefore, we can estimate the general term to be $a_n=-5n$.

$$\boldsymbol{a_n=-5n}$$

check

On the scientific calculator, use the Statistics function to confirm the general term of the sequence.

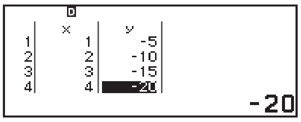


Press (HOME), select [Statistics], press (OK), select [2-Variable], press (OK)

Press (⋯), select [Edit], press (OK), select [Delete All], press (OK)

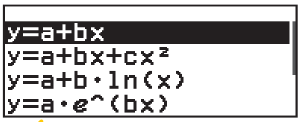
Input 1, 2, 3, and 4 in the x column, and −5, −10, −15, and −20 in the y column, respectively.

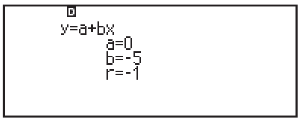
(1) (EXE) (2) (EXE) (3) (EXE) (4) (EXE) (∨) (>)　(−) (5) (EXE) (−) (1) (0) (EXE) (−) (1) (5) (EXE) (−) (2) (0) (EXE) (EXE)



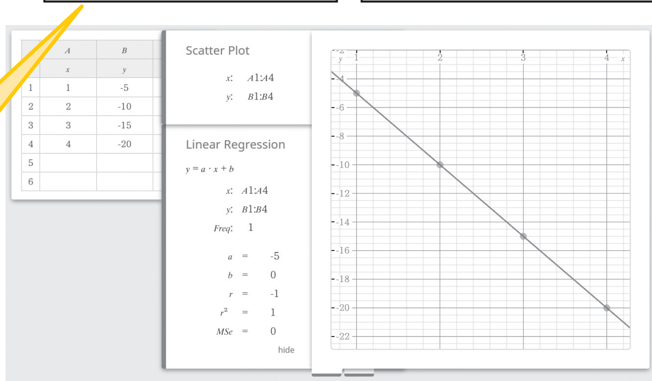
Select [Reg Results], press (OK), select [y=a+bx], press (OK)

We can confirm that $y=-5x$.





Press (↩) (⇧) (x), scan the QR code to display a graph.



This question introduces how students can use [Statistics] to guess the general term of a sequence. This subject is a little advanced, but it allows students to feel the connections between units of mathematics.

(2) 1, 4, 9, 16, ……

$a_1=1^2$, $a_2=2^2$, $a_3=3^2$, $a_4=4^2$, ……

Therefore, we can estimate the general term to be $a_n=n^2$.

$$a_n = n^2$$

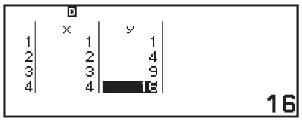
check

Press ⌂, select [Statistics], press OK, select [2-Variable], press OK

Press ⋯, select [Edit], press OK, select [Delete All], press OK

Input 1, 2, 3, and 4 in the x column, and 1, 4, 9, and 16 in the y column, respectively.

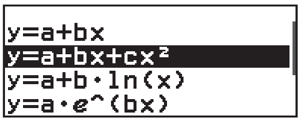
① EXE ② EXE ③ EXE ④ EXE ∨ > ① EXE ④ EXE ⑨ EXE ① ⑥ EXE EXE

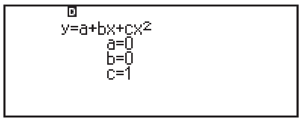


Select [Reg Results], press OK, select [y=a+bx+cx²],

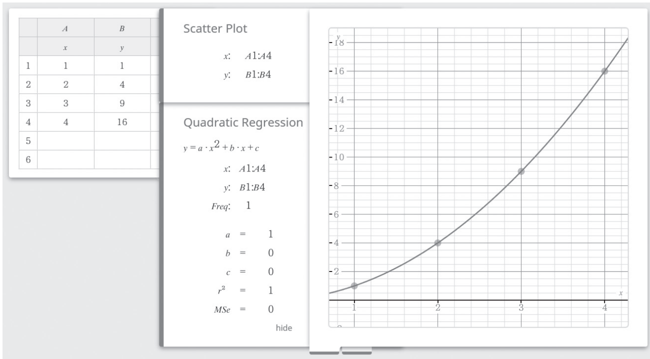
press OK

We can confirm that $y=x^2$.





Press ⮌ ⇧ ⓧ, scan the QR code to display a graph.



Recurrence formula (1)

TARGET

To understand recurrence formulas for arithmetic progressions, geometric progressions, and progressions of differences.

