

A Bridge Over Icy Waters

Teacher Notes

Topic Area: Graphs for sinusoidal and polynomial functions; statistical regression

NCTM Standards:

All students should be able to:

- Display a scatterplot and describe its shape.
- Determine regression coefficients and equations.

Objective

Each student will be able to determine sinusoidal models for a given set of points.
Each student will suggest and determine an alternate model for the points.

Getting Started

Consider letting the students work in pairs or in small groups to determine the points and the models. If each student or group selects points, the results will be different. An option for the first few questions is to demonstrate the point and have all students or groups use the same values.

Prior to using this activity:

- Students should have a basic understanding of sinusoidal graphs.
- Students should be familiar with transformations of functions.
- Students should know amplitude and period.
- Students should be familiar with basic calculator methods for regression.

Ways students can provide evidence of learning:

- Students should write equations for the functions.
- Students should display graphs that match the photograph.

Common mistakes to be on the lookout for:

- Students may be unclear on left and right horizontal shifts.
- Students may not realize that the vertical distance from a “middle” point to a “high” point is the amplitude (and not half the amplitude).
- Students may not realize that the horizontal distance from a “middle” point to a “high” point is one fourth of the period.
- Students may not recall how to compute the coefficient b from the period.
- Students may not be aware that all sinusoidal graphs can be written as sine and cosine functions.
- Students may not realize that only one coefficient changes when a sine function is rewritten as a cosine function.

Definitions

- | | |
|-------------|--------------------|
| • Sinusoid | • Horizontal shift |
| • Amplitude | • Vertical shift |
| • Period | • Regression |

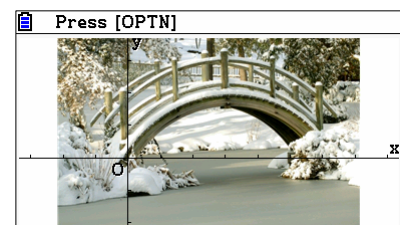
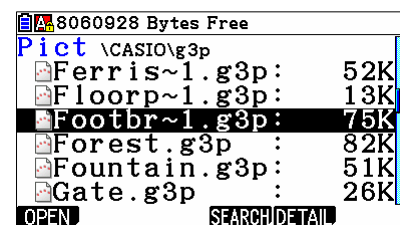
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“How To”

The following will walk you through the keystrokes and menus required to successfully complete this activity.

To open a background image in Picture Plot:

1. From the Main Menu, highlight the Picture Plot icon and press **EXE** or press **□**.
2. Press **F1** (OPEN) to open the CASIO folder.
3. The g3p folder contains 47 background images. Press **▼** **F1** (OPEN) to open the folder. Scroll down the list of images and highlight the desired picture. You will be using the “Footbr~1” image in this activity. Press **F1** (OPEN).



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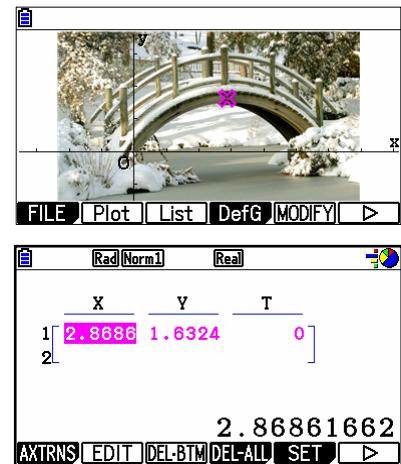
To plot points on the image:

1. The status bar at the top of the screen prompts what buttons you have to choose from. For this picture, you will need to press **OPTN**.
2. To plot a point, press **F2** (Plot). Use **◀** **▶** **▲** **▼** to move to the desired location and press **EXE** to plot a point there. Press **EXIT** when all points are plotted.