STUDY GUIDE

Recurrence formula

Recurrence formulas for arithmetic progressions and geometric progressions

Given we can derive that $\boldsymbol{a_{n+1}=2a_n+1}$ is between the 2 terms a_n and a_{n+1} that are adjacent to $\{a_n\}$, then by stating that $a_1=1$, we can determine that $a_2=2a_1+1=2\cdot1+1=3$, $a_3=2a_2+1=2\cdot3+1=7$, $a_4=2a_3+1=2\cdot7+1=15$,$\cdots\cdots$for each element in the sequence.

Equations that show us a rule that determines the 1 way to get the next term from the previous term in this way are called **recurrence formulas**.

Given a first term of a_1, a common difference of d, or a common ratio of r, we can express the general term and recurrence formula of arithmetic progressions and geometric progressions as follows.

	Recurrence formula	General term
Arithmetic progression	$a_{n+1}=a_n+d$	$a_n=a_1+(n-1)d$
Geometric progression	$a_{n+1}=ra_n$	$a_n=a_1r^{n-1}$

How to find the general term of a progression of differences

When a recurrence formula is stated as $a_{n+1}=a_n+f(n)$ (formula expressing n), then we can use a progression of differences in the following procedure to find the general term.

(1) Transform it to $a_{n+1}-a_n=f(n)$, and consider $f(n)$ to be the progression of differences $\{b_n\}$ of the sequence $\{a_n\}$.

(2) Find a_n from the equation $a_n=a_1+\sum_{k=1}^{n-1}b_k$ $(n\geq2)$ for finding the general term of the sequence $\{a_n\}$ from the progression of differences $\{b_n\}$.

(3) Confirm whether the formula found in (2) also holds when $n=1$, and then find the general term.

The content of this page is part of "10. Sequences".

EXERCISE

1 Find the 2nd to 5th terms given the first term and recurrence formula are as follows.

(1) $a_1=2,\ a_{n+1}=4a_n+5$

$a_2=4a_1+5=4\cdot2+5=13,\ a_3=4a_2+5=4\cdot13+5=57,\ a_4=4a_3+5=4\cdot57+5=233,\ a_5=4a_4+5=4\cdot233+5=937$

$a_2=13,\ a_3=57,\ a_4=233,\ a_5=937$

(2) $a_1=5,\ a_{n+1}=2a_n-3$

$a_2=2a_1-3=2\cdot5-3=7,\ a_3=2a_2-3=2\cdot7-3=11,\ a_4=2a_3-3=2\cdot11-3=19,\ a_5=2a_4-3=2\cdot19-3=35$

$a_2=7,\ a_3=11,\ a_4=19,\ a_5=35$

(3) $a_1=1,\ a_{n+1}=a_n{}^2+1$

$a_2=a_1{}^2+1=1^2+1=2,\ a_3=a_2{}^2+1=2^2+1=5,\ a_4=a_3{}^2+1=5^2+1=26,\ a_5=a_4{}^2+1=26^2+1=677$

$a_2=2,\ a_3=5,\ a_4=26,\ a_5=677$

check

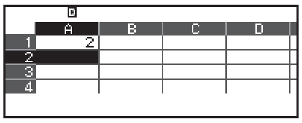
Press ⌂, select [Spreadsheet], press OK, then clear the previous data by pressing ⏻

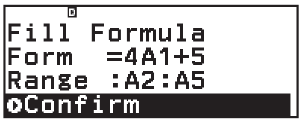
(1) After inputting [A1:2], press EXE

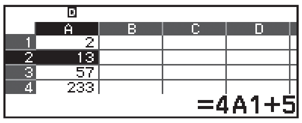
Press ⋯, select [Fill Formula], press OK

After inputting [Form=4A1+5], press EXE

After inputting [Range:A2:A5], press EXE, select [Confirm], press OK







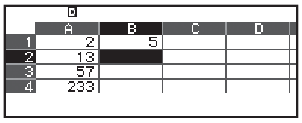
(2) When the sheet is displayed, move to [B1].

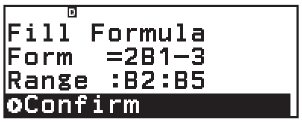
After inputting [B1:5], press EXE

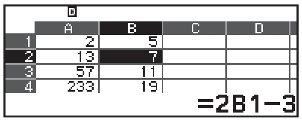
Press ⋯, select [Fill Formula], press OK

After inputting [Form=2B1−3], press EXE

After inputting [Range:B2:B5], press EXE, select [Confirm], press OK







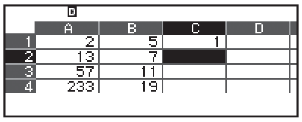
(3) When the sheet is displayed, move to [C1].

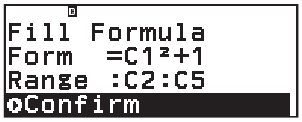
After inputting [C1:1], press EXE

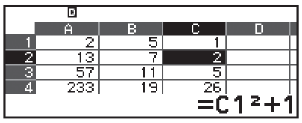
Press ⋯, select [Fill Formula], press OK

After inputting [Form=C1²+1], press EXE

After inputting [Range:C2:C5], press EXE, select [Confirm], press OK







On this page, the student is introduced to using [Spreadsheet] to find the value of each term from the recurrence formula of a sequence of numbers. [Spreadsheet] makes it possible to perform calculations using a 45-row × 5-column spreadsheet.

2 Find the general term the sequence $\{a_n\}$ given the first term and recurrence formula are as follows.

(1) $a_1=7,\ a_{n+1}=a_n-3$

The sequence $\{a_n\}$ is an arithmetic progression with a first term of 7 and a common difference of -3,

so $a_n=7+(n-1)\cdot(-3)=-3n+10$

$$a_n=-3n+10$$

(2) $a_1=5,\ a_{n+1}=2a_n$

The sequence $\{a_n\}$ is a geometric progression with a first term of 5 and a common ratio of 2, so $a_n=5\cdot 2^{n-1}$

$$a_n=5\cdot 2^{n-1}$$

(3) $a_1=3,\ a_{n+1}=a_n+2n$

From $a_{n+1}-a_n=2n$, then for the progression of differences $\{b_n\}$ of the sequence $\{a_n\}$, we get $b_n=2n$.

Given $n\geq 2$, then $a_n=a_1+\sum_{k=1}^{n-1}b_k=3+\sum_{k=1}^{n-1}2k=3+2\cdot\frac{1}{2}(n-1)n=n^2-n+3$

This also holds when $n=1$ and $1^2-1+3=3$.

Therefore, the general term is $a_n=n^2-n+3$

$$a_n=n^2-n+3$$

check

Press ⌂, select [Spreadsheet], press OK, then clear the previous data by pressing ⏻

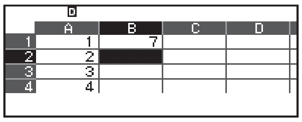
After inputting [A1:1, A2:2, A3:3, and A4:4] respectively, press EXE, move to [B1].

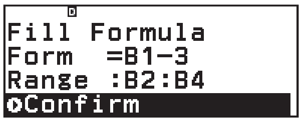
(1) After inputting [B1:7], press EXE

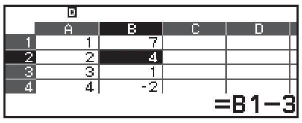
Press ⋯, select [Fill Formula], press OK

After inputting [Form=B1−3], press EXE

After inputting [Range:B2:B4], press EXE, select [Confirm], press OK







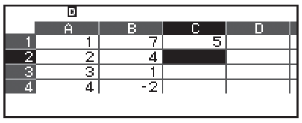
(2) When the sheet is displayed, move to [C1].

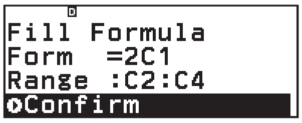
After inputting [C1:5], press EXE

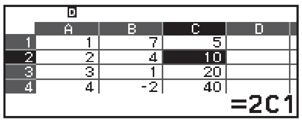
Press ⋯, select [Fill Formula], press OK

After inputting [Form=2C1], press EXE

After inputting [Range:C2:C4], press EXE, select [Confirm], press OK







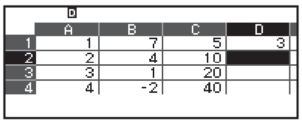
(3) When the sheet is displayed, move to [D1].

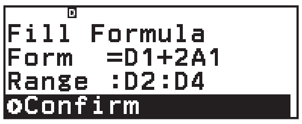
After inputting [D1:3], press EXE

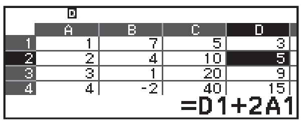
Press ⋯, select [Fill Formula], press OK

After inputting [Form=D1+2A1], press EXE

After inputting [Range:D2:D4], press EXE, select [Confirm], press OK



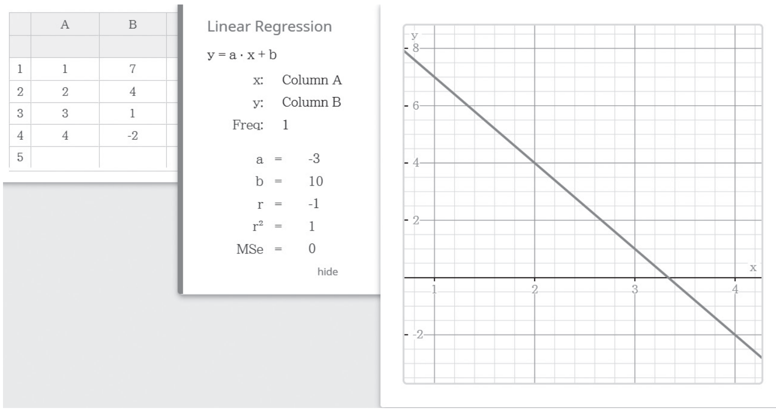




Press ⇧ 𝑥, scan the QR code to display the data. (Continued on the next page.)