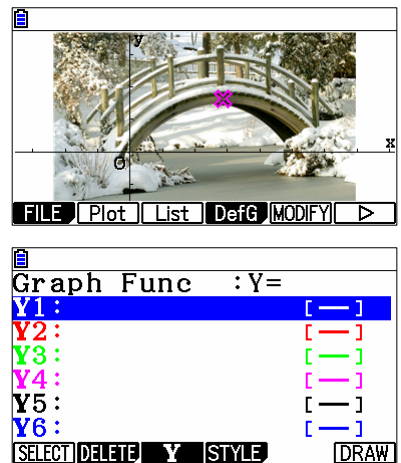


To utilize Image Backgrounds:

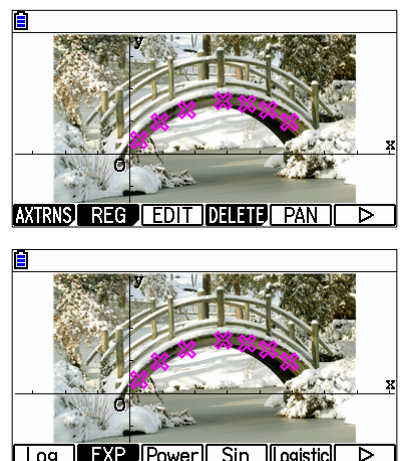
1. To see the coordinates of plotted points, press **F3** (List).



2. To edit (define) a function, press **F4** (DefG).



3. For regression, press **F6** (▷) and **F2** (REG).
For sinusoidal, press **F6** (▷) and **F4** (Sin).



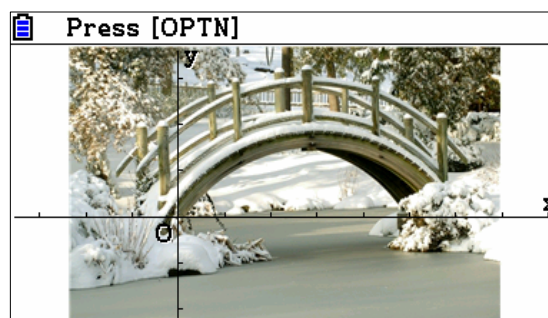
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Activity

Structures, such as the St. Louis Arch, Chicago's Millennium Bean, and bridges often have a shape that can be described by a mathematical model.

In this activity, you will determine models for two portions of a bridge, and review some useful concepts about functions.

Open the footbridge photo titled "Footbr~1.g3p". Notice that a window for the axes has already been created, so we will use that setting.



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Questions

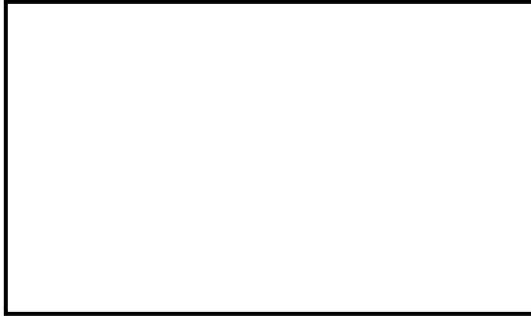
- Describe the shape of the bridge. What parts of the bridge did you focus on to describe the shape? Name several mathematical functions that could be used to model parts of the bridge.

- Let's use the bottom of the bridge and model it as a sine function. For now, we'll assume that the bottom passes through the origin and that the function has no vertical or horizontal shift. Plot a point at the highest point of the bottom of the bridge and write its coordinates.

- If the sine model has no vertical shift, what is the amplitude? Explain.

- If the sine model has no vertical or horizontal shifts, what is the period? Explain. If the model is a function, with equation $y = a \cdot \sin(b \cdot x)$, compute the coefficient b .

5. Graph the function.



6. It is also possible to model the bottom of the bridge as a cosine function. In this case, there will be a horizontal shift. Describe the shift in magnitude and direction. Write the model as a function, with equation $y = a \cdot \cos(b(x - c))$.