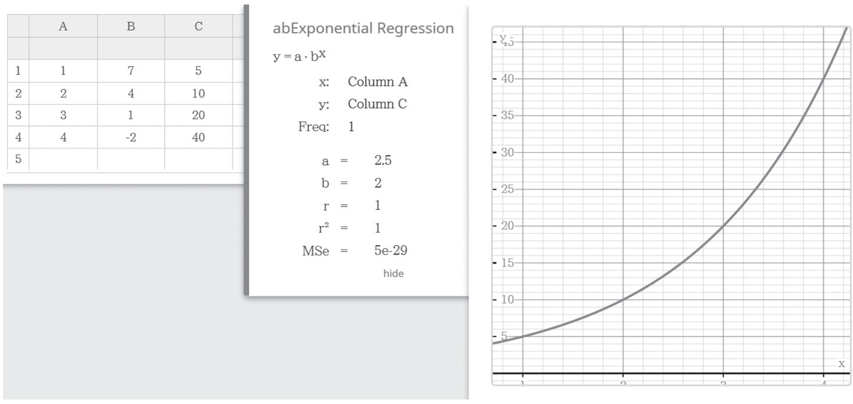
(1) Tap column A and column B, tap [Statistics], [Regression], and [Linear Regression] in this order.

We can confirm the functional relation $y=-3x+10$ of columns A and B.



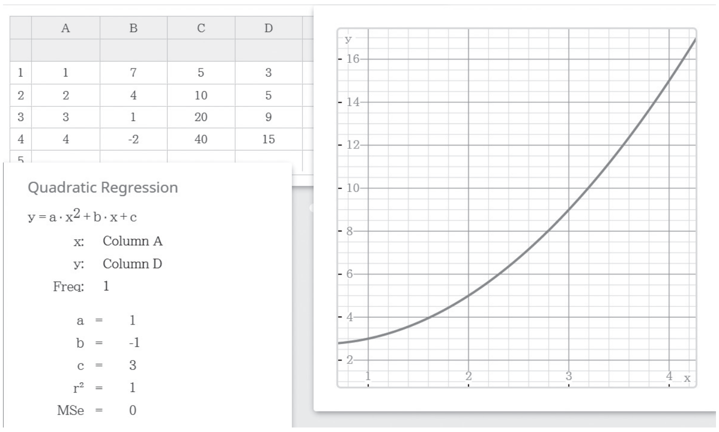
(2) Tap column A and column C, tap [Statistics], [Regression], and [abExponential Regression] in this order.

We can confirm the functional relation $y=\dfrac{5}{2}\cdot 2^{x}=5\cdot 2^{x-1}$ of columns A and C.



(3) Tap column A and column D, tap [Statistics], [Regression], and [Quadratic Regression] in this order.

We can confirm the functional relation $y=x^{2}-x+3$ of columns A and D.



PRACTICE

◆ The table on the right is the data for 2 variates x and y. Draw a scatter plot and find the correlation coefficient. Furthermore, determine if x and y have a correlation and whether it is positive or negative and strong or weak.

	x	y
A	3	4
B	8	7
C	12	11
D	21	14

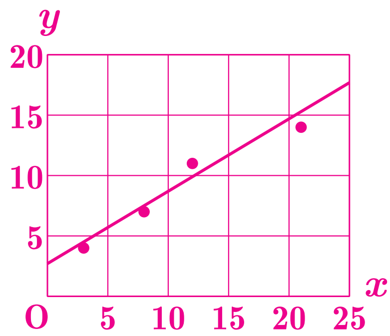
The average values of x and y are $\overline{x}$ and $\overline{y}$, the deviations are $x-\overline{x}$ and $y-\overline{y}$, the product of the deviations is $(x-\overline{x})(y-\overline{y})$, and the squares of the deviations are $(x-\overline{x})^2$ and $(y-\overline{y})^2$, which are calculated in order and summarized in the following table.

	x	y	$x-\overline{x}$	$y-\overline{y}$	$(x-\overline{x})(y-\overline{y})$	$(x-\overline{x})^2$	$(y-\overline{y})^2$
A	3	4	-8	-5	40	64	25
B	8	7	-3	-2	6	9	4
C	12	11	1	2	2	1	4
D	21	14	10	5	50	100	25
Total	44	36	0	0	98	174	58
Average	11	9	0	0	24.5	43.5	14.5

From $s_{xy}=24.5$, $s_x=\sqrt{43.5}\simeq 6.6$, $s_y=\sqrt{14.5}\simeq 3.8$, **we get**

$$r=\frac{s_{xy}}{s_x s_y}=\frac{24.5}{6.6\times 3.8}\simeq 0.98$$

The scatter plot is shown on the right.

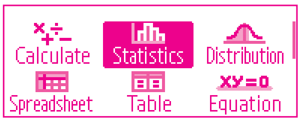


The correlation coefficient is $r=0.98$, and x and y have a strong positive correlation.

The content of this page is part of "12. Statistics".

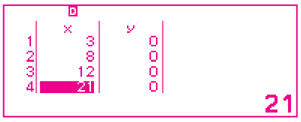
check

Press ⌂, select [Statistics], press OK, select [2-Variable], press OK



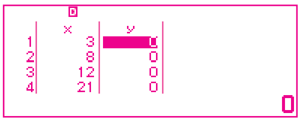
Input in the x column.

3 EXE 8 EXE 1 2 EXE 2 1 EXE



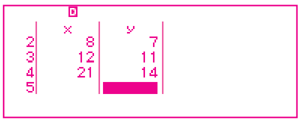
Return to row 1 in the y column.

∨ >



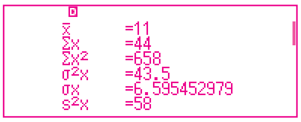
Input in the y column.

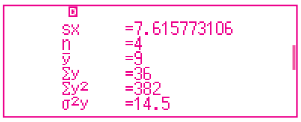
4 EXE 7 EXE 1 1 EXE 1 4 EXE

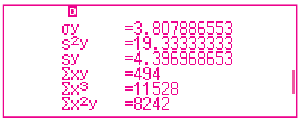


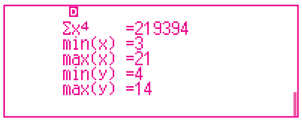
After inputting the final data, press EXE, select [2-Var Results], press OK

Standard deviation of x: $\sigma_x = 6.6$ Standard deviation of y: $\sigma_y = 3.8$









Since covariance is not shown in the above table, it is found by (covariance) = (average of product) − (product of averages).

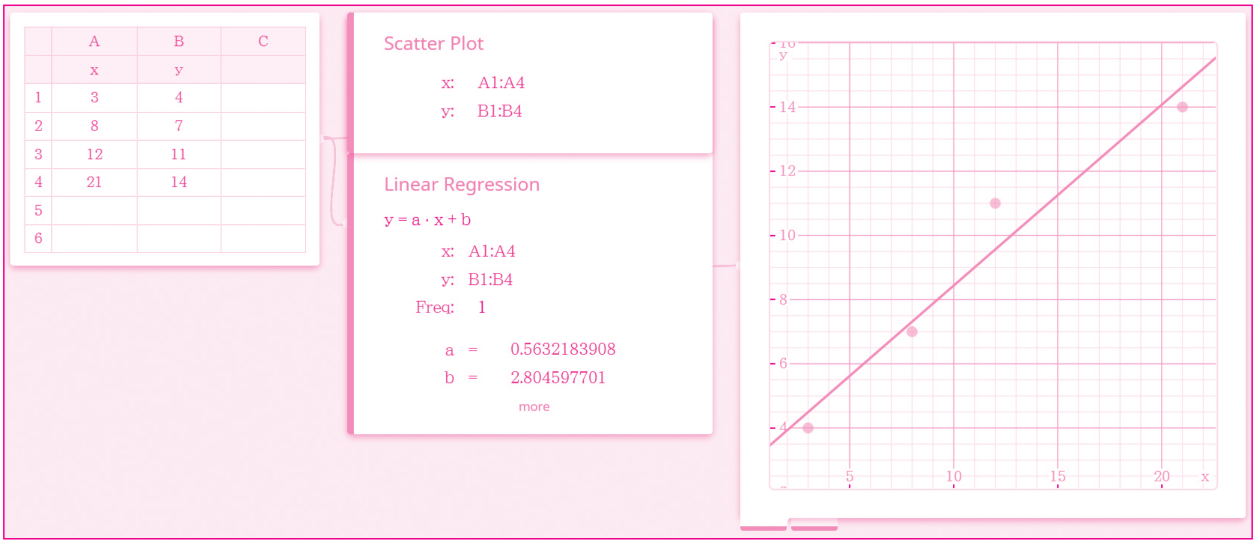
$$s_{xy} = \frac{\Sigma xy}{n} - \bar{x}\bar{y} = \frac{494}{4} - 11 \cdot 9 = 24.5$$

If students try to calculate the mean, variance, standard deviation, and other values by hand, it takes a very long time and they might make mistakes in their calculations. However, one of the fx-991CW's unique features is [Statistics], which students can use to find these values quickly, easily, and accurately. This allows students to focus on the process of analyzing the data and reading the trends in the data.