7. Graph the function. How does this graph compare to the first model?



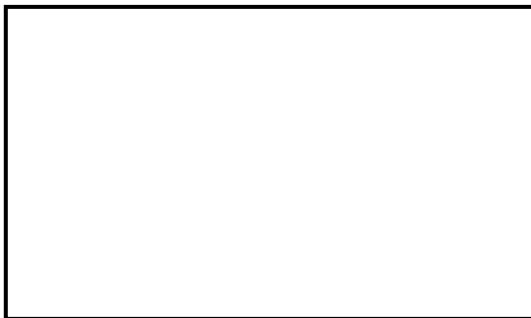
8. It is also possible to model the bottom of the bridge as a cosine function, shifted in the opposite direction. Describe the shift in magnitude and direction. Write the model as a function, with equation $y = a \cdot \cos(b(x + c))$. Check your result by graphing.

9. You have created this model using only two points. It is possible to use many additional points. Plot four to six additional points. Then use sinusoidal regression to write a model as a function with equation $y = d + a \sin(b(x - c))$. Check your result by graphing. Describe how this model is similar to the first model and how it is different than the first model.

10. Now, consider the upper portion of the bridge, that is, the curved railing. Plot five to seven points and use sinusoidal regression to write a model as a function with equation $y = d + a \sin(b(x - c))$. Check your result by graphing. Describe how this model is similar to the model for the lower portion and how it is different than that model.

Extensions

1. Use the graphing menu and shade the region between the models for the upper and lower parts of the bridge.
2. Use a different type of function to model the bridge. Determine the equation and make a graph.

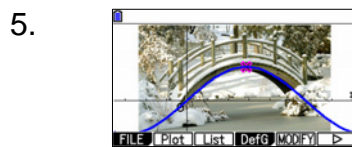


3. A quadratic regression will display a value for R^2 . For a certain set of points selected for the bridge, $R^2 = 0.9968$. Interpret the meaning of this value.

Solutions

1. The bridge is shaped like an arch, based on the main part where you walk, as well as the railings. The model could be a parabola, a sine curve, or a higher degree polynomial.
2. Answers will vary; for example, (2.8686, 1.6324).
3. If there is no vertical shift, the amplitude is the same as the y-coordinate of the highest point, about 1.6324.
4. The horizontal distance from (0, 0) to (2.8686, 1.6324) is one fourth of the period, so the period is about 11.4744. The coefficient $b = \frac{2\pi}{11.4744} \approx 0.5476$.

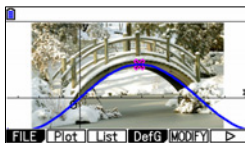
$$y = 1.6324 \cdot \sin(0.5476x)$$



6. The shift is 2.8686 units to the right.

$$y = 1.6324 \cdot \cos(0.5476(x - 2.8686))$$

7. It is the same graph.



8. If the period is 11.4744, then a horizontal shift left would be $11.4744 - 2.8686 = 8.6058$ units.

$$y = 1.6324 \cdot \cos(0.5476(x + 8.6058))$$

9. Points chosen will vary. For example,

	X	Y	T
1	2.8686	1.6324	0
2	0.2711	0.4284	1
3	0.9204	0.983	2
4	1.7554	1.3541	3
	2.86861662		

	X	Y	T
4	1.7554	1.3541	3
5	3.6107	1.5398	4
6	4.2801	1.3541	5
7	4.9095	0.983	6
	0.9830650401		

$$y = 11.6565 \cdot \sin(0.1693x + 1.0711) - 10.0326$$

This model is fairly close to our original model.

10. Points chosen will vary. For example,

	X	Y	T
1	0.6342	0.8902	0
2	-0.285	1.8324	1
3	1.2915	2.8528	2
4	2.7758	2.8384	3

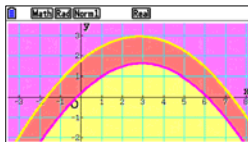
-0.8420909809

$$y = 24.0066 \sin(0.1114x + 1.2565) - 21.0560$$

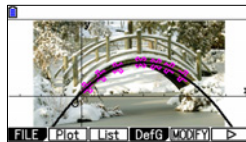
This model has a much larger vertical shift and amplitude, a larger period and horizontal shift.

Extensions

- 1.



2. Answers will vary. A quadratic function is a likely choice.



3. A value for $R^2 = 0.9968$, which is very close to 1, means there is a strong quadratic relationship for these points.