EXTRA Info.

Select the QR code function.

Read the QR code with your smartphone.



Events and probability

TARGET To understand the basics of probability.

STUDY GUIDE

Events and probability

Trials and events

An experiment or observation that can be repeated under the same conditions, such as rolling dice, is called a **trial**. The result of a trial is called an **event**, which is often expressed using a set. Also, an event that always happens in a trial is called a **sure event** for that trial, and it is expressed by U. Furthermore, an event that cannot be further divided is called a **root event**.

Ex. The sure events in a trial rolling 1 die is $U=\{1, 2, 3, 4, 5$, and $6\}$.

Probability

Given 1 trial, in which any root event is expected to occur equally, where N is the number of all possible events and a is the number of events A occurring, then we say $\frac{a}{N}$ is the **probability** of the event A, which we express as $P(A)$.

$$P(A)=\frac{\text{(Number of cases of event } A \text{ occurring)}}{\text{(Number of all cases that could occur)}}=\frac{a}{N}$$

When we consider multiple combinations, such as of coins or dice, to calculate their probability, we need to distinguish them to find the number of cases.

EXERCISE

1 Given 1 die is rolled, find the probability of an outcome of a 3 or less.

There are a total of 6 possible outcomes.

Furthermore, there are 3 ways, 1, 2, and 3, to roll a 3 or less.

Therefore, we find a probability of $\frac{3}{6}=\frac{1}{2}$

$\frac{1}{2}$

The content of this page is part of "14. Probability".

14. Probability 42

Students can simulate experiments, such as rolling dice or tossing coins, by using [Math Box]. This allows them to express their opinions through a comparison of experimental probability and theoretical probability.

In Math Box on the scientific calculator, you can use the Dice Roll or Coin Toss simulation to check probability (statistical probability).

Simulate rolling 1 die 250 times.

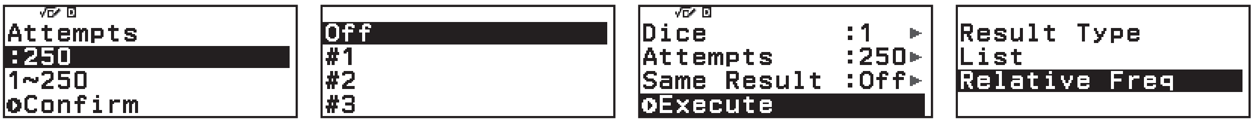
Press ⌂, select [Math Box], press OK, select [Dice Roll], press OK

Select [Dice], press OK, select [1 Die], press OK

Select [Attempts], press OK, after inputting 250 (number of attempts), select [Confirm], press OK

Select [Same Result], press OK, select [Off], press OK

Select [Execute], press EXE, select [Relative Freq], press OK

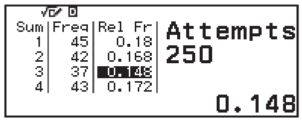


Read the values in the table for when the sum of the die was 1, 2, and 3 , then

add them to get $0.18+0.168+0.148=0.496$, and you can confirm that the

probability is approximately $0.5=\frac{1}{2}$.

(When Same Result is Off, the results of each trial are different.)



2 Given 2 coins being tossed at the same time, find the probability of 1 being heads and 1 being tails.

Consider the combinations of heads and tails of the 2 coins.

There are a total of 4 combinations: (Heads, Heads), (Heads, Tails), (Tails, Heads), and (Tails, Tails).

Furthermore, there are 2 ways in which 1 is heads and 1 is tails, as shown above.

Therefore, we find a probability of $\frac{2}{4}=\frac{1}{2}$

$\frac{1}{2}$

check

On the scientific calculator, simulate tossing 2 coins 250 times.

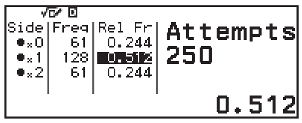
Press ⌂, select [Math Box], press OK, select [Coin Toss], press OK

Select [Coins], press OK, select [2 Coins], press OK

Select [Attempts], press OK, after inputting 250, select [Confirm], press OK

Select [Same Result], press OK, select [Off], press OK

Select [Execute], press EXE, select [Relative Freq], press OK



From the table, the value of heads coming up 1 time [●×1] is about 0.51, and you can confirm that the probability is approximately $\frac{1}{2}$.

Limit values

TARGET To understand limit values and how to find them.

STUDY GUIDE

Limit values

In the function $f(x)$, when the value x is infinitely approaching a, while remaining a different value than a, then the value of $f(x)$ is infinitely approaching b, which is expressed as $\lim_{x \to a} f(x) = b$. This value b is called the **limit value** of $f(x)$ when $x \to a$. lim is the abbreviated symbol for limit, and is read as "the limit of".

Limit values

$$\lim_{x \to a} f(x) = b \text{ or } f(x) \to b \text{ when } x \to a$$

Ex. In the function $f(x)=x+4$, when the value x is infinitely approaching 3, the limit value is $\lim_{x \to 3}(x+4) = 7$.

EXERCISE

◆ Find the limit values of the following.

(1) $\lim_{x \to 5}(x^2 - 3x + 2)$

When the value of x infinitely approaches 5, $x^2 - 3x + 2$ approaches $5^2 - 3 \times 5 + 2 = 12$.

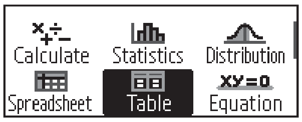
$\lim_{x \to 5}(x^2 - 3x + 2) = 12$

12

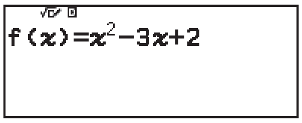
check

Use Table (to create a function table) to see how limit values are approached.

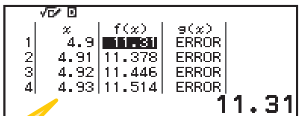
Press ⌂, select [Table], press OK, then clear the previous data by pressing ⏻

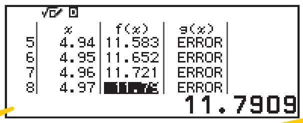


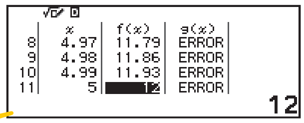
Press ⋯, select [Define f(x)/g(x)], press OK, select [Define f(x)], press OK, after inputting f(x)= $x^2 - 3x + 2$, press EXE



Press ⋯, select [Table Range], press OK, after inputting [Start:4.9, End:5, Step:0.01], select [Execute], press EXE







By using [Table], the student can check how the values of the function approaches the limit value.

...om the table, we can confirm that the value of $f(x)$ approaches 12 as the value of x approaches 5 from 4.9.

In this case, the value approaches 5 while remaining smaller than 5, but we get similar results even though it appr...

The content of this page is part of "15. Differential Calculus and Integral Calculus".

(2) $\lim_{x \to 2} \dfrac{3x+10}{x+2}$

When the value of x infinitely approaches 2, $\dfrac{3x+10}{x+2}$ approaches $\dfrac{3 \times 2 + 10}{2+2} = \dfrac{16}{4} = 4$.

$$\lim_{x \to 2} \frac{3x+10}{x+2} = 4$$

4

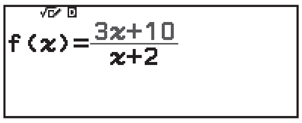
check

Use Table (to create a function table) to see how limit values are approached.

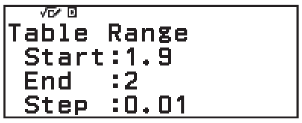
Press ⌂, select [Table], press OK, then clear the previous data by pressing ⏻

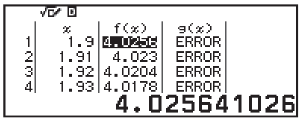
Press ⋯, select [Define f(x)/g(x)], press OK, select [Define f(x)], press OK,

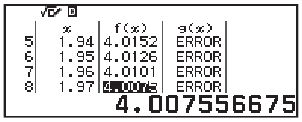
after inputting f(x)= $\dfrac{3x+10}{x+2}$, press EXE

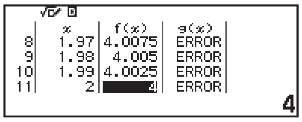


Press ⋯, select [Table Range], press OK, after inputting [Start:1.9, End:2, Step:0.01], select [Execute], press EXE









From the table, we can confirm that the value of $f(x)$ approaches 4 as the value of x approaches 2 from 1.9.

(3) $\lim_{h \to 0} \dfrac{h^2+3h}{h}$

$$\lim_{h \to 0} \frac{h^2+3h}{h} = \lim_{h \to 0} \frac{h(h+3)}{h} = \lim_{h \to 0}(h+3)$$

When the value of h infinitely approaches 0, $h+3$ approaches $0+3=3$.

$$\lim_{h \to 0} \frac{h^2+3h}{h} = 3$$

3

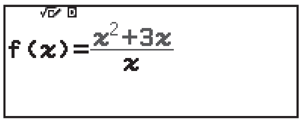
check

Use Table (to create a function table) to see how limit values are approached.

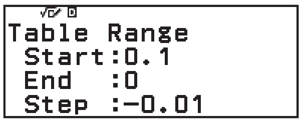
Press ⌂, select [Table], press OK, then clear the previous data by pressing ⏻

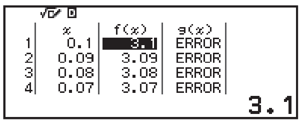
Press ⋯, select [Define f(x)/g(x)], press OK, select [Define f(x)], press OK,

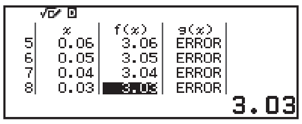
after inputting f(x)= $\dfrac{x^2+3x}{x}$, press EXE

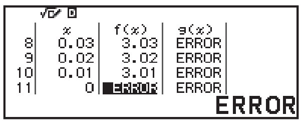


Press ⋯, select [Table Range], press OK, after inputting [Start:0.1, End:0, Step:−0.01], select [Execute], press EXE









From the table, we can confirm that the value of $f(x)$ approaches 3 as the value of x approaches 0 from 0.1.
In the table, $x=0$ appears as ERROR because there is no value.

PRACTICE

1 Let m be the slope of the tangent l for the point $\mathrm{P}(-1, 9)$ on the graph of the function $y=-x^2-6x+4$. Now, solve the following problems.

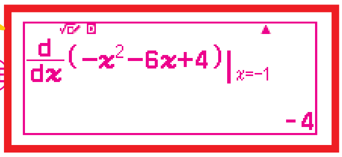
(1) Find the value of m.

Given $f(x)=-x^2-6x+4$, we can get $m=f'(-1)$.
By differentiating $f(x)$, we get $f'(x)=(-x^2-6x+4)'=-2x-6$.
Therefore, we get $m=f'(-1)=-2\cdot(-1)-6=-4$.

-4

Scientific calculators can also be used for calculations to find differential coefficients.

⊞ OK OK ⊖ x ■² ⊖ 6 x ⊕ 4 ▷ ⊖ 1 EXE



(2) Find the equation of tangent l.

The tangent l is a straight line passing through the point $\mathrm{P}\ (-1, 9)$ with a slope of $m=-4$.
Therefore, from $y-9=-4\{x-(-1)\}$, we get $y=-4x+5$.

$y=-4x+5$

check

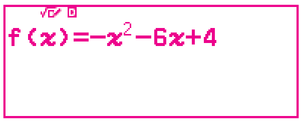
To better understand, use Table and the QR code to calculate and draw the relation of the tangent and graph of the function.

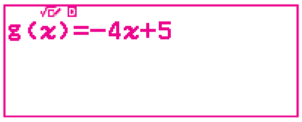
Press ⌂, select [Table], press OK, then clear the previous data by pressing ⏻

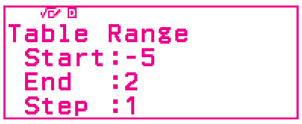
Press ◌, select [Define f(x)/g(x)], press OK, select [Define f(x)], press OK, after inputting f(x)$=-x^2-6x+4$, press EXE

In the same way, input g(x)$=-4x+5$.

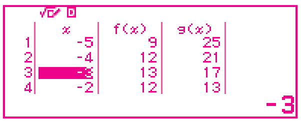
Press ◌, select [Table Range], press OK, after inputting [Start:−5, End:2, Step:1], select [Execute], press EXE

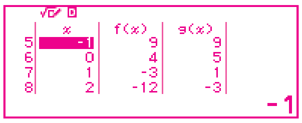




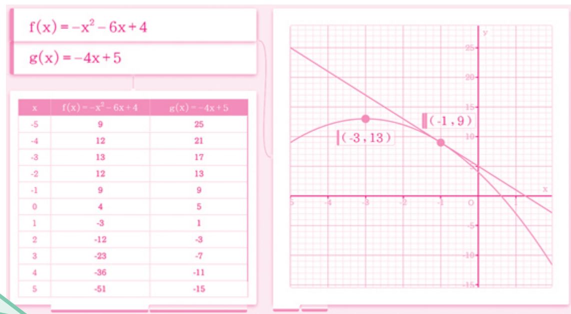


Press ⇧ x, scan the QR code to display a graph.





From the table and graph, we can see that the maximum value for $f(x)$ is $x=-3$.
$f(x)$ and $g(x)$ share a point $(-1, 9)$.



The content of this page is part of "15. Differential Calculus and Integral Calculus".

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